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"Determining the Singularities for the Observation of Three Image Lines"
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Singularity cases in the visual servoing of three general lines in space

We consider here the case where the three observed lines have a general configuration. We define the frame $\mathcal{F}_b : (Q, x_b, y_b, z_b)$ attached to the observed body $B$ such that $x_b$ is collinear to $U_1$, $y_b$ is lying in the plane $\mathcal{P}$ containing $L_1$ and $L_2$. We parameterize the lines as follows (see Fig. 1)

$$
\overrightarrow{OP}_1 = [(X - b) (Y - c) (Z - a)]^T, \ U_1 = [1 \ 0 \ 0]^T
$$

$$
\overrightarrow{OP}_2 = [(X - b) (Y - c) (Z + a)]^T, \ U_2 = [d \ e \ 0]^T
$$

$$
\overrightarrow{OP}_3 = [(X + b) (Y + c) Z]^T, \ U_3 = [f \ g \ h]^T
$$

where $d$, $e$, $f$, $g$ and $h$ are variables parameterizing the direction of the lines $L_2$ and $L_3$.

Then, we have

$$
f_{i1} \propto U_1 \times \overrightarrow{OP}_1, \ m_{i2} \propto U_1 \times \overrightarrow{f_{i1}}
$$

which, from the singularity conditions which are recalled here for reasons of clarity,

$$
f_1 = f_{i1}^T (f_{21} \times f_{31}) = 0 \text{ or } f_2 = m_{i2}^T (m_{22} \times m_{32}) = 0
$$

leads to

$$f_1 = 0 \Leftrightarrow a_{2z} Z^2 + a_{1z} Z + a_{0z} = 0
$$

$$f_2 = 0 \Leftrightarrow b_{2z} Z^2 + b_{1z} Z + b_{0z} = 0
$$

where

$$a_{2z} = 2(e \ c f - b g)
$$

$$a_{1z} = c_{1y} Y + c_{0y}
$$

$$a_{0z} = d_{2y} Y^2 + d_{1y} Y + d_{0y}
$$

$$b_{2z} = e_{1z} X + e_{1y} Y + e_{0z}
$$

$$b_{1z} = f_{2xy}(Y^2 + Y^2) + f_{1xy}XY + f_{1x}X + f_{1y}Y + f_{0y}
$$

$$b_{0z} = g_{3y} Y^3 + g_{21xy} X^2 Y + g_{22xy} XY^2 + g_{2z} X^2
$$

$$+ g_{2y} Y^2 + g_{1xy} XY + g_{1x} X + g_{1y} Y + g_{0y}
$$

in which

$$c_{1y} = 2adg - acf + 2bch
$$

$$c_{0y} = abeg - 2acdg + acf - (2ch - ag)e X
$$

$$d_{2y} = -2adh
$$

$$d_{1y} = 2e h a X - e f a^2
$$

$$d_{0y} = a^2 beg - a^2 cef + 2ac^2 dh - 2abc rh + a^2 e g X
$$

and

$$e_{1x} = (g he + df h) e
$$

$$e_{1y} = (f he - d q h) e
$$

$$e_{0} = -e(h b f + beg - cd g + c e f)
$$

$$f_{2xy} = -e(d g^2 - e f g + dh^2)
$$

$$f_{1xy} = -2 e^2 h^2
$$

$$f_{1x} = -e(2 c e^2 f^2 + a d f h - 2 c d f g + a c g h)
$$

$$f_{1y} = 2 a d h d f + 2 c e f g + 2 a c d g^2 + a h d g + a h e^2 f - 2 b c e^2 g^2
$$

$$f_{0} = b^2 d e g^2 + b^2 d e h^2 - b^2 e^2 f g + 2 b c e^2 f^2
$$

$$+ 2 b c e^2 h^2 + 2 b c e^2 h^2 + a b d h f + a b c e g h
$$

$$- 2 c^2 d e f^2 - c^2 d e^2 g^2 - c^2 d e h^2 - c^2 e^2 f g
$$

$$- 2 a c d f h - a c d g h - a c e^2 f h
$$

$$g_{3y} = d e g h
$$

$$g_{21xy} = -e^2 f h
$$

$$g_{22xy} = d f h e - g h e^2
$$

$$g_{2x} = c f e^2 h + a d e g^2 + a d e h^2 - a f e^2 g
$$

$$g_{2y} = 2 a d g^2 f - a d e f^2 + b d e f h - a d e h^2
$$

$$- c g d h + a g e^2 f + b g e^2 h
$$

$$g_{1xy} = -2 a c d^2 g^2 - 2 d a d^2 h^2 - 2 c d e f h + a e^2 f^2
$$

$$- a e^2 g^2
$$

$$g_{1x} = c^2 d e f h + c^2 e^2 g^2 + 2 a c d^2 g^2 + 2 a c d^2 h^2
$$

$$- 2 a c d e f g + a c e^2 f^2 + a c e^2 g^2 + 2 a c e^2 h^2
$$

Fig. 1. Observation of three general lines in space
\[ g_{1y} = b^2 e^2 fh - 2bcdef h - 2abd^2 g^2 - 2abd^2 h^2 \\
+ 2abdef g - abc^2 f^2 - abc^2 g^2 - 2abc^2 h^2 \\
- c^2 degh \\
g_0 = b^2 ce^2 fh - ab^2 deg^2 - ab^2 deh^2 + ab^2 e^2 fg \\
+ bc^2 def h - bc^2 e^2 gh + 2abcd^2 g^2 + 2abcd^2 h^2 \\
- abce^2 f^2 + abce^2 g^2 + c^3 degh - 2ac^2 d^2 fg \\
+ ac^2 de f^2 + ac^2 de h^2 - ac^2 e^2 fg \]