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Towards a unified bayesian geometric framework for template estimation in Computational Anatomy

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Template estimation in Computational Anatomy

Computational Medicine relying on medical images

Intra-subject
- Aging model of the brain
- Brain tumors

Inter-subjects
- Vascular network
- Brain MRI segmentation

Computational Anatomy

Computational Physiology
- Electromechanical models of the heart
- Aging model of the brain

First step: template shape computation

Second step: analysis

Template shape

Template estimation as a non-linear model of Errors-in-Variables

Generative model of organs’ shapes

\[ X_i = \rho(g_i) + \varepsilon_i \]
where \( g_i \sim \mathcal{N}(\mathbf{g}_0, \Sigma) \) i.i.d. and \( \varepsilon_i \sim \mathcal{N}(0, \sigma) \) i.i.d.

Goal: Estimate the template \( T \)

Non-linear model of Errors-in-Variables

\[ X_i = \rho(g_i) + \varepsilon_i \]
where \( g_i \sim \mathcal{N}(\mathbf{g}_0, \Sigma) \) i.i.d. and \( \varepsilon_i \sim \mathcal{N}(0, \sigma) \) i.i.d.

Regression curve parameterized by \( T \)

Goal: Estimate the curve parameterized by \( T \)

Different estimators of the template’s shape

Functional model: \( g_i \)'s are parameters

Structural model: \( g_i \)'s are random variables

Maximum-Likelihood (MLE-F)

\[ \begin{align*}
\mathbf{g}_i & = \arg \min_{g_i} \sum_{i=1}^n d_\mathcal{L}(\rho(g_i), X_i) \\
T & = \arg \min_T \sum_{i=1}^n d_\mathcal{L}(\rho(T, g_i), X_i)
\end{align*} \]

Frechet mean in the shape space

Maximum-Likelihood (MLE-S)

\[ \begin{align*}
(1) & \quad \mathcal{L} = \prod_{i=1}^n \int_{\mathcal{L}} \exp \left( -\frac{d_\mathcal{L}(\rho(T, g_i), X_i)^2}{2\sigma^2} \right) d\gamma \\
(2) & \quad \mathbf{g}_i = \arg \min_{g_i} \int_{\mathcal{L}} d_\mathcal{L}(\rho(g_i), X_i) d\gamma
\end{align*} \]

No closed form solution

Maximum-Likelihood: Expectation-Maximization algorithm (MLE-S)

(1) Expectation
(2) Maximization

Maximum-a-Posteriori (MAP-F)

MLE-S: Consistent but slow
MLE-F: Fast but inconsistent

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References: