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To cite this version:

HAL Id: hal-01396716
https://hal.archives-ouvertes.fr/hal-01396716
Submitted on 14 Nov 2016

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Towards a unified bayesian geometric framework for template estimation in Computational Anatomy

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Computational Anatomy aims to model and analyze the variability of the human anatomy. Given a set of medical images of the same organ, the first step is the estimation of the mean organ’s shape. This mean anatomical shape is called the template in Computer vision or Medical imaging. The estimation of a template/atlas is central because it represents the starting point for all further processing or analyses. In view of the medical applications, evaluating the quality of this statistical estimate is crucial. How does the estimated template behave for varying amount of data, for small and large level of noise? We present a geometric Bayesian framework which unifies two estimation problems that are usually considered distinct: the template estimation problem and manifold learning problem - here associated to estimating the template’s orbit. We leverage this to evaluate the quality of the template estimator.

Template estimation in Computational Anatomy

First step: template shape computation
Second step: analysis

Template estimation as a non-linear model of Errors-in-Variables

\[ X_i = \rho(T, g_i) + \varepsilon_i \]
where \( g_i \sim N(\mu_i, \sigma_i^2) \) i.i.d. and \( \varepsilon_i \sim N(0, \sigma_i^2) \) i.i.d.

Goal: Estimate the template \( T \)

Different estimators of the template’s shape

Functional model: \( g_i \)'s are parameters
Structural model: \( g_i \)'s are random variables

Likelihood:
\[ L = \prod_{i=1}^{n} \exp \left( -\frac{d^2(\rho(T, g_i), X_i)}{2\sigma^2} \right) \]

- Modal approximation in (1): \( \exp \left( -\frac{d^2(\rho(T, g_i), X_i)}{2\sigma^2} \right) = \delta_{g_0}, i.e. \eta = 0 \)
- Adding regularization in (1): \( \frac{1}{2n^2} d^2(g_i, g_0) \)

Maximum-Likelihood (MLE-F)
\( \hat{T} = \arg\min_{T} \sum_{i=1}^{n} d^2(\rho(T, g_i), X_i) \)
Frechet mean in the shape space

Maximum-Likelihood: Expectation-Maximization algorithm (MLE-S)
(1) Expectation
(2) Maximization
No closed form solution

Adding priors: \( p(T) = \text{cte} \cdot \exp \left( -\frac{d^2(T, T_0)}{2\sigma^2} \right) \) reweights metric in shape space; \( p(g_i) = \text{cte} \cdot \exp \left( -\frac{d^2(g_i, g_0)}{2\sigma^2} \right) \) reweights metric in the orbit; \( p(\gamma) = \text{cte} \cdot \exp \left( \frac{-\gamma^2}{2\sigma^2} \right) \) : \( p(\gamma) = \text{cte} \cdot \exp \left( \frac{-\gamma^2}{2\sigma^2} \right) \)

Maximum-A-posteriori (MAP-F)

Comparison of the estimators

MLE-S: Consistent but slow
MLE-F: Fast but inconsistent

Acknowledgment: Participation in this conference was supported by the "NSF @ISBA junior travel support"

References: