On a Singular Solution in Higgs Field

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A formula for mass of Standard Model Higgs boson is derived by considering certain asymptotic behavior for singular solution of equation of motion (EOM) of Higgs field via Euler-Lagrange equation, in which M_H^0 is shown as a rest mass of Higgs boson mass of the field, which maintains Lorentz invariance. Where the asymptotic formula extracts a proper information near the singular solution (vacuum expectation value (vev)) from EOM. By modifying the mass formula to 'mass triangle' with H^0 production scheme of W/Z-fusion process and by obtaining mass representation at a stationary point, the value of M_H^0 is determined at 120.611 GeV/c^2, which is not excluded by latest experimentally preferred mass, and is consistent with simulation result for vector boson fusion.

1. INTRODUCTION

The value of Higgs boson mass has long been sought by both theoretically and experimentally until now. At this time, the values of $100 < M_H < 130$ GeV/c^2 with radiative correction (theoretically [1] for MSSM Higgs boson) and $114 < M_H^0 < 123$ GeV/c^2 (experimentally [2] (68%CL), for SM Higgs boson), are known. However any theoretically exact formula or definite value for SM Higgs boson mass has not been shown yet, that is mainly from difficulty of obtaining the value of quartic (self-coupling) constant λ.

Therefore, here we try to give a formula of SM Higgs boson mass by studying, at first, asymptotic behavior of singular solution for equation of motion (EOM) near vev, which is derived from Lagrangian density of Higgs scalar field (φ). This EOM is, mathematically, one of nonlinear Klein-Gordon equation (NLKG).[3] Since EOM should have a unique singular solution $(\varphi = 0)$ of the field at vev where the Higgs scalar field has been extended, we study its behavior near the solution by considering certain asymptotic formula for it. Then we will extract an information without λ from EOM as the asymptotic behavior, introducing an infinitesimal Grassmann number. And we express a formula of Higgs boson mass of the field (m_φ), keeping Lorentz invariance, in which M_H^0 is shown as a rest mass. The formula is modified to 'mass triangle' with the relation of H^0 production scheme of W/Z-fusion processes which have been already described by Feynman diagram, to formulate a mass equation by certain parameter. Then by differentiating the mass formula regarding the parameter to obtain the mass representation at a stationary point, we can now get an expected solution $\left(M_{H^0} = 2M_W M_Z \Big/ \sqrt{M_W^{\,2} + M_Z^{\,2}} \equiv \sqrt{2\lambda_0}\; v \right)$ which shows that the mass value is at 120.611 GeV/c^2, then $\lambda_0 = 0.119975\, c^{-4}$, which is not excluded by Large Electron Positron Collider (LEP)'s latest preferred value and also is consistent with simulation results of A Toroidal LHC Apparatus (ATLAS), etc. for vector boson fusion (VBF).[4, 5] And we compute respective W_μ- and Z_μ-gauge boson fields with the value of this singular solution, and describe the potential V with Higgs scalar fields. Finally, canonical quantization and renormalizability of Higgs field are briefly reviewed.

2. FORMULATION AND THE RESULT

2.1. LAGLANGIAN DENSITY OF GAUGE FIELD AND EOM OF HIGGS FIELD

Since we will later treat the case of VBF in which only weak bosons relate, we start with well known Lagrangian density for gauge field of $\mathrm{SU(2)_W \times U(1)_Y}$ after spontaneous symmetry breaking and with unitary condition as follows [6], to make a gauge invariant formulation of the theory where the gauge-boson masses arise.

$$L = (D^\mu\phi)^\dagger(D_\mu\phi) - \lambda\left(\phi^\dagger\phi - \frac{\mu^2}{2\lambda}\right)^2 - \frac{1}{4}(\partial_\mu W_\nu{}^a - \partial_\nu W_\mu{}^a + gf^{abc}W_\mu{}^b W_\nu{}^c)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 \tag{1}$$

$$\begin{aligned}
&= \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{2}m_\varphi{}^2\varphi^2 - \lambda v\varphi^3 - \frac{\lambda}{4}\varphi^4 + W_\mu{}^+W^{-\mu}(M_W + \frac{g}{2}\varphi)^2 - \frac{1}{2}\left|\partial_\mu W_\nu{}^+ - \partial_\nu W_\mu{}^+\right|^2 \\
&+ \frac{1}{2}(\mathrm{Z}_\mu)^2(M_Z + \frac{G}{2}\varphi)^2 - \frac{1}{4}(\partial_\mu \mathrm{Z}_\nu - \partial_\nu \mathrm{Z}_\mu)^2 - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{ig}{2}(W^{+\mu}W^{-\nu} \\
&- W^{-\mu}W^{+\nu})\left[\partial_\mu(\mathrm{Z}_\nu\cos\theta_W + A_\nu\sin\theta_W) - \partial_\nu(\mathrm{Z}_\mu\cos\theta_W + A_\mu\sin\theta_W)\right] + \frac{ig}{2}(\partial^\mu W^{+\nu} \\
&- \partial^\nu W^{+\mu})\left[W_\mu{}^-(\mathrm{Z}_\nu\cos\theta_W + A_\nu\sin\theta_W) - W_\nu{}^-(\mathrm{Z}_\mu\cos\theta_W + A_\mu\sin\theta_W)\right] + \textit{Hermite conjugates} \\
&+ \frac{g^2}{4}(W_\mu{}^+W_\nu{}^- - W_\nu{}^+W_\mu{}^-)^2 + \frac{g^2}{2}\left|W_\mu{}^+(\mathrm{Z}_\nu\cos\theta_W + A_\nu\sin\theta_W) - W_\nu{}^+(\mathrm{Z}_\mu\cos\theta_W + A_\mu\sin\theta_W)\right|^2,
\end{aligned} \tag{2}$$

where $W_\mu{}^\pm = \frac{1}{\sqrt{2}}(W_\mu{}^1 \mp iW_\mu{}^2)$, $M_W = \frac{g}{2}v$, $\mathrm{Z}_\mu = \frac{gW_\mu{}^3 - g'B_\mu}{G}$, $M_Z = \frac{G}{2}v$

$$A_\mu = \frac{g'W_\mu{}^3 + gB_\mu}{G}, \quad e = \frac{gg'}{G} \equiv g\sin\theta_W, \quad m_\varphi{}^2 = 2\lambda v^2, \quad G = \sqrt{g^2 + g'} \tag{3}$$

and

$D \equiv \partial_\mu - igW_\mu{}^aT^a - ig'B_\mu Y/2$; $\mu = 0 \sim 3$

$W_\mu{}^a,\ B_\mu$: gauge fields which belong to $SU(2)$, $U(1)$ respectively

$T^a = (1/2)\tau^a$: $a = 1 \sim 3$,

τ^a : 2×2 Hermite matrices which have same form of Pauli matrices

$g,\ g'$: gauge coupling constants of $SU(2)$, $U(1)$ respectively

$Y = 1$, for complex scalar field ϕ

f^{abc} : structure constant of the Lie group $SU(2)$

$v \equiv \sqrt{\mu^2/2\lambda}$: cf. eq.(1), λ : self-coupling constant of ϕ, θ_W : Weinberg angle (3)'

As it is hard to directly solve m_φ from eq.(2) itself, let us apply Euler-Lagrange equation onto L of eq.(2) regarding φ, and then try to solve it; i.e.,

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial L}{\partial(\partial\varphi/\partial x_\mu)}\right) - \frac{\partial L}{\partial\varphi} = 0, \tag{4}$$

Thus, after calculation, we get an EOM of NLKG for Higgs scalar field (φ);

$$\lambda\varphi^3 + 3\lambda v\varphi^2 + \left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + \left\{m_\varphi{}^2 - \frac{g^2}{2}W_\mu{}^+W^{-\mu} - \frac{1}{4}G^2(\mathrm{Z}_\mu)^2\right\}\right]\varphi - \left\{gM_W W_\mu{}^+W^{-\mu} + \frac{1}{2}GM_Z(\mathrm{Z}_\mu)^2\right\} = 0 \tag{5}$$

2.2. DERIVATION OF HIGGS MASS FORMULA

Though EOM (eq.(5)) above is consistent with the type of PDE form: Klein-Gordon equation, which describes equation of motion of boson particle, still now there is no mathematically systematic method to calculate an exact solution for the NLKG yet.[3)] Therefore we shall from now on study an asymptotic behavior near its certain singular solution. While we will later see that this approach is sufficient within our purpose. Since EOM should have a singular solution of $\varphi = 0$ as explained above, let us take an asymptotic form of the solution near vev point to be connected smoothly to it as follows. (Where though we will find another singular solution that $\varphi = -3v$ by factorization of eq.(5), we shall abandon it because of its inconsistency with Lorentz invariance of Lagrangian (eq.(2)).)

$$\varphi(s) \sim a v s^3 \left\{ 1 - \exp\left(-s_0/s\right) \right\}\Big|_{s\to 0} \tag{6}$$

where $s \equiv \sqrt{c^2 t^2 - x_i x^i}$: relativistically invariant distance from world origin , c : velocity of light ,

t : time, $x_i,\ x^i$: coordinates of Minkowski space , v : cf. eq,(3)' , $a,\ s_0$: constants

So we may expect that $\varphi(0) \sim 0,\ \varphi'(0) \sim 0$, and having a finite value at infinitely far point. (7)

Thus eq.(6) asymptotically satisfies eq.(5) at world origin. Then by expanding near $s \to 0$,

$$\varphi(s)\big|_{s\to 0} \approx a v s^3 \left\{1 - \left(1 - s_0/s\right)\right\}\Big|_{s\to 0} \approx a_1 v s^2\big|_{s\to 0}$$

$$= \varepsilon^2 v\big|_{\varepsilon\to 0}, \quad \text{where } \varepsilon^2 \equiv a_1 s^2,\ a_1 \equiv a s_0,\ 0 < s_0 \ll 1. \tag{8}$$

Hence we can take an asymptotic form near singular solution ($\varphi = 0$) as

$$\varphi \sim \varepsilon^2 v,\ (\ \varepsilon \to 0) \tag{9}$$

By inserting eq.(9) into eq.(5) and using Higgs mass definition, etc. of eq.(3), we are able to have a Higgs mass formula without λ as

$$m_\varphi{}^2 = 2\left\{ \left(\sqrt{W_\mu{}^+ W^{-\mu}}\right)^2 \Big/ \left(\sqrt{\varepsilon'}/g\right)^2 \right\} + \left\{ (\mathrm{Z}_\mu)^2 \Big/ \left(\sqrt{\varepsilon'}/G\right)^2 \right\}, \quad \text{where } \sqrt{\varepsilon'} \equiv \varepsilon \sqrt{2\left(\varepsilon^2 + 2\right)}\Big|_{\varepsilon\to 0} \tag{10}$$

Here we understood that eq.(10) expresses one of elliptic curves with coordinates of $\left(\sqrt{W_\mu{}^+ W^{-\mu}},\ \mathrm{Z}_\mu\right)$. It is noteworthy that $\sqrt{\varepsilon'}$ still has a finite and larger value than ε^2 at very near vev point ($\varepsilon^2 \sim 0$), because the power of $\sqrt{\varepsilon'}$ is always a half power of ε^2 (that is, equals to 1), describing a micro elliptic mass curve even by very near this point. Further, if we introduce an infinitesimal Grassmann number (ε_λ) for ε by putting $\varepsilon \equiv \varepsilon_\lambda$, then eqs.(9),(10) are elegantly represented as

$$\varphi(\varepsilon_\lambda) = (\tag{9'}$$

$$m_\varphi{}^2 = 2\left\{ \left(\sqrt{W_\mu{}^+ W^{-\mu}}\right)^2 \Big/ \left(2\varepsilon_\lambda/g\right)^2 \right\} + \left\{ (\mathrm{Z}_\mu)^2 \Big/ \left(2\varepsilon_\lambda/G\right)^2 \right\}, \text{ from nilpotent property that } \varepsilon_\lambda{}^2 = 0. \tag{10'}$$

After all, we now could have an asymptotic behavior of eq.(10') for eq.(9'), as shown in Fig.B1.

Since dimensions of two terms in right-side of eq.(10') are to be both square of mass, we can put as

$$c_W \cdot \left(\sqrt{W_\mu{}^+ W^{-\mu}}\right)_0^2 \Big/ \left(2\varepsilon_\lambda/g\right)^2 \equiv m_W{}^2, \tag{11}$$

$$c_Z \cdot (\mathrm{Z}_\mu)_0{}^2 \Big/ \left(2\varepsilon_\lambda/G\right)^2 \equiv m_Z{}^2, \quad \text{where } c_W,\ c_Z : \text{constant} \tag{12}$$

at certain point (designated by 0) in respective gauge field. Mathematically, eq.(10') is one of the elliptic curves with $\left(\sqrt{W_\mu{}^+ W^{-\mu}},\ \mathrm{Z}_\mu\right)$-coordinates. So eqs.(11) and (12) are understood as they fix a coordinate of certain point on the micro elliptic mass curve in the first quadrant. Thus we rewrite eq.(10');

$$m_\varphi = \sqrt{2\left\{ \left(\sqrt{W_\mu{}^+ W^{-\mu}}\right)_0^2 \Big/ \left(2\varepsilon_\lambda/g\right)^2 \right\} + \left\{ (\mathrm{Z}_\mu)_0{}^2 \Big/ \left(2\varepsilon_\lambda/G\right)^2 \right\}}$$

$$= \sqrt{\frac{2m_W{}^2}{c_W} + \frac{m_Z{}^2}{c_Z}} = \sqrt{k'^2\left(2m_W{}^2 + \kappa^2 m_Z{}^2\right)}\,, \quad \text{where we put as} \quad c_W \equiv \kappa^2 c_Z \equiv \frac{1}{k'^2}\,, \tag{13}$$

where κ: constant. Furthermore we can write as [7]

$$m_W = M_W\gamma_W\,,\ \ \gamma_W \equiv \frac{1}{\sqrt{1-\left(u_W/c\right)^2}}; \quad m_Z = M_Z\gamma_Z\,,\ \ \gamma_Z \equiv \frac{1}{\sqrt{1-\left(u_Z/c\right)^2}}\,, \tag{14), (15}$$

where M_W, M_Z : rest mass of vector bosons,
$u_W,\ u_Z$: velocity of vector bosons,
c: velocity of light.

Then we are able to write as

$$m_\varphi = \sqrt{k'^2\left\{2\left(M_W\gamma_W\right)^2 + \left(\kappa M_Z\gamma_Z\right)^2\right\}}$$
$$= \sqrt{k'^2\left\{2M_W{}^2\left(\gamma_W/\gamma_Z\right) + \kappa^2 M_Z{}^2\left(\gamma_Z/\gamma_W\right)\right\}}\ \sqrt{\gamma_W\gamma_Z} \equiv M_H{}'\gamma_H\,, \tag{16}$$

To maintain Lorentz invariance of m_φ in eq.(16), it should be that; $\gamma_W = \gamma_Z \equiv \gamma.$ (17)

Then we can write as,

$$m_\varphi = \sqrt{k'^2\left(2M_W{}^2 + \kappa^2 M_Z{}^2\right)}\cdot\gamma \equiv \mathrm{M_{H^0}}\gamma, \quad \text{where} \quad \mathrm{M_{H^0}} \equiv \sqrt{k'^2\left(2M_W{}^2 + \kappa^2 M_Z{}^2\right)} \tag{18), (19}$$

Since the value of $\mathrm{M_H}^0$ above is supposed to be in the range of 'intermediate mass' from the results of LEP, let us consider H^0 production scheme of W/Z-fusion processes which is most expected to meet with above mass formula.[8),9)] As these processes are described by Feynman diagram as shown in Fig.1, we here study the case of that $\mathrm{n_w}$-W fusions and $\mathrm{n_z}$-Z fusions are simultaneously occurred.

Then $(\mathrm{N} \equiv \mathrm{n_w} + \mathrm{n_z})\,\mathrm{H}^0$'s may be produced after these graphs. On the other hand, from eq.(19);

$$\left(1\cdot \mathrm{M_{H^0}}\right)^2 = \left(\sqrt{2}k'M_W\right)^2 + \left(\kappa k'M_Z\right)^2 \tag{20}$$

Where eq.(20) forms 'mass triangle'. We will see that two Feynman diagrams (Fig.1) give each factor for three sides of mass triangle (Fig.2) by recognizing that we can apply eq.(20) onto two fusion diagrams, as explained in APPENDIX -A. So, by comparing Figs.2 and Fig.A1(b) of APPENDIX -A,

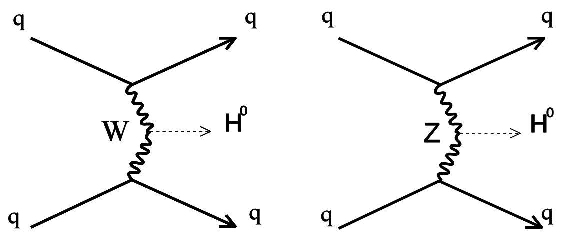

Fig.1 H^0 production scheme of W/Z fusion

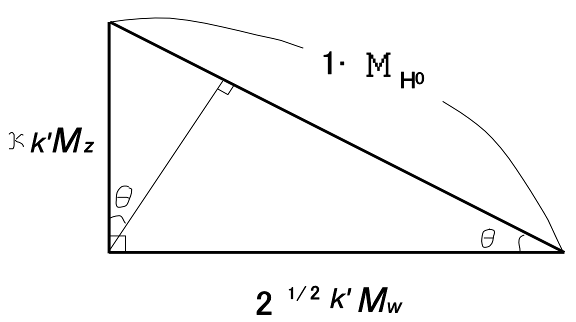

Fig.2 Mass triangle.

$$2\eta_\mathrm{w} \to \sqrt{2}k', \quad 2\eta_\mathrm{z} \to \kappa k' \qquad \text{Thus,} \quad \eta_\mathrm{w} + \eta_\mathrm{z} \to \left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2}\right)k' \tag{21), (22}$$

Also, for H^0 particle, from above discussion and mass triangle it must be kept that

$$(\eta_w + \eta_z) \rightarrow 1 \qquad \text{Thus,} \quad k' \rightarrow \left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2} \right)^{-1} \tag{23), (24}$$

Inserting eq.(24) into eq.(19), we obtain Higgs mass curve;

$$\mathrm{M}_{\mathrm{H}^0}(\kappa) = \sqrt{ \left(\sqrt{2} \left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2} \right)^{-1} \right)^2 M_W{}^2 + \left(\kappa \left(\frac{1}{\sqrt{2}} + \frac{\kappa}{2} \right)^{-1} \right)^2 M_Z{}^2 } \tag{25}$$

As shown in Fig.3, where the value of $\mathrm{M_H}^0(\kappa)$ runs from $2M_W\left(\kappa=0\right)$ to $2M_Z\left(\kappa\rightarrow\infty\right)$ via the value of stationary point. Here κ is physically interpreted as a parameter related to effective rate of W and Z boson masses into Higgs boson mass in VBF process. Therefore, we will find κ-value at stationary point to differentiate eq.(25) regarding κ, and to put zero;

$$\frac{d\mathrm{M}_{\mathrm{H}^0}(\kappa)}{d\kappa} \equiv 0, \quad \text{then} \quad \kappa = \sqrt{2}\left(\frac{M_W}{M_Z}\right)^2 = \sqrt{2}\cos^2\theta_W = 1.09934\cdots, \tag{26}$$

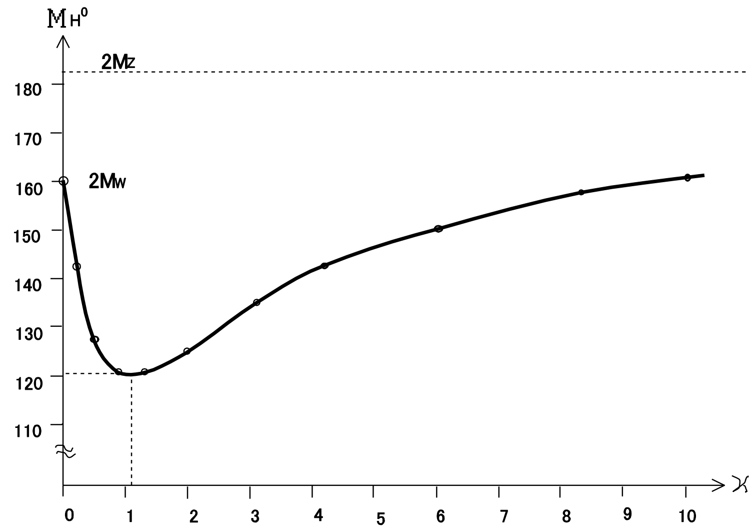

Fig.3 Higgs mass $(\mathrm{M}_{\mathrm{H}^0}(\kappa))$ curve.

as, θ_W : Weinberg angle.

Hence we now have a form of stationary value as an expected Higgs mass solution from eqs.(25),(26) and then we get the value of rest mass for SM Higgs boson by inserting experimental values [2)], of M_W and M_Z :

$$\mathrm{M}_{\mathrm{H}^0} = \frac{2M_W}{\sqrt{1+\cos^2\theta_W}} = \frac{2M_W M_Z}{\sqrt{M_W{}^2 + M_Z{}^2}}$$

$$= 120.611^{+0.023}_{-0.022}\ \left[\mathrm{GeV}/c^2\right], \tag{27}$$

which is not excluded by LEP's latest preferred value for SM Higgs boson mass [2)], and is in accordance with the simulation results for VBF of ATLAS and CMS which appear to show around 120 GeV/c^2 the best significance both in Higgs boson decays into τ-pairs and into γ-pairs respectively, in the low-mass region: $110 < \mathrm{M_H}^0 < 140$ GeV/c^2. [4),5)] Then, from eq.(27) and the value of G_F in v [10)],

$$\lambda_0 \equiv \left(\frac{\mathrm{M}_{\mathrm{H}^0}}{\sqrt{2}\,v}\right)^2 = 0.119975^{+0.000047}_{-0.000045}\ \left[c^{-4}\right], \qquad \text{where} \quad \lambda = \lambda_0 \gamma^2, \tag{28); (29}$$

from eqs.(3),(18) and (28).

Respective W_μ- and Z_μ-gauge boson fields are computed at a point of the singular solution φ_0, and the potential V is described with scalar Higgs ϕ and φ fields, in APPENDICES-B and -C. Finally, canonical quantization and renormalizability of Higgs field are shortly touched on in APPENDIX-D.

3. CONCLUDING REMARKS

So far, we have derived a formula and shown a value of mass of SM Higgs boson via an asymptotic behavior of a singular solution for the Euler-Lagrange equation, which extracts a proper information without λ from EOM. The result is to be strongly expected to examine under the forthcoming experiments. And, SM Higgs Mass form (eq.(27); symmetrical between W and Z with a factor of 2 (twice)) appears to show the possibility that the Higgs particle is to be composite, as supposed, which is now proceeded to study, and will be discussed elsewhere.

4. APPENDICES

-A: Relation between Mass triangle and VBF triangles

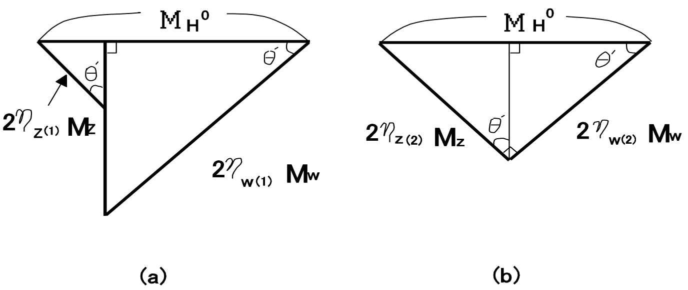

Fig.A1 VBF triangles.

Since we have seen that W, Z and H^0 particles should have both equal γ to maintain Lorentz invariance as eq.(17), we also should consider the case in which they still have both equal γ in VBF process. Therefore we shall hereafter discuss with their rest masses, dropping out γ 's from their relativistic masses, in VBF triangles below, etc. with remembering eqs.(14) and (15).

Because numbers of consuming W and Z particles are $2n_W M_W$ and $2n_Z M_Z$ in Feynman diagram (Fig.1) at each event time and all related γ 's are both equal, we can write next formulae after $N \equiv n_W + n_Z$ events;

-consuming mass: $2n_W M_W;\ 2n_Z M_Z$, -producing mass: $n_W M_{H^0} + n_Z M_{H^0} = N\, M_{H^0}$.

From Fig.2, it will be understood that each mass quantity of W and Z boson sides, to which $\cos\theta$ and $\sin\theta$ are multiplied respectively, contributes to Higgs mass. Therefore we can generally write VBF process for producing one Higgs mass as, with referring to Fig.A1(a),

$$M_{H^0} = (2\eta_{W_{(1)}} M_W)\cos\theta' + (2\eta_{Z_{(1)}} M_Z)\sin\theta', \qquad \text{where} \quad \eta_{W_{(1)}} \equiv \frac{n_{W_{(1)}}}{N};\ \eta_{Z_{(1)}} \equiv \frac{n_{Z_{(1)}}}{N} \tag{A1}$$

When these $(\eta_{W_{(1)}}, \eta_{Z_{(1)}})$ are equal to $(\eta_{W_{(2)}}, \eta_{Z_{(2)}})$ as shown in Fig.A1(b), the Mass triangle (Fig.2) and VBF triangle should be equivalent since these right-angled triangles both should have an equal angle $\theta = \theta'$ to make same rate of contribution to Higgs mass, and have an equal side (M_{H^0}). Thus the remainder sides should also be equal. In fact, once we later see that

$$\theta' \equiv \theta = \tan^{-1}\left(\frac{\kappa k' M_Z}{\sqrt{2}k' M_W}\right) = \tan^{-1}\left(\cos\theta_W\right) \cong 41.402\ [\deg], \tag{A2}$$

as 'VBF angle', then we will get always an equal value: 120.611 GeV/c^2 for $M_H{}^0$ regarding all-$\left(\eta_{W_{(1)}}, \eta_{Z_{(1)}}\right)$ values from eq.(A1).

-B: Calculation of W_μ - and Z_μ - boson fields

We now understand that W_μ - and Z_μ -boson field each takes such an asymptotic coordinate value at this singular solution for φ as follows. From eqs.(11)-(15),

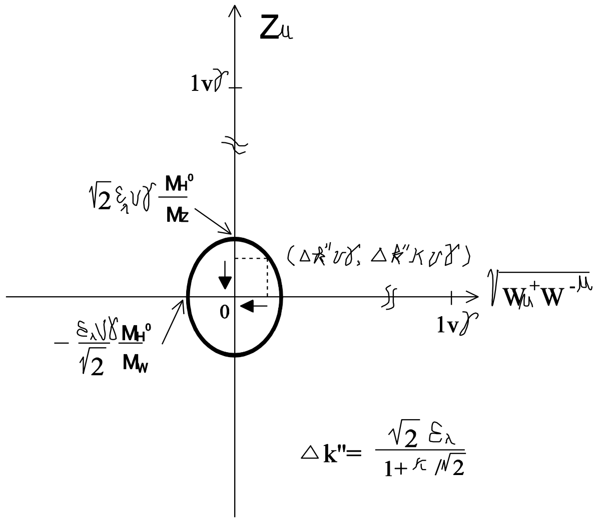

Fig.B1 Micro elliptic mass curve in W_μ- and Z_μ-field.

$$m_W = \frac{g\sqrt{(W_\mu{}^+W^{-\mu})_0}}{2\varepsilon_\lambda\ k'} \equiv M_W\gamma_W, \qquad \text{(B1)}$$

$$m_Z = \frac{G\ (\mathrm{Z}_\mu)_0}{2\varepsilon_\lambda\kappa k'} \equiv M_Z\gamma_Z \qquad \text{(B2)}$$

Therefore,

$$\sqrt{(W_\mu{}^+W^{-\mu})_0} = \sqrt{\frac{1}{2}\left\{\left(W_\mu{}^1\right)^2 + \left(W_\mu{}^2\right)^2\right\}_0}$$

$$\equiv \Delta k''\left(\frac{2M_W\gamma_W}{g}\right) = \Delta k'' v\gamma_W,$$

Hence, $\left(W_\mu{}^1\right)_0 = \left(W_\mu{}^2\right)_0 = \Delta k'' v\gamma_W$

(B3), (B4)

$$\text{Similarly,}\quad (\mathrm{Z}_\mu)_0 = \left(\frac{gW_\mu{}^3 - g'B_\mu}{\sqrt{g^2+g'^2}}\right)_0 \equiv \kappa\Delta k''\left(\frac{2M_Z\gamma_Z}{G}\right) = \kappa\Delta k'' v\gamma_Z, \qquad \text{(B5)}$$

$$\text{where}\quad \Delta k'' \equiv \varepsilon_\lambda k' = \varepsilon_\lambda\left(\frac{1}{1/\sqrt{2}+\kappa/2}\right) = \left(\frac{\sqrt{2}}{1+\kappa/\sqrt{2}}\right)\varepsilon_\lambda \qquad \text{(B6)}$$

Finally γ_W, γ_Z above should be equal ($=\gamma$) so as eq.(17) at this singular solution for φ. Hence the mass formula of eq.(10') is described as Fig.B1, in which W and Z bosons should have energies of $\Delta k'' v\gamma$ and $\kappa\Delta k'' v\gamma$ respectively at the condition of eq.(13).

-C: Description of potential V with scalar Higgs (ϕ_1 and φ) fields

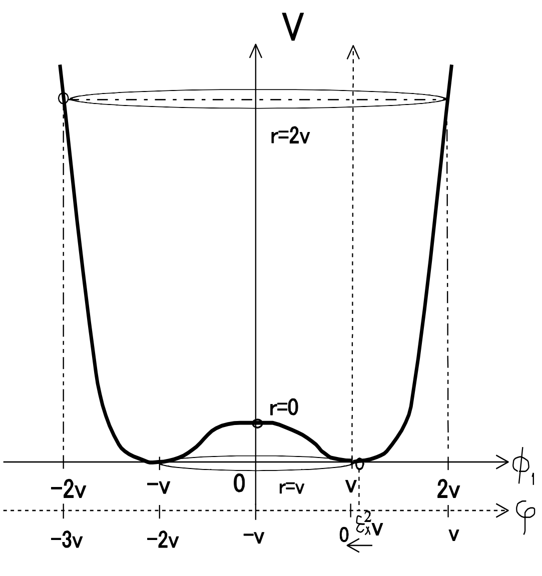

Fig.C1 Potential V with ϕ_1 and φ fields.

We here describe the potential V with scalar Higgs fields in which the position of singular solution $(\varphi = \varepsilon_\lambda{}^2 v = 0)$ is shown. We consider an isospinor scalar field (the Higgs field) ϕ as [11)]

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)\\ \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)\end{pmatrix}, \qquad \text{(C1)}$$

Then $\phi^\dagger\phi$ in potential V of eq.(1) is

$$\phi^\dagger\phi = (\phi^+)^*(\phi^+) + (\phi^{0})^*(\phi^0)$$

$$= \frac{1}{2}(\phi_1{}^2 + \phi_2{}^2 + \phi_3{}^2 + \phi_4{}^2) \qquad \text{(C2)}$$

When we choose as $\phi_2 = \phi_3 = \phi_4 = 0$ at lowest energy state,

$$\phi_1 = v, \quad \text{as} \quad (\phi^\dagger\phi)_{\text{Lowest Energy State}} = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \tag{C3}$$

Therefore we can write for $\phi(x)$, under local symmetry, at every point as

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+\varphi(x) \end{pmatrix} \tag{C4}$$

After all, we may describe potential V with ϕ_1 and φ fields as Fig.C1, in which

$$V_{r=0(\varphi=-v)} = \frac{\lambda v^4}{4}, \quad V_{r=2v(\varphi=-3v)} = 9\left(\frac{\lambda v^4}{4}\right) \tag{C5}$$

where r is cross-sectional radius of potential V at each point of ϕ_1. Thus we see that the point of e.g. $\varphi = v$ has twice of radius for the Lowest Energy Point $(\varphi = 0)$.

-D: Canonical quantization and renormalizability of Higgs field

Canonical quantization of Higgs scalar field $(\varphi(x))$ which derives the NLKG above can be furnished by computing the canonically conjugate momentum $(\pi(x) = \pi(t,x^j) \equiv \partial L/\partial\dot{\varphi}(t,x^j) = \partial L/\partial(\partial_0\varphi(x)) = \partial_0\varphi(x) = \dot{\varphi}(x))$, and by describing the Hamiltonian (*H*) with the Lagrangian density (*L*). After setting the same-time canonical commutation relation between $\varphi(x)$ and $\pi(x)$, we are able to calculate the Heisenberg's equation of motion $\left(i\partial_0\varphi(x) = [\varphi(x), H],\ i\partial_0\pi(x) = [\pi(x), H]\right)$ which re-produces the NLKG and the momentum $(\pi(x))$ above.[12)]

Renormalizability (and unitary property) of massive vector field under gauge symmetry $(\mathrm{SU}(2)\times\mathrm{U}(1))$ has been firstly confirmed by t' Hooft.[13)] Later, Fujikawa et al.[14)] developed R_ξ-gauge theory. The renormalizability of Higgs field under BRS-symmetry[15)] was shown, using the R_ξ-gauge, also by Fujikawa.[16)]

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