# Hopf bifurcation for one-phase flow in model porous media

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## 1. Introduction

Inertial one-phase flow, in the laminar regime, within porous media is of interest for a wide range of applications including flow in filters, in columns of reactors or near well during hydrocarbon recovery. Formalized existing macroscopic models for this type of flow are restricted to steady conditions [1]. Their validity is hence restricted to a Reynolds number smaller than the critical value at which the flow switches from steady to unsteady condition, a transition that is known as the first Hopf bifurcation [2]. While the critical Reynolds number at the bifurcation was mainly determined for flow around an isolated cylinder [3], the analysis remains to be done for more complex structures.

In this work, we address the determination of the critical Reynolds number characterizing the Hopf bifurcation on 2D porous model structures made of cylinders of square cross section by means of direct numerical simulation. This type of structure can be thought of as a relevant representation of streets/buildings network or clusters of trees for which the characterization of steady/unsteady inertial flow is of crucial importance for particle dissemination (pollutant, pollen), prediction of fluid-structure interaction, etc.

## 2. Physical model and critical Reynolds number determination

The initial boundary value problem, defined on a periodic domain $V$ (a Representative Elementary Volume (REV) of the structure) corresponds to the mass conservation equation in its incompressible form, together with the unsteady Navier-Stokes equation where gravity is neglected and in which the pointwise pressure of the fluid phase $\beta$ is decomposed according to $p_\beta = \langle p_\beta \rangle^\beta + \tilde{p}_\beta$. Here we use the definition $\langle \psi_\beta \rangle^\beta = \frac{1}{V_\beta} \int_{V_\beta} \psi_\beta dV = \frac{\langle \psi_\beta \rangle}{\epsilon} = \frac{1}{\epsilon V} \int_{V_\beta} \psi_\beta dV$, $\epsilon = V_\beta / V$ being the porosity and $V_\beta$ the volume of the $\beta$-phase within the computational domain $V$. Periodic boundary conditions are considered on $\tilde{p}_\beta$ and on the velocity, $\mathbf{v}_\beta$. Both $\mathbf{v}_\beta$ and $\tilde{p}_\beta$ are zero at $t = 0$. The problem is made dimensionless as in [1] and involves a Reynolds number, $Re^*$ given by $Re^* = \frac{\rho_\beta l^3}{\mu_\beta^2} \left| \nabla \langle p_\beta \rangle^\beta \right|$, $\rho_\beta$ and $\mu_\beta$ being respectively the fluid density and dynamic viscosity while $l$ is the size of the geometrical unit cell of the periodic structure. In this work 2D structures made of ordered (OS) or disordered (DS) arrays of parallel cylinders of square cross sections were considered (see Fig 1a and d) with flow orthogonal to the cylinders' axes. For practical purposes, two other Reynolds numbers are employed, namely $Re_d = \left| \left\langle \mathbf{v}_\beta^* \right\rangle \right| d^* Re^*$ where $d^* = d/l$ is the dimensionless size of the solid inclusion and $Re_k = Re_d \frac{\sqrt{k}}{d}$, $k$ being the intrinsic permeability of the structure.

The problem was solved numerically using the CFD OpenFoam toolbox using the standard solver icoFoam. Convergence and mesh sensitivity were carefully addressed so as to ensure accuracy. The critical Reynolds number was characterized by determining an interval whose lower bound corresponds to the largest value for which the flow remains stationary and the upper bound to the smallest value at which non stationary flow is observed.

## 3. Results

The critical Reynolds number was determined for $0.15 \leq \epsilon \leq 0.96$ for the OS (see Fig 1a) and $0.36 \leq \epsilon \leq 0.75$ for the DS (see Fig 1d) and a macroscopic pressure gradient oriented along the x-axis.

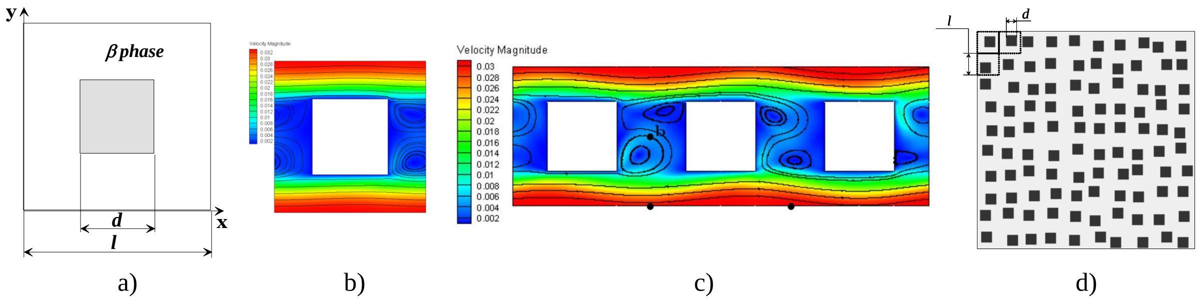


Figure 1: a) Geometrical periodic unit cell for an ordered array of parallel cylinders of square cross section. b) Streamlines and velocity color map for a REV corresponding to geometrical periodic unit cell. $Re^* = 29000$, $\epsilon = 0.75$. c) Streamlines and velocity color map for a REV corresponding to 3×1 geometrical periodic unit cells. $Re^* = 25000$, $\epsilon = 0.75$. d) REV used for a disordered array of parallel cylinders of square cross section.

Results on the critical Reynolds number ($Re_c^*$ and $Re_{dc}$) obtained in these configurations can be summarized as follows. For the OS, the size of the REV must be larger than the geometrical periodic unit cell. It was found that a REV made of 3×1 geometrical unit cells in the x and y directions respectively was necessary to reach a consistent value of $Re_{dc}$. The physical explanation lies in the fact that the periodic boundary condition, when applied on a single geometrical periodic unit cell forces the structure of the vortices in between successive cylinders in the pressure gradient direction, a structure that develops differently when the REV is larger (see Fig. 1b and c). This effect amplifies with $\epsilon$. This is illustrated in Fig. 2a representing the dependence of $Re_c^*$ and $Re_{dc}$ on $\epsilon$. This feature was not observed for a pressure gradient not aligned with the principal axes of the structure (e.g. along $\mathbf{e}_x + \mathbf{e}_y$). When structural disorder is introduced like in the DS, the critical Reynolds number strongly decreases, the smaller the porosity, the larger the difference, as shown in Fig 2b. The origin of this strong modification for small values of $\epsilon$ lies in the local enlargements and constrictions together with obstacles misalignment that create tortuous channels as a result of disorder.

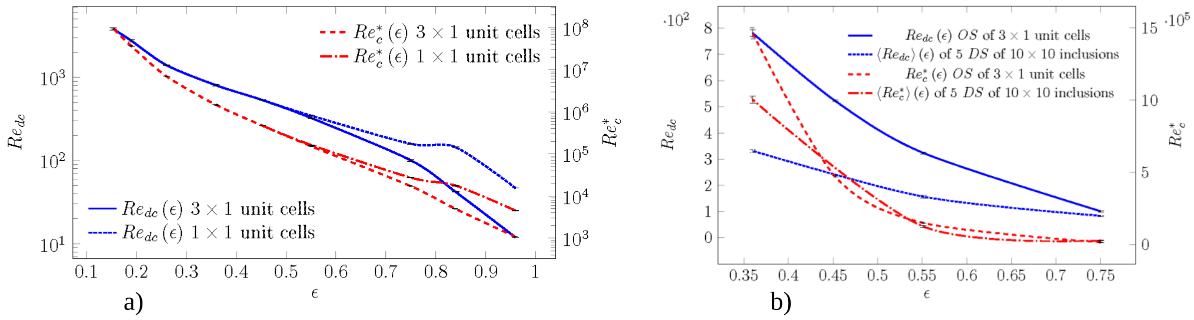


Figure 2: Critical Reynolds number versus the porosity $\epsilon$ for: a) OS. REVs made of 1×1 and 3×1 geometrical unit cells; b) 5 different realizations of DS together with results in the case of the OS.

## References


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