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▶ **To cite this version:**

Ioana Ilea, Lionel Bombrun, Christian Germain, Yannick Berthoumieu. Texture image classification with Riemannian fisher vectors issued from a Laplacian model. IEEE Image, Video, and Multidimensional Signal Processing Workshop, Jul 2016, Bordeaux, France. pp.1 - 5, 10.1109/IVMSPW.2016.7528231 . hal-01385369

HAL Id: hal-01385369
https://hal.science/hal-01385369

Submitted on 21 Oct 2016

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TEXTURE IMAGE CLASSIFICATION WITH RIEMANNIAN FISHER VECTORS ISSUED FROM A LAPLACIAN MODEL

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ABSTRACT

Many signal and image processing applications are based on the classification of covariance matrices. These latter are elements on a Riemannian manifold for which many generative models have been developed in the literature. Recently, the Riemannian Laplace distribution (RLD) has been proposed to model the within-class variability of images. In this context, the present paper proposes an application of RLDs to the definition of Riemannian Fisher vectors issued from this Laplacian model. The expression of these descriptors is derived for mixtures of RLDs and their relation with the Riemannian vectors of locally aggregated descriptors is shown. Some comparisons with the bag of Riemannian words model are also performed. All these aforementioned descriptors are applied to texture image classification to find the most discriminating one. Moreover, to determine the best model for fitting the data, the classification performances are compared to those given by the Riemannian Gaussian distribution.

I. INTRODUCTION

Covariance matrices are used in a wide variety of applications in signal and image processing, including EEG signal classification [1], object detection [2] and recognition [3], texture analysis, etc. Being elements in the space $\mathcal{P}_m$ of $m \times m$ real, symmetric and positive definite matrices, several distributions have been introduced to model them, such as the Wishart distribution [4] and those issued from the so-called product model [5]. Recently, the Riemannian Gaussian distribution (RGD) has been proposed in [6] to model the within-class variability of images. This probability density function is characterized by two parameters, its central element and its dispersion around this central element. For this model, the maximum likelihood estimator (MLE) of the central value corresponds to the Riemannian center of mass. While being efficient to model the mean element, this latter is easily influenced by the presence of aberrant data [7], [8], [9]. In practice, outliers may arise from faulty measurements, or they may be explained by the inherent variability of data. To overcome this problem, a robust estimator of the central element can be considered. For example, one may use the Huber M-estimator [10] or the Riemannian median [11], [12], [13]. A generative model for which the MLE of the central element is the Riemannian median has recently been introduced in [14]: the Riemannian Laplace distribution (RLD).

This paper proposes an application of the RLD to model local descriptors able to capture the information lying in signals, images or videos. Starting from a generative model, local descriptors, such as Fisher vectors (FVs) can be extracted [15], [16], [17]. These FVs are descriptors derived from the Fisher kernels [18] and they represent a way to measure if samples are correctly fitted by a given model. Introduced initially in the context of Gaussian mixture models (GMM) [15], FVs have been recently generalized for Riemannian manifolds, based on the Riemannian Gaussian distributions [19]. The obtained descriptors have been called Riemannian Fisher vectors (RFVs). Motivated by the recalled interest of the RLD [14], the main contribution of this paper is to extend the definition of RFVs to RLDs. Next, the connection between the RFVs and the Riemannian version of the conventional vectors of locally aggregated descriptors (R-VLAD) [3] is also analyzed. Both models are then applied in the context of texture image classification. In addition, their behavior is compared to another local descriptor, already generalized for the Riemannian case, the bag of Riemannian words (BoRWs) [20]. All the tested methods are implemented for the RLDs, but also for the RGDs, in order to find the most suited model able to fit the data.

The paper is structured as follows. Section II recalls the definition of RLD and of mixtures of RLDs, and it details the parameter estimation procedure. Section III introduces the RFVs for RLDs and their relation with R-VLAD. Section IV presents an application of the proposed method to texture image classification and section V reports some conclusions and future works.

II. RIEMANNIAN LAPLACE DISTRIBUTIONS

Inspired from the well-known Laplace distribution on $\mathbb{R}$, the RLD has been introduced in [14] on the space $\mathcal{P}_m$ of $m \times m$ real, symmetric and positive definite matrices. For a set

$\boldsymbol{\Upsilon} = \{\mathbf{Y}_t\}_{t=1:T}$ of T independent and identically distributed (i.i.d.) samples, the probability density function of the RLD with respect to the Riemannian volume element is defined as:

$$p(\mathbf{Y}_t|\bar{\mathbf{Y}}, \sigma) = \frac{1}{\zeta_m(\sigma)} \exp\Big\{ - \frac{d(\mathbf{Y}_t, \bar{\mathbf{Y}})}{2\sigma^2} \Big\}, \qquad (1)$$

where $\bar{\mathbf{Y}} \in \mathcal{P}_m$ and $\sigma > 0$ are the location and the dispersion parameters. ζ_m is a normalizing constant independent of $\bar{\mathbf{Y}}$ and d is the Riemannian distance given by $d(\mathbf{Y}_1, \mathbf{Y}_2) = \left[\sum_i (\ln \lambda_i)^2\right]^{\frac{1}{2}}$, with λ_i, $i = 1, \ldots, m$ being the eigenvalues of $\mathbf{Y}_2^{-1}\mathbf{Y}_1$.

Starting from (1), the RLD definition has been extended to the case of mixtures of RLDs [14]. For a mixture of K RLDs, the probability density function becomes:

$$p(\mathbf{Y}_t|\lambda) = \sum_{j=1}^{K} \varpi_j \; p(\mathbf{Y}_t|\bar{\mathbf{Y}}_j, \sigma_j), \qquad (2)$$

where $\lambda = \{(\varpi_j, \bar{\mathbf{Y}}_j, \sigma_j)_{1 \leq j \leq K}\}$ is the parameter vector. ϖ_j are the positive weights, with $\varpi_j \in (0, 1)$ and $\sum_{j=1}^{K} \varpi_j = 1$, while $p(\mathbf{Y}_t|\bar{\mathbf{Y}}_j, \sigma_j)$ is given by (1).

For each cluster c_j, $j = 1, \ldots, K$, the elements of the parameter vector λ can be estimated through the maximum likelihood estimation. Thus, for the cluster c_j, the estimated location $\hat{\bar{\mathbf{Y}}}_j$ is the Riemannian median defined as the solution of:

$$\hat{\bar{\mathbf{Y}}}_j = \text{argmin}_{\bar{\mathbf{Y}}_j} \sum_{n=1}^{N_j} d(\bar{\mathbf{Y}}_j, \mathbf{Y}_{j_n}), \qquad (3)$$

with $\mathbf{Y}_{j_n}$, $n = 1, \ldots, N_j$ being the set of elements $\mathbf{Y}_j$ in the cluster c_j and N_j representing the cardinal of $\mathbf{Y}_{j_n}$. In addition, the estimated dispersion $\hat{\sigma}_j$ is obtained as the solution of:

$$\sigma_j^2 \times \frac{d}{d\sigma_j} \log \zeta_m(\sigma_j) = \frac{1}{N_j} \sum_{n=1}^{N_j} d(\hat{\bar{\mathbf{Y}}}_j, \mathbf{Y}_{j_n}), \qquad (4)$$

while the estimated weights $\hat{\varpi}_j$ are given by:

$$\hat{\varpi}_j = \frac{N_j}{\sum_{j=1}^{K} N_j}. \qquad (5)$$

In practice, the location $\hat{\bar{\mathbf{Y}}}_j$ is determined by means of a gradient descent algorithm detailed in [11], while the dispersion $\hat{\sigma}_j$ is obtained by a classical Newton-Raphson algorithm [14]. Furthermore, it has been shown that $\hat{\bar{\mathbf{Y}}}_j$ and $\hat{\sigma}_j$ are unique and $\hat{\bar{\mathbf{Y}}}_j$ is a consistent estimator of $\bar{\mathbf{Y}}_j$ [14].

Based on the elements recalled in this section, the next part introduces the RFVs for RLDs.

III. FISHER VECTORS FOR RIEMANNIAN LAPLACE DISTRIBUTIONS

Fisher vectors have been recently generalized for Riemannian Gaussian distributions [19]. Starting from their initial definition, the Fisher vectors are extended here to RLDs.

III-A. Definition

Let $\boldsymbol{\Upsilon} = \{\mathbf{Y}_t\}_{t=1:T}$ be a sample of T i.i.d observations following a mixture of K RLDs. Under the independence assumption, the probability density function of $\boldsymbol{\Upsilon}$ is given by:

$$p(\boldsymbol{\Upsilon}|\lambda) = \prod_{t=1}^{T} p(\mathbf{Y}_t|\lambda), \qquad (6)$$

where $\lambda = \{(\varpi_j, \bar{\mathbf{Y}}_j, \sigma_j)_{1 \leq j \leq K}\}$ is the parameter vector and $p(\mathbf{Y}_t|\lambda)$ is the probability density function given in (2).

In order to obtain the FV, the gradient of the probability density function characterizing the data has to be determined. In practice, this is achieved by computing the gradient of the log-likelihood with respect to the model parameters, called the Fisher score [18]. For the sample $\boldsymbol{\Upsilon}$, the Fisher score $U_{\boldsymbol{\Upsilon}}$ is given by:

$$U_{\boldsymbol{\Upsilon}} = \nabla_\lambda \log p(\boldsymbol{\Upsilon}|\lambda) = \nabla_\lambda \sum_{t=1}^{T} \log p(\mathbf{Y}_t|\lambda). \qquad (7)$$

In classification problems, the gradient of the log-likelihood is often normalized by using the Fisher information matrix F_λ. For this purpose, F_λ is given by [18]:

$$F_\lambda = E_{\boldsymbol{\Upsilon}}[U_{\boldsymbol{\Upsilon}} U_{\boldsymbol{\Upsilon}}^T] \qquad (8)$$

and the normalized Fisher score becomes [15]:

$$F_\lambda^{-1/2} \nabla_\lambda \log p(\boldsymbol{\Upsilon}|\lambda). \qquad (9)$$

Up to our knowledge, there is no closed-form expression for this Fisher information matrix. Practically, it can be estimated by carrying out a Monte Carlo integration. In this case, N i.i.d samples from the mixture of K RLDs should be generated. The interested reader is referred to [14] for a generation algorithm. Nonetheless, due to the computation cost of this approach, the Fisher information matrix is often approximated by the identity matrix [15]. In the following, the Riemannian Fisher Vectors are derived by computing the Fisher score $U_{\boldsymbol{\Upsilon}}$. For that, closed-form expressions of the derivatives of the log-likelihood function with respect to λ can be computed based on the following observations:

- the probability $\gamma_i(\mathbf{Y}_t)$ that the observation $\mathbf{Y}_t$ is generated by the i^{th} RLD is computed as:

$$\gamma_i(\mathbf{Y}_t) = \frac{\varpi_i \; p(\mathbf{Y}_t|\bar{\mathbf{Y}}_i, \sigma_i)}{\sum_{j=1}^{K} \varpi_j \; p(\mathbf{Y}_t|\bar{\mathbf{Y}}_j, \sigma_j)}; \qquad (10)$$

- to ensure the constraints of positivity and sum to one for the weights, the derivative of the log-likelihood with respect to this parameter needs the following parametrization [16]:

$$\varpi_i = \frac{\exp(\alpha_i)}{\sum_{j=1}^{K} \exp(\alpha_j)}. \qquad (11)$$

As a result, the expressions of the derivatives are:

$$\frac{\partial \log p(\boldsymbol{\Upsilon}|\lambda)}{\partial \bar{\mathbf{Y}}_i} = \sum_{t=1}^{T} \gamma_i(\mathbf{Y}_t)\ \frac{\mathrm{Log}_{\bar{\mathbf{Y}}_i}(\mathbf{Y}_t)}{2\ \sigma_i^2\ d(\mathbf{Y}_t, \bar{\mathbf{Y}}_i)}, \tag{12}$$

$$\frac{\partial \log p(\boldsymbol{\Upsilon}|\lambda)}{\partial \sigma_i} = \sum_{t=1}^{T} \gamma_i(\mathbf{Y}_t) \left\{ -\frac{Z'(\sigma_i)}{Z(\sigma_i)} + \frac{d(\mathbf{Y}_t, \bar{\mathbf{Y}}_i)}{\sigma_i^3} \right\}, \tag{13}$$

$$\frac{\partial \log p(\boldsymbol{\Upsilon}|\lambda)}{\partial \alpha_i} = \sum_{t=1}^{T} \gamma_i(\mathbf{Y}_t)\ (1 - \varpi_i), \tag{14}$$

where $\mathrm{Log}_{\bar{\mathbf{Y}}_i}(\cdot)$ is the Riemannian logarithm mapping.

In the end, the FVs for the RLD is obtained by concatenating some, or all the derivatives in (12), (13) and (14).

III-B. Interpretation

FVs can be viewed as methods for determining if samples are correctly fitted by a given model. Using these descriptors, a sample is characterized by its deviation from the model [16], measured by the Fisher score. More precisely, a large value for the gradient of the log-likelihood implies that the model does not correctly fit the data.

By taking into consideration only the derivatives with respect to the central value, a special case of FVs can be obtained: the VLAD features, or the R-VLAD features for the Riemannian manifold [3].

To build the R-VLAD features for the RLDs, several steps are needed. First, only the derivatives with respect to $\bar{\mathbf{Y}}_i$ (12), are considered. Next, a hard assignment scheme is applied:

$$\gamma_i(\mathbf{Y}_t) = \begin{cases} 1, & \text{if } \mathbf{Y}_t \in c_i \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

with $\mathbf{Y}_t \in c_i$ being the elements $\mathbf{Y}_t$ assigned to the cluster c_i, $i = 1, \ldots, K$. In the end, the assumption of homoscedasticity is added, that is $\sigma_i = \sigma\ , \forall i = 1, \ldots, K$.

IV. APPLICATIONS TO IMAGE CLASSIFICATION

In this section, the RFVs are applied to texture image classification by using both RGDs and RLDs to model the space of covariance matrices. The purpose of the experiments is to find the most suited distribution to fit the data and to determine the most discriminating RFVs.

IV-A. Databases

The experiments reported in this paper are carried out using the VisTex [21] database. This database consists in 40 texture classes, each of them having 64 images of size 64×64 pixels. In the following, the feature extraction and classification steps are detailed.

IV-B. General framework

As previously mentioned, the experimental workflow consists in two stages. First, the descriptors modeling the textural information are extracted and the Riemannian feature vectors are computed by considering (12) to (14). Second, a supervised classification algorithm is used to classify those RFVs.

In this paper, the textural information is captured by using region covariance descriptors (RCovDs) based on simple features. Thus, for an image I, characteristics like the image intensity and the norms of the first and second order derivatives are extracted for each pixel $(x, y) \in I$. As a result, a vector $\mathbf{v}$ of 5 elements is obtained [23]:

$$\mathbf{v}(x,y) = \left[I(x,y),\ \left|\tfrac{\partial I(x,y)}{\partial x}\right|,\ \left|\tfrac{\partial I(x,y)}{\partial y}\right|,\ \left|\tfrac{\partial^2 I(x,y)}{\partial x^2}\right|,\ \left|\tfrac{\partial^2 I(x,y)}{\partial y^2}\right| \right]^T, \tag{16}$$

where $I(x, y)$ is the image intensity of pixel $(x, y) \in I$.

Starting from these vectors, the RCovDs are defined as being the estimated covariance matrices of vectors $\mathbf{v}(x, y)$ computed on a sliding patch of size 15×15 pixels. In addition, an overlap of 8 pixels is considered for the patches. Therefore, each texture class in the VisTex database is represented by a set of 36 covariance matrices of size 5×5. To speed-up the computation time, the fast covariance computation algorithm based on integral images presented in [23] has been implemented. In the end, each texture class is represented by a set $\mathbf{Y}_1, \ldots, \mathbf{Y}_T$ of T covariance matrices, with $\mathbf{Y}_t \in \mathcal{P}_5$, $t = 1, \ldots, T$.

Knowing that supervised classification methods are considered later, the database is equally and randomly divided in order to obtain the training and the testing sets. Further on, the patches in the training set are used to create a codebook. For this step, a within-class approach is implemented. More precisely, each texture class is modeled by a mixture of K Riemannian distributions (RGD or RLD) and the estimated parameters $\{\hat{\bar{\mathbf{Y}}}_j, \hat{\sigma}_j, \hat{\varpi}_j\}_{1 \leq j \leq K}$ represent the codewords. The codebook is obtained by concatenating the codewords previously extracted for each class. The estimation procedure is carried out here by using the intrinsic k-means algorithm detailed in Section II, with K being set to 3.

Once that the codebook is determined, the BoRWs, RFV and R-VLAD models can be derived for both RGD [19] and RLD (see Section III) distributions. After their computation, a normalization step is required. In the RFV framework, the classical power and ℓ_2 normalizations are applied [20]. The ℓ_2 normalization has been proposed in [24] to minimize the influence of the background information on the image signature, while the power normalization corrects the independence assumption made on the patches [25]. The same normalization scheme is applied for R-VLAD models. For the BoRW algorithm, only ℓ_2 normalization is performed, as recommended in [26].

For the final classification step, the SVM algorithm with a Gaussian kernel and the k-NN method have been considered.

In the next section, only the best results are reported, that are the ones given by the SVM approach. For this method, the dispersion parameter in the Gaussian kernel is optimized by considering a cross-validation procedure on the training set.

IV-C. Results and discussion

In this section, the classification results obtained on the VisTex database are discussed. Table I reports the classification performances in terms of overall accuracy. In order to find these values, the database has been partitioned 10 times in training and testing sets. In addition, the Fisher information matrix given in (8) is considered to be the identity matrix.

In this table, the first column specifies the descriptor's type (BoRW, RFV, or R-VLAD). The second column (Homosced.) refers to the homoscedasticity assumption. If this assumption is true, all the clusters c_j have the same dispersion parameter σ_j. The third column (Prior) corresponds to the weights ϖ_j. If this parameter is set to false, the same weight is given to all the clusters of the mixture model. The last two columns present the classification performances when mixtures of RGDs and RLDs model the space of estimated covariance matrices.

In this experiment, the contribution of each parameter (weight, dispersion and centroid) to the classification accuracy is also analyzed. For example, the row "RFV : ϖ" indicates the classification results when only the derivatives with respect to the weights are considered to calculate the RFV (see (14)), etc.

The carried out experiments have multiple purposes. First, the RGD and RLD models are analyzed in order to discover the most suitable distribution for data modeling. Second, the descriptors are compared to find the most accurate one for the present problem. Third, for the RFVs, the contribution of each parameter (weight, dispersion and centroid) to the classification accuracy is tested.

By observing the classification results, the following conclusions can be noticed. First, for these experiments, the use of RLDs brings little improvements in terms of classification accuracy. The most important raises can be spotted by considering the "RFV: σ" (about 7%) and the "RFV: σ, ϖ" (about 5%) features. Moreover, combining the RFV associated to the centroid $\bar{\mathbf{Y}}$ with those associated to the weight and dispersion parameters yields to a gain of about 3% for both RGDs and RLDs. In addition, the proposed RFVs outperform significantly the state-of-the-art BoRW and R-VLAD descriptors [3]. A significant gain of 3 to 4% is observed. This gain is quite logical since the RFVs can be interpreted as a generalization of R-VLAD features.

Table I. Classification results on the VisTex database in terms of overall accuracy.

Method	Homosced.	Prior	RGD	RLD
BoRW	false	true	87.22 ± 1.19	87.70 ± 1.75
BoRW	false	false	87.51 ± 0.92	88.10 ± 1.42
BoRW [20]	true	false	87.20 ± 1.55	87.69 ± 0.93
BoRW	true	true	76.67 ± 2.35	69.01 ± 5.39
RFV : ϖ	false	true	90.31 ± 0.94	90.84 ± 0.91
RFV : σ	false	true	81.42 ± 1.12	88.51 ± 0.87
RFV : $\bar{\mathbf{Y}}$	false	true	87.22 ± 1.15	87.71 ± 1.06
RFV : σ, ϖ	false	true	83.05 ± 1.15	88.59 ± 0.56
RFV : $\bar{\mathbf{Y}}, \varpi$	false	true	87.85 ± 0.97	88.43 ± 1.11
RFV : $\bar{\mathbf{Y}}, \sigma$	false	true	90.41 ± 0.86	**91.07 ± 0.53**
RFV : $\bar{\mathbf{Y}}, \sigma, \varpi$	false	true	**90.43 ± 0.84**	90.98 ± 0.89
R-VLAD [3]	true	false	87.94 ± 0.58	87.38 ± 0.73

V. CONCLUSION

In this paper, a new local model for image classification in the Riemannian space has been introduced. Recently, this space has been modeled by several distributions, including the Wishart distribution, the Riemannian Gaussian distribution, or the Riemannian Laplace distribution. In order to have a good representation of the data, the choice of an appropriate probabilistic generative model is very important. For instance, in real life, datasets usually contain outliers that modify their structure. Therefore, a robust model, like the RLD, may be needed to control the impact of these aberrant values. Motivated by the distribution's robustness properties, this paper proposes the use of RLDs in the definition of Fisher vectors. Thus, the main contribution of this work is the extension of the Riemannian Fisher vectors to the Laplacian model. First, the definition of the RLD has been recalled and the new descriptors have been introduced for mixtures of RLDs. Next, their relation with the R-VLAD model has been shown. Once that the theoretical background has been fixed, the proposed RFVs for Riemannian Laplace distribution have been applied to texture image classification on the VisTex database. Moreover, another descriptor of the state-of-the-art, the bag of Riemannian words, has been considered. In the end, the classification results have been compared to those obtained for the same descriptors derived for the RGD model. As a conclusion, it can be mentioned that RLDs may bring little improvement to the classification accuracy. In addition, the proposed RFVs outperforms the BoRW and R-VLAD descriptors.

Further works on this subject will concern the derivation of an analytical expression of the Fisher information matrix of the Riemannian Laplace distribution.

ACKNOWLEDGMENT

This study has been carried out in the frame of the Investments for the future Programme IdEx Bordeaux - CPU (ANR-10-IDEX-03-02) of the French National Research Agency (ANR). It has also been supported by the French Foreign Affairs and International Development Ministry and by the Executive Agency for Higher Education, Research, Development and Innovation Funding Romania, under the projects 32619VL and PNII Capacitati 779/27.06.2014.

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