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Abstract : I give a resolution of the Navier-Stokes [2] equation by using the series of Fourier.

Résumé: Je donne une résolution de l'équation de Navier-Stokes [2] par les séries de Fourier.

Keywords : Navier-Stokes, Fourier, Séries de Fourier.

I . Introduction

The Navier–Stokes equations is considered to be the first step to understanding the elusive phenomenon of turbulence, the Clay Mathematics Institute in May 2000 made this problem [2] one of its seven Millennium Prize problems in mathematics.

In this article I will prove that the Navier-Stokes equation have a solutions and I will give techniques to resolve this beautiful equation.

The Navier-Stokes equation, established in the nineteenth century by the French Navier and the British Stokes. It is an equation that describes the velocity field of a fluid. More specifically, it is a differential equation whose velocity field is unknown.

The Navier-Stokes equation is also used to predict the weather, the oceans simulate, optimize aircraft wings ...

Knowing that a link between the Boltzmann equation and the Navier-Stokes equation was established, by studying the latter problem, I found that for to solve it we can reduce the problem of the heat-equation which is known can be solved by several methods : one of the first methods of solving the heat-equation was proposed by Joseph Fourier in his treatise analytical Theory of heat [1] in 1822 .

After giving a specific solutions to the Navier-Stokes equation, I will demonstrate how to find all solutions of this equation if they exist, and I give the necessary and sufficient conditions for their existence.

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It will be seen in a remark that if the turbulence function is negligible, then the fluid will tend to behave like an ideal gas.

II. Recall, notations and definitions

Here are the Navier-Stokes equation:

$$\rho\left(\frac{\partial u}{\partial t}+(u.\nabla u)\right)=-\nabla p+\mu\nabla^2 u$$
$$\operatorname{div} u=0$$

Where u is the velocity field, p is the pressure ,the density of the fluid , and μ its viscosity.

And :

$$\nabla=\left(\frac{\partial}{\partial x_1},\dots,\frac{\partial}{\partial x_n}\right)$$

$$(u.\nabla u)=\sum_1^n u_i\frac{\partial u}{\partial x_i}$$
$$\nabla^2 u=\sum_1^n \frac{\partial^2 u}{\partial x_i^2}$$
$$\operatorname{div} u=\sum_1^n \frac{\partial u}{\partial x_i}$$

In the following, by dividing by ρ , the Navier-Stokes equations is of the form :

$$\frac{\partial u}{\partial t}+(u.\nabla u)=\alpha\nabla p+\beta\nabla^2 u$$
$$\operatorname{div} u=0$$

III- Existence of the solutions for the Navier-Stokes equation :

On each axis i, try to find the solutions of the form :

$$\frac{\partial u_i}{\partial t}+\left(u_i\frac{\partial u_i}{\partial x_i}\right)=\alpha\frac{\partial p_i}{\partial x_i}+\beta\frac{\partial^2 u_i}{\partial x_i^2}$$

This is equivalent to:

$$\frac{\partial u_i}{\partial t}+\frac{\partial\left(\frac{1}{2}u_i^2-\alpha\, p_i\right)}{\partial x_i}=\beta\frac{\partial^2 u_i}{\partial x_i^2}$$

If u_i is a solution of the equation :

$$\frac{\partial u_i}{\partial t}=\beta\frac{\partial^2 u_i}{\partial x_i^2}$$

Such solutions u_i exist because the equation is analogous to the heat-equation which is resolvable by the Fourier series [1].

If p_i is such $\frac{1}{2}u_i^2-\alpha\, p_i=f_i(t)$, then $\frac{\partial\left(\frac{1}{2}u_i^2-\alpha\, p_i\right)}{\partial x_i}=0$, and the equation is solved.

We do the same for all axes i until $i=n-1$.

For the axe $i=n$:

Let be $u_n=-\sum_1^{n-1}u_i$.

We have : $\frac{\partial u_n}{\partial t}=\beta\frac{\partial^2 u_n}{\partial x_i^2}$, and if p_n is such $\frac{1}{2}u_n^2-\alpha\, p_n=f_n(t)$, then

$$\frac{\partial\left(\frac{1}{2}u_n^2-\alpha\, p_n\right)}{\partial x_n}=0$$, and the equation is solved for the axe n.

It is clear that if e_i is the vector for the axes i, then $u=\sum_1^n u_i e_i$ is one solution of the Naviers-Stokes equation if $\operatorname{div} u=0$.

Else, to have $\operatorname{div} u=0$, we take u_i of the form :

$$u_i=e^{\beta t+\left(\sum_1^n x_i\right)}, \forall i\in\{1,\ldots,n-1\}$$

So we have solutions of the Navier-Stokes equation.

IV- Necessary conditions :

Any solution (u,p) of the Navier-Stokes equation verifies that :

$$u_i^2-\alpha\, p_i=f_i(t), \forall i\in\{1,\ldots,n\}$$

Indeed :

If (u,p) is a solution of the Navier-Stokes equation , we must have :

$$\frac{\partial u_i}{\partial t}+\frac{\partial\left(\frac{1}{2}u_i^2-\alpha\, p_i\right)}{\partial x_i}=\beta\frac{\partial^2 u_i}{\partial x_i^2}$$

Therefore :

$$-\frac{\partial u_i}{\partial t}=\frac{\partial\left(\frac{1}{2}u_i^2-\alpha\, p_i-\beta\frac{\partial u_i}{\partial x_i}\right)}{\partial x_i}$$

And :

$$-\partial x_i\frac{\partial u_i}{\partial t}=\partial\left(\frac{1}{2}u_i^2-\alpha\, p_i-\beta\frac{\partial u_i}{\partial x_i}\right)$$

When the fluid flows in one direction, then the space-time flows in the opposite direction with the same speed value:

We deduce that :

$$-u_i\partial u_i=\partial\left(\frac{1}{2}u_i^2-\alpha\, p_i-\beta\frac{\partial u_i}{\partial x_i}\right)$$

Therefore :

$$0=\partial\left(u_i^2-\alpha\, p_i-\beta\frac{\partial u_i}{\partial x_i}\right)$$

And :

$$u_i^2-\alpha\, p_i-\beta\frac{\partial u_i}{\partial x_i}=f_i(t)$$

So :

$$\sum_1^n u_i^2-\alpha\, p_i-\beta\frac{\partial u_i}{\partial x_i}=f(t)$$

And consequently $\sum_1^n u_i^2-\alpha\, p_i=g(t)$ because $\operatorname{div} u=0$.

We deduce therefore that :

$u_i^2-\alpha\, p_i=h_i(t), \forall i\in\{1,\ldots,n\}$ because : $\forall i\in\{1,\ldots,n\}, u_i^2-\alpha\, p_i=l_i(x_i,t)$, and we must have $\frac{\partial(u_i^2-\alpha\, p_i)}{\partial x_j}=0\,\forall j\in\{1,\ldots,n\}$.

V- Conclusion :

Theorem:

The Navier-stokes equation have a solution, Moreover, any solution (u, p) must check: $u_i^2-\alpha\, p_i=f_i(t)$ and $\frac{\partial u_i}{\partial t}=\beta\frac{\partial^2 u_i}{\partial x_i^2}$, $\forall i\in\{1,\ldots,n\}$. where $u=\sum_1^n u_i e_i$ and $p=\sum_1^n p_i e_i$.

Conversely any pair (n, p) satisfying these conditions with $\operatorname{div} u=0$ is solution of the Navier-Stokes equation.

Remarks :

1- We note in the above equations the dependence between pressure, density, speed vector fields and viscosity

2- By dividing by α in the equation $u_i^2-\alpha\, p_i=f_i(t)$ we deduce that : $\rho\, u_i^2-p_i=\rho f_i(t)$ where $\frac{1}{\alpha}=\rho$ is the density of the fluid.

Let's $\rho=\frac{m}{V}$, we will have : $m u_i^2-Vp_i=mf_i(t)$.

And the equation $m u_i^2-Vp_i=mf_i(t)$ is linking energy, mass, pressure, temperature, volume and time ... This may not be surprising since a link between the Boltzmann equation and the

Navier-Stokes has been established.

When $f_i(t)$ tends to 0, we will have $mu_i^2 \simeq Vp_i$, there is therefore a tendency towards the law of an ideal gas, and the function $f_i(t)$ can be regarded as a turbulent function.

References

[1] Joseph Fourier, Théorie analytique de la chaleur, Firmin Didot Père et Fils (Paris-1822). Réédition Jacques Gabay, 1988 (ISBN 2-87647-046-2)

[2] http://www.claymath.org/sites/default/files/navierstokes.pdf