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Selma Belhadj Amor, Samir Perlaza

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Abstract In this paper, the fundamental limits of *decentralized* information transmission in the K-user Gaussian multiple access channel (G-MAC), with $K \geqslant 2$, are fully characterized. Two scenarios are considered. First, a game in which only the transmitters are players is studied. In this game, the transmitters autonomously and independently tune their own transmit configurations seeking to maximize their own transmission rates, $R_1, \ldots, R_K$, respectively. On the other hand, the receiver adopts a fixed receive configuration that is known a priori to the transmitters. The main result consists of the full characterization of the set of rate tuples $(R_1, \ldots, R_K)$ that are achievable and stable in the G-MAC when stability is considered in the sense of the η-Nash equilibrium (NE), with $\eta \geqslant 0$ arbitrarily small. Second, a sequential game in which the two categories of players (the transmitters and the receiver) play in a given order is presented. For this sequential game, the main result consists of the full characterization of the set of rate tuples $(R_1, \ldots, R_K)$ that are stable in the sense of an η-sequential equilibrium, with $\eta \geqslant 0$ arbitrarily small.

1 Problem Formulation

1.1 K-User Centralized Gaussian Multiple Access Channel

Consider the K-user memoryless Gaussian multiple access channel (G-MAC) with $K \geqslant 2$ users. Let $n \in \mathbb{N}$ be the blocklength. At each time $t \in \{1, \ldots, n\}$ and for any $i \in \{1, \ldots, K\}$, let $X_{i,t}$ denote the real input symbol sent by transmitter i. The receiver observes the real channel output $Y_t = \sum_{i=1}^{K} h_i X_{i,t} + Z_t$, where h_i, for all $i \in \{1, \ldots, K\}$, is a constant nonnegative real channel coefficient. The noise

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terms Z_t are independent and identically distributed (i.i.d.) realizations of a zero-mean unit-variance real Gaussian random variable. Let R_i denote the information transmission rate at transmitter i, for all $i \in \{1, \ldots, K\}$. The goal of the communication is to convey the message index M_i, uniformly distributed over the set $\mathscr{M}_i \triangleq \{1, \ldots, \lfloor 2^{nR_i} \rfloor\}$, from transmitter i, with $i \in \{1, \ldots, K\}$ to the common receiver. The message indices $(M_1, \ldots, M_K)$ are independent of each other and of the noise terms $Z_1, \ldots, Z_n$. At each time t, the t-th symbol of transmitter i, for all $i \in \{1, \ldots, K\}$, depends solely on its message index M_i, i.e., $X_{i,t} = f_{i,t}^{(n)}(M_i)$, $t \in \{1, \ldots, n\}$, for some encoding functions $f_{i,t}^{(n)}\colon \mathscr{M}_i \to \mathbb{R}$. The receiver produces an estimate $(\hat{M}_1^{(n)}, \ldots, \hat{M}_K^{(n)}) = \Phi^{(n)}(Y_1, \ldots, Y_n)$ of the message-tuple $(M_1, \ldots, M_K)$ via a decoding function $\Phi^{(n)}\colon \mathbb{R}^n \to \mathscr{M}_1 \times \ldots \times \mathscr{M}_K$, and the average probability of error is given by

$$P_{\text{error}}^{(n)}(R_1, \ldots, R_K) \triangleq \Pr\left\{(\hat{M}_1^{(n)}, \ldots, \hat{M}_K^{(n)}) \neq (M_1, \ldots, M_K)\right\}. \tag{1}$$

The symbols $X_{i,1}, \ldots, X_{i,n}$ satisfy an expected average *input power constraint*

$$\frac{1}{n}\sum_{t=1}^{n} \mathrm{E}\left[X_{i,t}^2\right] \leqslant P_{i,\max}, \quad i \in \{1, \ldots, K\}, \tag{2}$$

where the expectation is over the message indices and where $P_{i,\max}$ denotes the maximum average power of transmitter i in energy units per channel use. This channel is fully described by the signal to noise ratios (SNRs): SNR_i, with $i \in \{1, \ldots, K\}$, which are defined as: $\mathrm{SNR}_i \triangleq |h_i|^2 P_{i,\max}$.

1.2 Achievable Rates and Capacity Region

The K-tuple $(R_1, \ldots, R_K) \in \mathbb{R}_+^K$ is said to be *achievable* if there exists a sequence of encoding and decoding functions $\left\{\{f_{1,t}^{(n)}\}_{t=1}^{n}, \ldots, \{f_{K,t}^{(n)}\}_{t=1}^{n}, \Phi^{(n)}\right\}_{n=1}^{\infty}$ such that the average error probability tends to zero as the blocklength n tends to infinity. That is,

$$\limsup_{n \to \infty} P_{\text{error}}^{(n)}(R_1, \ldots, R_K) = 0. \tag{3}$$

The closure of the union of all achievable rate tuples is called the *capacity region* and is denoted by $\mathscr{C}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$. From [5, 14], it follows that

$$\mathscr{C}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K) = \Bigg\{(R_1, \ldots, R_K) \in \mathbb{R}_+^K : \\ \sum_{j \in \mathscr{U}} R_j \leqslant \frac{1}{2}\log_2\big(1 + \sum_{j \in \mathscr{U}} \mathrm{SNR}_j\big), \forall \mathscr{U} \subseteq \{1, \ldots, K\}\Bigg\}. \tag{4}$$

Note that $\mathscr{C}(\mathrm{SNR}_1,\ldots,\mathrm{SNR}_K)$ is a K-dimension polyhedron with $K!$ corner points. Each corner point corresponds to a decoding order among the users.

1.3 K-User Decentralized Gaussian Multiple Access Channel

In a decentralized K-user G-MAC, the aim of transmitter i, for all $i \in \{1,\ldots,K\}$, is to autonomously choose its transmit configuration s_i in order to maximize its information rate R_i. The transmit configuration s_i can be described in terms of the information rates R_i, the block-length n, the channel input alphabet $\mathscr{X}_i$, the encoding functions $\{f_{1,t}^{(n)}\}_{t=1}^{n},\ldots,\{f_{K,t}^{(n)}\}_{t=1}^{n}$, etc. The receiver autonomously chooses a receive configuration s_0 in view of maximizing the sum-rate. Let $\mathscr{P}_K$ denote the set of all permutations (all possible decoding orders) over the set $\{1,\ldots,K\}$. For any $\pi \in \mathscr{P}_K$, the considered decoding order $\pi(1),\pi(2),\ldots,\pi(K)$ is such that user $\pi(1)$ is decoded first, user $\pi(2)$ is decoded second, etc. The receive configuration can be described in terms of the decoding function $\Phi^{(n)}$, which in this paper is restricted to single-user decoding (SUD), successive interference cancelation (SIC(π)) with a given order $\pi \in \mathscr{P}_K$, or any time-sharing (TS) combination of the previous schemes. However, the choice of the transmit configuration of each transmitter depends on the choice of the other transmitters as well as the decoding scheme at the receiver. The input signal of one transmitter is interference to the others. Thus, the rate achieved by transmitter i depends on all transmit configurations $s_1,\ldots,s_K$ as well as the configuration of the receiver s_0. The utility function of transmitter i, for all $i \in \{1,\ldots,K\}$, is $u_i : \mathscr{A}_0 \times \ldots \times \mathscr{A}_K \to \mathbb{R}_+$ and it is defined as its own rate,

$$u_i(s_0,\ldots,s_K) = \begin{cases} R_i(s_0,\ldots,s_K), & \text{if } P_{\text{error}}^{(n)}(R_1,\ldots,R_K) < \varepsilon \\ 0, & \text{otherwise,} \end{cases} \tag{5}$$

where $\varepsilon > 0$ is an arbitrarily small number and $R_i(s_0,\ldots,s_K)$ denotes a transmission rate achievable with the configurations $(s_0,\ldots,s_K)$. Often, the information rate $R_i(s_0,\ldots,s_K)$ is written as R_i for simplicity. However, every nonnegative achievable information rate is associated with a particular transmit-receive configuration $(s_0,\ldots,s_K)$ that achieves it. It is worth noting that there might exist several transmit-receive configurations that achieve the same tuple $(R_1,\ldots,R_K)$ and distinction between the different transmit-receive configurations is made only when needed. The utility function of the receiver is $u_0 : \mathscr{A}_0 \times \ldots \times \mathscr{A}_K \to \mathbb{R}_+$ and it is defined as the sum-rate,

$$u_0(s_0,\ldots,s_K) = \begin{cases} \sum_{i=1}^{K} R_i(s_0,\ldots,s_K), & \text{if } P_{\text{error}}^{(n)}(R_1,\ldots,R_K) < \varepsilon \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

In the absence of a central controller which dictates the transmit/receive configurations to the various network components, only stable rate tuples are possible operating points of the network. Within this context, stability is considered in the sense that none of the components is able to increase its utility by unilaterally changing its own transmit/receive configuration. From this perspective, in the capacity region $\mathscr{C}(\mathrm{SNR}_1,\ldots,\mathrm{SNR}_K)$, any rate tuple $(R_1,\ldots,R_K)$ for which

$$R_i < \frac{1}{2}\log_2\left(1+\frac{\mathrm{SNR}_i}{1+\sum_{j=1;j\neq i}^{K}\mathrm{SNR}_j}\right), \tag{7}$$

at least for one $i \in \{1,\ldots,K\}$, is not stable. This is true when the receiver is constrained to choose among the decoding strategies mentioned above (SUD, SIC, or TS) because the considered transmitter can always increase its rate and achieve

$$R_i = \frac{1}{2}\log_2\left(1+\frac{\mathrm{SNR}_i}{1+\sum_{j=1;j\neq i}^{K}\mathrm{SNR}_j}\right) - \delta, \tag{8}$$

with $\delta > 0$ arbitrarily small. The remaining achievable rate tuples $(R_1,\ldots,R_K) \in \mathscr{C}(\mathrm{SNR}_1,\ldots,\mathrm{SNR}_K)$ which satisfy

$$R_i \geqslant \frac{1}{2}\log_2\left(1+\frac{\mathrm{SNR}_i}{1+\sum_{j=1;j\neq i}^{K}\mathrm{SNR}_j}\right), \quad \forall i \in \{1,\ldots,K\}, \tag{9}$$

can be stable or not, depending on the actions of the receiver.

In the following, two games are considered. First, a game in which only the transmitters are players is studied in Sec. 2. For this game, the set of stable rate tuples is fully characterized when stability is considered in the sense of η-Nash equilibrium [10], with $\eta \geqslant 0$ arbitrarily small. Second, a sequential game in which the two categories of players (the transmitters and the receiver) play in a given order. For this sequential game, the set of stable rate tuples in the sense of the *η-sequential equilibrium*, with $\eta \geqslant 0$ arbitrarily small, is derived in Sec. 3.

2 Game I: Only the Transmitters are Players

Under the assumption that the receiver adopts a fixed receive configuration $\tilde{s}_0$ that is known a priori to all terminals, the competitive interaction of the K transmitters in the decentralized G-MAC can be modeled by the following game in normal form:

$$\mathscr{G}_1 = \left(\mathscr{K}, \{\mathscr{A}_k\}_{k\in\mathscr{K}}, \{u_k\}_{k\in\mathscr{K}}\right). \tag{10}$$

The set $\mathscr{K} = \{1,\ldots,K\}$ is the set of players, i.e., the transmitters. For all $i \in \mathscr{K}$, the set $\mathscr{A}_i$ is the set of actions of player i. An action $s_i \in \mathscr{A}_i$ of player i is basically its transmit configuration as described above. The utility function of transmitter i, for all $i \in \{1,\ldots,K\}$, is u_i defined in (5). Note that since the receiver is not a player, its action $\tilde{s}_0$ is kept fixed, but it remains being an argument of the utility function.

A formal definition of an η-NE is provided below.

Definition 1 (η-NE [10]). In the game $\mathscr{G}_1$, under the fixed receive configuration $\tilde{s}_0$, an action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ is an η-NE if for all $i \in \mathscr{K}$ and for all $s_i \in \mathscr{A}_i$, it holds that

$$u_i(\tilde{s}_0, s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_K^*) \leqslant u_i(\tilde{s}_0, s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_K^*) + \eta. \quad (11)$$

Under the fixed receive configuration $\tilde{s}_0$, from Def. 1, it becomes clear that if $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ is an η-NE, then none of the transmitters can increase its own rate by more than η bits per channel use by unilaterally changing its own transmit configuration while keeping the average error probability arbitrarily close to zero. Thus, at a given η-NE, every transmitter achieves a utility that is η-close to its maximum achievable rate given the transmit configuration of the other transmitters. Note that if $\eta = 0$, then the definition of NE is obtained [9].

Remark 1. Note that the definition of the utilities in (5) and (6) is parametrized by the choice of the error probability threshold ε. Within this context, considering NE instead of η-NE with an arbitrary slack $\eta \geqslant 0$ would require the difficult task of deriving a coding scheme that achieves the optimal rate with exactly ε error probability. The slack $\eta \geqslant 0$, which can be made arbitrarily small, allows to remove this difficulty [3] and [11]. Note also that there is a slight abuse of notation in the equalities defining the utilities and it is assumed that the blocklength is sufficiently high to neglect the asymptotically small slack due to the fixed blocklength.

The following investigates the rate region that can be achieved at an η-NE. This set of rate tuples is known as the η-NE region.

Definition 2 (η-NE Region). Let $\eta \geqslant 0$ be arbitrarily small. An achievable rate tuple $(R_1, \ldots, R_K) \in \mathscr{C}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$ is said to be in the η-NE region of the game $\mathscr{G}_1$ under the fixed receive configuration $\tilde{s}_0$, if there exists an action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*) \in \mathscr{A}_0 \times \mathscr{A}_1 \times \ldots \times \mathscr{A}_K$ that is an η-NE and the following holds:

$$u_i(\tilde{s}_0, s_1^*, \ldots, s_K^*) = R_i, \quad \forall i \in \{1, \ldots, K\}. \quad (12)$$

The following section studies the η-NE region of the game $\mathscr{G}_1$, with $\eta \geqslant 0$ arbitrarily small, for several decoding strategies adopted by the receiver.

2.1 η-NE Region With Single User Decoding (SUD)

The η-NE region of the game $\mathscr{G}_1$ when the receiver uses SUD, denoted by $\mathscr{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$, is described by the following theorem.

Theorem 1 (η-NE Region With SUD). *Let $\eta \geqslant 0$ be arbitrarily small. Then, the set $\mathscr{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$ of η-NEs of the game $\mathscr{G}_1$ contains only the nonnegative rate tuple $(R_1, \ldots, R_K)$ that satisfies*

$$R_i = \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_i}{1 + \sum_{j=1; j\neq i}^{K} \mathrm{SNR}_j}\right), \forall i \in \{1, \ldots, K\}. \quad (13)$$

Proof: The proof of Theorem 1 is provided in [1]. ■

2.2 η-NE Region With Successive Interference Cancelation (SIC)

The η-NE region of the game $\mathscr{G}_1$ when the receiver uses $\mathrm{SIC}(\pi)$ with a fixed decoding order $\pi \in \mathscr{P}_K$, denoted by $\mathscr{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$, is described by the following theorem.

Theorem 2 (η-NE Region of the Game $\mathscr{G}_1$ With $\mathrm{SIC}(\pi)$). *Let $\eta \geqslant 0$ be arbitrarily small and let $\pi \in \mathscr{P}_K$ be a fixed decoding order. Then, the set $\mathscr{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$ contains only the nonnegative rate tuple $(R_1, \ldots, R_K)$ satisfying:*

$$R_{\pi(i)} = \frac{1}{2}\log_2\left(1 + \frac{\mathrm{SNR}_{\pi(i)}}{1 + \sum_{j=i+1}^{K}\mathrm{SNR}_{\pi(j)}}\right), \forall i \in \{1, \ldots, K\}. \quad (14)$$

Proof: The proof of Theorem 2 is provided in [1]. ■

Remark 2. Note that for every decoding order $\pi \in \mathscr{P}_K$, the region contains a unique rate tuple. When considering SIC at the receiver under any decoding order, the η-NE region $\mathscr{N}_{\mathrm{SIC}}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$ contains $K!$ rate tuples and is given by

$$\mathscr{N}_{\mathrm{SIC}}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K) = \bigcup_{\pi \in \mathscr{P}_K} \mathscr{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K). \quad (15)$$

2.3 η-NE Region With Time-Sharing (TS)

Let $\mathscr{N}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$ denote the η-NE region of the game $\mathscr{G}_1$ when the receiver might use any time-sharing between the previous decoding techniques. This region is described by the following theorem.

Theorem 3 (η-NE Region of the Game $\mathscr{G}_1$). *Let $\eta \geqslant 0$ be arbitrarily small. Then, the set $\mathscr{N}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$ equals the convex hull of*

$$\mathscr{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K) \cup \Big(\bigcup_{\pi \in \mathscr{P}_K} \mathscr{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K) \Big). \quad (16)$$

Proof: The proof is based on Theorem 1, Theorem 2, and a time-sharing argument. The details are omitted. ■

If the receiver performs any time-sharing combination between any of the considered decoding strategies, then the transmitters can use the same time-sharing combination between their corresponding η-NE strategies to achieve any point inside $\mathscr{N}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$. Note that every time-sharing strategy of the receiver induces a unique rate tuple inside $\mathscr{N}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K)$. However, several time-sharing schemes might achieve the same rate tuple.

3 Game II: A Sequential Game

In this section, the decentralized information transmission in the K-user G-MAC is modeled as a sequential game in which there are two groups of players: one group, the leaders, in which all players play simultaneously before the players of the other group, the followers. The followers, simultaneously play after the leaders under the assumption that the actions of the leaders are perfectly known by all the followers. Let $\{\mathscr{K}_{21}, \mathscr{K}_{22}\}$ be a partition of $\mathscr{K} \cup \{0\}$, such that $\mathscr{K}_{21}$ is the set of leaders and $\mathscr{K}_{22}$ is the set of followers. The competition between the different users (the transmitters and the receiver) in the G-MAC can be modeled as follows:

$$\mathscr{G}_2 = \left(\mathscr{K} \cup \{0\}, \{\mathscr{K}_{21}, \mathscr{K}_{22}\}, \{\mathscr{A}_k\}_{k\in\mathscr{K}}, \{u_k\}_{k\in\mathscr{K}}\right). \tag{17}$$

Backward induction is used in order to characterize a *sequential equilibrium* of this game. First, the leaders simultaneously play knowing that the followers will simultaneously play their *best responses*. Instead of seeking an exactly optimal solution, each player allows a *tolerance* $\eta \geqslant 0$ and seeks a strategy that is η-close to the optimal reward. The set of these η-close optimal strategies of player k is given by its best η-response set defined as follows:

Definition 3 (Set of Best η-Response of Player k). The set of best η-responses of a given player $k \in \{0, 1, \ldots, K\}$ is

$$\mathrm{BR}_k^{(\eta)}(\mathbf{s}_{-k}) = \left\{ s_k \in \mathscr{A}_k : u_k(s_k, \mathbf{s}_{-k}) \geqslant \max_{\tilde{s}_k \in \mathscr{A}_k} u_k(\tilde{s}_k, \mathbf{s}_{-k}) - \eta \right\}. \tag{18}$$

Definition 4 (η-Sequential Equilibrium (η-SE)). Let $\eta \geqslant 0$ be arbitrarily small. In the game $\mathscr{G}_2$, an action profile $(s_0^\dagger, \ldots, s_K^\dagger)$ is an η-SE if it satisfies:

1. $\forall i \in \mathscr{K}_{21}, \quad s_i^\dagger \in \mathrm{BR}_i^{(\eta)}\left(\mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}\right)$ with

$$\mathrm{BR}_i^{(\eta)}\left(\mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}\right) \triangleq \Big\{ s_i \in \mathscr{A}_i : u_i(s_i, \mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}, \mathbf{s}_{\mathscr{K}_{22}}) \geqslant \max_{\tilde{s}_i \in \mathscr{A}_i} u_i(\tilde{s}_i, \mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}, \tilde{\mathbf{s}}_{\mathscr{K}_{22}}) - \eta$$
$$\text{subject to} \quad \mathbf{s}_{\mathscr{K}_{22}} \in \mathrm{BR}^{(\eta)}_{\mathscr{K}_{22}}\left(s_i, \mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}\right) \text{ and } \tilde{\mathbf{s}}_{\mathscr{K}_{22}} \in \mathrm{BR}^{(\eta)}_{\mathscr{K}_{22}}\left(\tilde{s}_i, \mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}\right) \Big\},$$

with $\quad \mathrm{BR}^{(\eta)}_{\mathscr{K}_{22}}\left(s_i, \mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}\right) \triangleq \prod_{j\in\mathscr{K}_{22}} \mathrm{BR}_j^{(\eta)}\left(\mathbf{s}_{\mathscr{K}_{22}\setminus\{j\}}, s_i, \mathbf{s}^\dagger_{\mathscr{K}_{21}\setminus\{i\}}\right).$

2. $\forall j \in \mathscr{K}_{22}, \quad s_j^\dagger \in \mathrm{BR}_j^{(\eta)}\left(\mathbf{s}^\dagger_{\mathscr{K}_{22}\setminus\{j\}}, \mathbf{s}^\dagger_{\mathscr{K}_{21}}\right).$

Note that when $\eta = 0$ and when for all the action profile $\mathbf{s}_{\mathscr{K}_{21}} \in \mathscr{A}_{\mathscr{K}_{21}}$ of the leaders, the set $\mathrm{BR}^{(0)}_{\mathscr{K}_{22}}(\mathbf{s}_{\mathscr{K}_{21}})$ is unitary, the definition of a Stackelberg equilibrium [13] is obtained. Note also that the η-SE in Def. 4 can be seen as a generalization of the sequential Stackelberg equilibrium in [4] for two-person games and it results in a two-stage η-NE. A first η-NE is established among the leaders under the assumption that the followers are playing their η-best responses and second η-NE is observed among the followers under the assumption that the actions played by the leaders are perfectly known by the followers.

Definition 5 (η-Sequential Equilibrium Region). An achievable rate tuple $(R_1,\ldots,R_K)$ is said to be in the η-SE region of the game $\mathscr{G}_2$, if there exists an action profile $(s_0^{\dagger},\ldots,s_K^{\dagger}) \in \mathscr{A}_0 \times \ldots \times \mathscr{A}_K$ that is an η-SE and such that

$$u_i(s_0^{\dagger},\ldots,s_K^{\dagger}) = R_i, \quad \forall i \in \{1,\ldots,K\}, \tag{19}$$

$$u_0(s_0^{\dagger},\ldots,s_K^{\dagger}) = \sum_{i=1}^{K} R_i. \tag{20}$$

3.1 η-Sequential Equilibrium Region With the Receiver as Leader

Consider the game in which the receiver chooses first a receive configuration (is the leader) and the transmitters adapt their transmit configurations to the choice of the decoding rule in order to maximize their utilities (are the followers), i.e., $\mathscr{K}_{21} = \{0\}$ and $\mathscr{K}_{22} = \{1,\ldots,K\}$. Let $\mathscr{S}_R(\mathrm{SNR}_1,\ldots,\mathrm{SNR}_K)$ denote the η-SE region of the game $\mathscr{G}_2$ when the receiver is the leader and is described by the following theorem.

Theorem 4 (η-SE Region of the Game $\mathscr{G}_2$ With the Receiver as Leader). *The set $\mathscr{S}_R(\mathrm{SNR}_1,\ldots,\mathrm{SNR}_K)$ contains all nonnegative rate tuples $(R_1,\ldots,R_K)$ satisfying*

$$\sum_{i=1}^{K} R_i = \frac{1}{2}\log_2\left(1+\sum_{i=1}^{K}\mathrm{SNR}_i\right). \tag{21}$$

Proof: The proof of Theorem 4 is provided in [1]. ■

3.2 η-Sequential Equilibrium Region With Transmitter i as Leader

Consider the game in which transmitter i, for a given $i \in \{1,\ldots,K\}$, chooses first its transmit configuration and the receiver and the remaining transmitters follow, i.e., $\mathscr{K}_{21} = \{i\}$ and $\mathscr{K}_{22} = \{0,\ldots,K\} \setminus \{i\}$. Let $\eta \geqslant 0$ be arbitrarily small and let $\mathscr{S}_i(\mathrm{SNR}_1,\ldots,\mathrm{SNR}_K)$ denote the η-SE region of the game $\mathscr{G}_2$ when the transmitter i is the leader. This region is described by the following theorem.

Theorem 5 (η-SE Region of the Game $\mathscr{G}_2$ With Transmitter i as Leader). *The set $\mathscr{S}_i(\mathrm{SNR}_1,\ldots,\mathrm{SNR}_K)$ contains all tuples $(R_1,\ldots,R_K) \in \mathbb{R}_+^K$ satisfying*

$$R_i = \frac{1}{2}\log_2(1+\mathrm{SNR}_i), \tag{22}$$

$$\sum_{j=1;j\neq i}^{K} R_j = \frac{1}{2}\log_2\left(1+\sum_{j=1}^{K}\mathrm{SNR}_j\right) - \frac{1}{2}\log_2(1+\mathrm{SNR}_i). \tag{23}$$

Proof: The proof of Theorem 5 is provided in [1]. ■

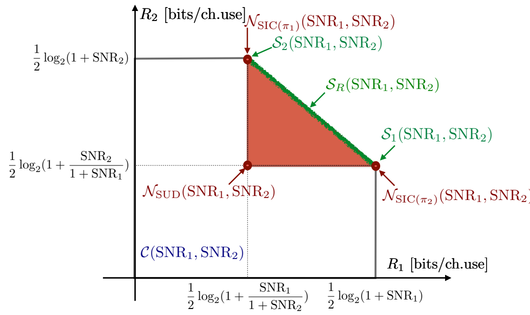

Fig. 1 η-NE and η-SE regions, with $\eta \geqslant 0$ arbitrarily small, for the games $\mathscr{G}_1$ and $\mathscr{G}_2$ in the two-user G-MAC. Here π_i refers to the decoding order in which transmitter i is decoded first, $\forall i \in \{1,2\}$. The η-NE regions in Theorems 1-3 are plotted in red and the η-SE regions in Theorems 4-5 are plotted in green.

4 Example and Observations

In the two-user G-MAC, the regions described in Theorems 1-5 are illustrated in Fig. 1, with the capacity region plotted as a reference.

Existence of η-NE and η-SE: For any nonnegative $\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K$, the existence of an η-NE and an η-SE, with η arbitrarily small, is always guaranteed as the regions in Theorems 1-5 are nonempty. Note in particular that $\mathscr{N}_{\mathrm{SUD}}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K) \neq \emptyset$ and $\mathscr{N}_{\mathrm{SIC}(\pi)}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K) \neq \emptyset$ for any $\pi \in \mathscr{P}_K$. Thus, $\mathscr{N}(\mathrm{SNR}_1, \ldots, \mathrm{SNR}_K) \neq \emptyset$, which ensures the existence of at least one action profile $(\tilde{s}_0, s_1^*, \ldots, s_K^*)$ that is an η-NE, under any fixed receive strategy $\tilde{s}_0$.

Cardinality of η-NE and η-SE: In both games $\mathscr{G}_1$ and $\mathscr{G}_2$, the unicity of a given η-NE or η-SE is not ensured even in the case in which the cardinality of the equilibrium region is one. This is mainly due to the fact that a given rate tuple can be achieved by various transmit and receive configurations. When the set of actions is more restricted, i.e., power control, then the unicity is ensured [8].

Optimality: In $\mathscr{G}_1$, depending on the choice of the receiver, the η-NE rate tuples are not necessarily Pareto-optimal. On the other hand, in $\mathscr{G}_2$, the η-SE rate tuples are Pareto-optimal. This suggests that, under the assumption that the players are able to properly choose the operating equilibrium action profiles for instance via learning algorithms, there is no loss of performance in the decentralized G-MAC case with respect to the fully centralized case. Furthermore, in $\mathscr{G}_2$, the utility of the leader is always maximized, and thus it is always better to move first. Note that the definition of the sequential games in this paper allows for a non-unitary set of leaders. Even though the analysis here is restricted only to the game with unitary sets of leaders, the above statement continues to hold for non-unitary sets of leaders.

Potential Games: The definition of the utilities of the transmitters and the receiver in (5) and (6), respectively, does not impose any restriction on the action sets, which can be complex objects. From this perspective, it is hard to cast the games presented here as potential games. If the actions of the players are restricted for instance to power allocation policies, the results on power allocation games in [8, 2, 7, 6, 12] can be seen as special cases of the results presented in this paper.

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