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Jihad Elnaboulsi

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JIHAD C. ELNABOULSI

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Environmental Regulation and Policy Design: The Impact of the Regulator’s Ecological Conscience on the Tax Setting Process

J. C. Elnaboulsi*

CRESE EA3190, Univ. Bourgogne Franche-Comté, 45 D Av. de l’Observatoire, F-25000 Besançon, France.
E-mail: jihad.elnaboulsi@univ-fcomte.fr

This paper presents an analysis of environmental policy in imperfectly competitive markets. We investigate how environmental taxes should be optimally levied in a pre-commitment policy game and their effects on social welfare. The paper also examines the potential impacts of the regulator’s environmental conscience on policy setting. We start the analysis with a benchmark model where all players are environmentally dirty in the marketplace. We then extend the model to the case in which the market is composed of a mix of dirty and clean strategic players. We show that, in both cases, the regulator must necessarily trade off between regulation of environmental quality and the industry production inefficiency problems. Furthermore, the results show how higher levels of concern for environmental issues outweigh the under taxation problem that arises in order to avoid further reductions in welfare. Finally, we show that the existence of clean players produces positive social externalities. Under an ex ante environmental policy game, higher social welfare outcomes are possible.

Key Words: Environmental Policy, Emissions Tax, Environmental Conscience, Social Welfare, Strategic Behavior, Oligopoly Competition.

Jel: D60, D82, L13, Q28.

Introduction

Much environmental economic research efforts have been put into studying environmental instruments as a mean of improving and protecting the environment. Market-based instruments are attracting increasing attention and provide stronger long term incentives than other environmental policy instruments (OECD, 2006a, 2010). Environmental taxes

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are one of the major features of the market-based instruments in designing environmental policies, and are one of the most widely used and historically experienced instruments. However, there remains a high potential for a wider use of these instruments especially to slow down global warming, provided that they are properly designed (Nordhaus, 2007; Kerkhof et al. 2008).

In theory, environmental taxes have many advantages when compared to other instruments and policies. They allow least-cost abatement, raise governmental revenues in part for fiscal consolidation, provide incentives to polluters to internalize the negative effects of their activities, etc. Therefore, it is well known today that the proper design of environmental taxation does not only depend on the environmental damage caused by economic activities but it also depends on other economic variables and distortions (Bovenberg and de Mooij, 1994).

The structure and the efficiency properties of emissions taxes have been widely analyzed under perfect competition and monopoly. More recently, there has been a growing interest in the analysis of market-based instruments under oligopoly conditions, especially duopolistic markets. Emissions taxes are not immune to market power concerns and to strategic behavior. In addition, under imperfectly competitive conditions, environmental regulation affects products’ environmental features and the performance of the market.

In setting environmental policy, Pigouvian taxation is regarded as a benchmark. Under perfect competition, the desired internalization of the external damages is complete, and the optimal Pigouvian tax is the same for all polluters. Buchanan (1969) was the pioneer in challenging the Pigouvian tax by considering the other polar case, the monopoly. He suggested that the monopolist’s sub-optimal level of output is the source of a basic dilemma for the formulation of policy to regulate externalities. Emissions taxation provides an incentive for pollution abatement, but, at the same time, raises the firm’s marginal cost and thereby induces a reduction in output. Thus, the monopoly power distorts and affects heavily the tax optimality since the monopoly will simply hold down its output. Monopolistic market structure leads to a second best solution due to the loss in efficiency from the contraction in output: the optimal emissions tax is less than marginal external damages. As a result, when public authorities implement an environmental tax, it is well known today that the proper design of environmental taxation does not only depend on the environmental damage caused by economic activities but it also depends on other economic variables and distortions (Bovenberg and de Mooij, 1994).

Since 1991, there has been an increase in the number of countries which implement market-based instruments, and it looks like there is considerable scope for their much wider use. The OECD (2006b, 2010) shows empirically that many market-based instruments, including taxes, have a positive influence on the environmental quality.

Different OECD Member States impose emissions taxes on several industries to fund the cleanup of highly polluted activities such as inactive hazards and to partially subsidize the development of renewable energies. For example, a regulatory fee on lead paint manufacturers imposed by the State of California was used in part to fund government programs that addressed the health risks of children exposed to lead paint.
policy to protect the environment, they have to take into account the structure of the market since the implementation may differ from one market structure to the other and since competition and output level may be affected by environmental policy.

The second best solution pioneered by Buchanan is sustainable in polluting industries performing in more or less imperfectly competitive markets (Ulph, 1996). The optimal taxation has to be modified because environmental taxes are not neutral to the production decisions of oligopolists. On the one hand, the taxation reduces the emissions harm and enhances social welfare. But on the other hand, it raises production costs and consequently reduces the supply of final products. Since oligopoly undersupplies the market even without distortional taxes, the emission levy would contract the output further and diminish welfare in the form of deadweight loss. Optimal environmental taxation needs to trade-off these effects.

Levin (1985) investigates various forms of taxation in order to control pollution in the case of Cournot oligopoly market. The author showed that the industry output will fall due to the tax, but output will also be reallocated across firms as they reach the new equilibrium. Damania (1996) studied the effects of an emissions tax on the incentives for oligopolists to acquire clean technologies. The author showed that there are situations in which the firms may reject the option of acquiring the pollution abatement equipment, even when this lowers their production costs.

In the case of identical firms, it has been demonstrated that the optimal tax rate is less than the marginal external damage of pollution (Lee, 1999; Katsoulacos and Xepapadeas, 1996). In related works, it is found that when the market structure is endogenous, the optimal tax rate under symmetric pollution oligopoly with fixed costs is likely to exceed external damages (Katsoulacos and Xepapadeas, 1995) because free entry may result in an excessive number of firms.

In markets with homogeneous products, environmental regulation induces an over internalization of pollution as polluters may react by reducing their output levels (Moraga-Gonzales and Padron-Fumero, 2002). Thus, environmental policies such as standards (Farzin, 2003) and Pigouvian taxes fall short of marginal environmental damage due to the additional reductions of output levels indirectly induced by the policy (Schoonbeek and de Vries, 2009). Long and Soubeyran (2005) examines asymmetric firms with respect to their production and abatement costs. They considered an oligopolistic pollution game in a differential setting with symmetric information. The optimal tax under cost

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4For an interesting review of the link between environmental regulation and competition, see Heyes (2009). Requate (2005) gives a complete discussion on the performance of environmental instruments in the presence of imperfectly competitive output markets.
asymmetry differs across firms and may facilitate strategic interactions\(^5\).

It is clear today that market structures and strategic behaviors, which are a prevalent real world phenomena, introduce another source of distortion in setting environmental policy and have substantial implications. In this paper, we move a step further by highlighting a way in which the design and implementation of environmental taxes can be improved by considering the potential impacts of the regulator’s environmental conscience on policy design. In order to provide recommendations with respect to optimal taxation, it is important to understand and acknowledge how changes in the level of concern for environmental issues affect the tax setting process. To this end, we consider a precommitment environmental policy game in which the regulator, who usually possesses sovereign authority, occupies a position of leader and commits to a specific emissions tax rule. This implies that the regulator decision once made remains in force for an extended period of time while \(h\)—players respond in the marketplace. This is true in different policy contexts where the regulator cannot change its policy decision periodically but must enforce it for a fixed period of time. We determine the optimal policy in two cases: one in which all players are environmentally dirty; and the second case in which dirty and clean players are active in the marketplace. We compare taxes and welfare levels under these two different outcomes. We then analyze how the results change depending on the various potential levels of the regulator’s ecological conscience. Finally, a comparative static analysis is performed. The intention of these simulations is to provide an illustration of our analytical results since environmental taxation depends closely on some parameters of the model which sometimes are inherently difficult to estimate.

Our setting is not a convenient simplification but may characterize strategic interaction in many types of markets where policy changes require long administrative and legal procedures. Such policy instrument could be potentially applied to regulate some environmental externalities where a complex international negotiation must take place and cannot be readily changed in response to a mix of dirty-clean players’ actions. An international carbon market is a good example of such negotiations. For instance, the energy sector in many countries around the world is at a crossroads. Environmental regulation needs to give appropriate incentives for major investments in the energy sector in order to insure the growing concern of supply and, most importantly, reduce greenhouse-gas emissions. A wide carbon-energy tax could be a key part of a successful regulation at the international level. This policy setting is also relevant in the utility industries where play-

\(^5\)Other market structures have been analyzed in the context of pollution markets where asymmetry has been mainly treated in a static and duopolistic simultaneous manner (Espinola-Arredondo and Munez-Garcia, 2013; Bárcena-Ruíz and Garzón, 2006, 2003; Amir and Nannerup, 2005; Carlsson, 2000; Xepapadeas, 1995; and van der Ploeg and de Zeeuw, 1992).
ers generates a negative externality for an extensive region or country. Thus, such policy instrument could be potentially adjusted to deal with greenhouse gases in the U.S. energy sector, where electricity is produced by firms engaged in a competition à la Cournot (SO$_2$ emissions market or the CO$_2$ emissions in California). Other examples are effluents from some industries polluting surface water such as rivers or lacs. This was the case with the Rhine river between France and Germany or the Elbe river in Germany.

The remainder of the paper is organized as follows. In section 2, we present the model. The optimal environmental policy is examined in section 3. Welfare implications are discussed in section 4. Conclusions are in section 5. Technical details are given in the Appendices.

1. The model

Since taxing emissions tends to exacerbate the preexisting distortion in an imperfect market, the regulator needs to overcome the conflict between regulation of environmental quality and market failures. Policy makers are always facing such situations. In order to provide recommendations with respect to optimal tax design, we formulate an analytically tractable model to examine the interlinkages between environmental regulation and the interindustry production inefficiency problems as closely as possible without questioning the robustness of our results in general settings. We consider positive fees which can be sustained only when the market inefficiency emerging from pollution dominates that stemming from underproduction. We determine the tax rule that induces players to choose the socially-desirable level of output and emissions, and examine the extent to which the market structure as well as the level of concern for environmental issues affect the magnitude of the optimal tax rule.

The assumptions of our model are in line with the related literature (Espinola-Arredondo and Munez-Garcia, 2013; Antelo and Loureiro, 2009(a), 2009(b)). We assume a game in which $h$–players are active in the output market and are engaged in a Cournot competition. The model is flexible and admits several interpretations in terms of policy implication.

Let $Z = \{1, \ldots, h\}$ denotes the set of $h$ players producing a homogeneous final good. The term players can be interpreted as firms or countries. In the first case, a leading application of the model to goods markets is to wholesale energy market. The regulator is then a national agency. In the second case, the model may be applied to the current talks about Kyoto II Agreement. For instance, it is true that we do not know yet the type of regulatory institutions, including policy instruments and participants, that will succeed the post-2012 Kyoto Protocol in the multinational efforts to stabilize Carbon
Emissions and Concentrations in the atmosphere\(^6\). Therefore, the plausible architecture may include an industry-specific Global Carbon Tax such as on the energy sector. In this case, regulation can be performed under the auspices of a Supranational Authority. The social planner is concerned about environmental harm and uses per-unit tax rule to maximize the total welfare. For expositional purposes, we assume that players are facing the following inverse demand function\(^7\):

\[
p(Q) = 1 - Q
\]  

\(p(Q)\) denotes the unit price of the good and \(Q = \sum_{z=1}^{h} q_z\) is the total output of the industry. The assumption of linearity is widely employed in the literature on environmental regulation and yields analytical tractability.

Emissions are given by \(e_z\) and depend on the technology of production used by each player. They can be low or high and thus the corresponding player is considered relatively clean or dirty. A dirty agent is identified as the player that has a low marginal cost of production while a clean agent who pollutes less per unit of output, is the one with high marginal cost of production since it faces higher abatement costs. Emissions are supposed to be proportional to production levels, \(q_z\), in such a way \(e_z(q_z) = q_z, z \in Z\), if player \(z\) uses a dirty technology, and \(e_z(q_z) = \phi_z q_z, z \in Z\), with \(0 \leq \phi_z \leq 1\) if player \(z\) uses a clean technology. In general, \(\phi_z \neq \phi_k\) for \(z \neq k\). Thus, we can write:

\[
e_{z \in Z} = \begin{cases} 
q_z, & \text{if the technology used is dirty} \\
\phi_z q_z, & \text{if the technology used is clean}
\end{cases}
\]  

In the following, it is convenient to assume that \(\phi_{z \in Z} = \phi_{k \in Z} = 0\) for any \(z, k \in Z, z \neq k\) for the sake of simplicity. We also assume that a dirty player \(z\) must pay a tax per unit of emissions, \(\tau_z\), which must be set optimally by the regulator. The environmental damage generated by the production activity is given by the following quadratic convex function:

\[
D = \frac{1}{2} d(E)^2 > 0
\]  

where \(E = \sum_{z \in Z} e_z(q_z)\) represents the aggregate level of emissions or total pollution level. A marginal increase in output, hence, entails a positive and increasing environmental damage, i.e., pollution is convex in output. The positive parameter \(d\) is an exogenous variable.

\(^6\)Nordhaus (2007) shows that price-type instruments, such as internationally harmonized carbon taxes, have major advantages for slowing global warming.

\(^7\)This demand function is the result of a representative consumer maximization problem with quasi-linear utility function.
that captures the regulator’s valuation of the environment or the ecological conscience of the regulator\(^8\). This type of damage function is commonly used in the literature and assumes that this damage is exogenous for consumers: they do not take into account the effect of their consumption decisions on the environment.

We also assume that total costs of production and abatement of each player are linear, \(c_z q_z\). Thus, we suppose that the technology used exhibits constant returns to scale, namely for a given state of the nature the marginal cost of production is a constant equal to \(c_z\). A player \(z \in Z\) can adopt either a low or a high value of \(c_z\). Thus, an agent may employ a dirty production technology harming the environment which implies cheap production costs of the final good, or may adopt a clean environmental technology by acquiring costly equipment to reduce the pollution generated by its production process. Such equipment reduces emissions per unit of output and alters its marginal and average production and abatement costs. Thus, for each \(z\), the marginal cost of production and abatement in each period is given by \(c_z \in \{0, c_z\}\) with

\[
c_z = \begin{cases} 
0, & \text{if the technology used is dirty} \\
c_z, & \text{if the technology used is clean} 
\end{cases}
\]  

(4)

The regulator follows a mechanical but natural rule for setting the optimal environmental policy. The regulator believes that polluters will behave strategically in the marketplace and there is a need to understand and acknowledge the potential impacts and limitations of environmental taxes. To achieve this goal, the regulator maximizes an un-weighted social welfare function which includes consumer surplus (\(CS\)), the players’ expected profits (\(\sum_{z \in Z} \pi_z\)), and the regulator’s total expected revenue generated by emission taxes (\(R\)), minus the value of environmental damages (\(D\)). Thus, we consider the following social welfare function to evaluate and discuss the effects of these distinct environmental measures on welfare grounds:

\[
W = CS + \sum_{z \in Z} \pi_z + R - D
\]  

(5)

where revenues are given by\(^9\):

\[
R = \sum_{z \in Z} \tau z e_z (q_z) = \begin{cases} 
\sum_{z \in Z} \tau z q_z, & \text{if the used technology is dirty} \\
0, & \text{otherwise} 
\end{cases}
\]  

(6)

\(^8\)\(d\) can also represent the marginal social damage of environmental pollution or equivalently the degree of convexity of the damage function.

\(^9\)Administrative costs associated with environmental taxes are supposed to be negligible.
Under the specification adopted here, since the social welfare function incorporates consumers surplus, players’ profits, government revenues, and externalities together, then the optimal environmental tax is able to balance corrections for both negative externalities and sub-optimal production.

2. The optimal environmental policy

Our game examines an oligopolistic structure with $h$ players under full information. Since our main goal is to examine the role of strategic behavior in large Cournot oligopoly and the impact of the regulators’ ecological conscience on the tax setting process, this assumption is quite acceptable\(^{10}\). For instance, the regulators of most OECD member countries have more accurate information about the technology used in some industries such as the energy sector, and the resulting environmental damage (OECD, 2006b). This situation could be considered as a symmetric information context in our setting.

In this section we describe output and environmental taxes in the subgame perfect equilibrium of the game where all players ($h = n + m$) are supposed to be dirty, $z \in Z, c = 0$. The benchmark case will be used for comparison purposes. Then, we consider the fragmentation of the market by allowing a group of $m \leq n$ players to adopt a clean and environment-friendly technology, $\forall m \in Z, c_m > 0$. Since a clean player does not pollute, it will never be taxed whether it competes with a clean or a dirty rival.

2.1. The benchmark case: all players are dirty

In this \textit{ex ante} pollution-tax game we assume that the regulator acts as a Stackelberg leader and polluters are the followers. This implies that the regulatory policy must be put in place at the first stage, and remains static while players respond in the marketplace. In the first stage, before observing the players’ output decisions, the regulator announces and commits to a per-unit environmental tax. Thus, the regulator sets $\tau_z, z \in Z$ in order to maximize $W$. In the second period, for any $z \in Z$ and given $\tau_z$, polluters compete as Cournot rivals and decide the level of production $q_z$ in order to maximize their profits. The quantity produced generates a negative externality $e_z$ that affects the environmental quality.

If all players are dirty, then $\forall z \in Z, c_z = 0$. In the first stage, the regulator sets an environmental tax defined in the following Proposition.

\(^{10}\)As shown in Kurz (2008), asymmetric information in Cournot repeated games leads, under independence properties and the law of large numbers, to full revelation of the true value of outputs. In the limit, all forecasts converge with probability 1 to the true value of production levels. The revelation may also result from direct regulatory oversight, or through other mechanisms such as internal whistleblowers, disclosures by the media or environmental watchdog groups, or simply due to random events that bring information into the public domain.
**Proposition 1**  
In the context of complete information, the optimal environmental tax imposed on each dirty player if its rival is also dirty is given by:

\[ \tau^*_z = \frac{hd - 1}{h(d + 1)} \]  

(7)

**Proof.** See Appendix A. ■

Note that, if all players are dirty and if their strategies are confined to quantity decisions, then the fiscal policy that induces the optimal output level is increasing in \( d \):

\[ \frac{\partial \tau^*_z}{\partial d} (\cdot) = \frac{h(h + 1)(h(d + 1))}{(h(d + 1))^2} > 0 \quad \text{and} \quad \frac{\partial^2 \tau^*_z}{\partial d^2} < 0 \]  

(8)

This simply means that, as the environmental damages become more severe, the equilibrium emissions tax increases. This is evident since \( d \) represents a higher ecological conscience of the regulator. Note that for sufficiently low values of \( d \), i.e. \( d \leq \frac{1}{h} \), the emission tax collapses to zero. We also can show that the optimal environmental tax is clearly an increasing function in \( h \), the number of active players in the marketplace, which is economically intuitive:

\[ \frac{\partial \tau^*_z}{\partial h} (\cdot) = \frac{d + 1}{(h(d + 1))^2} > 0 \quad \text{and} \quad \frac{\partial^2 \tau^*_z}{\partial h^2} < 0 \]  

(9)

**Lemma 1**  
If all players are dirty in the marketplace, then equilibrium oligopoly values are:

\[ Q^*_z = \frac{1}{(1 + d)}; \quad e^*_z = \frac{1}{h(1 + d)}; \quad p^*_z = \frac{d}{(1 + d)}; \quad \pi^*_z = \left( \frac{1}{h(1 + d)} \right)^2 \]

\[ CS^*_z = \frac{1}{2(1 + d)^2}; \quad R^*_z = \frac{hd - 1}{h(1 + d)^2}; \quad D^*_z = \frac{d}{2(1 + d)^2} \]

**Proof.** Straightforward using equation (7). ■

It is easy to see that the industry output level, \( Q^*_z \), is independent of \( h \), the number of dirty players in the market. The intuition behind this finding is that the regulator somehow seems to correct the market structure in order to restore efficiency. Furthermore, the socially optimal output level, \( Q^*_z \), is decreasing in \( d \), which is quite intuitive, given the fact that all players are dirty and the parameter \( d \) measures the regulator’s level of concern for environmental issues. Finally, we can show that, if all players are dirty in the marketplace, then the social planner sets an environmental tax below the marginal
Equation (10) shows that, since the tax reduces environmental damage and induces dirty players to reduce the industry output level in the second stage, the regulator sets an environmental tax below the marginal environmental damage in order to prevent dirty players from reducing their output further. This under-taxation is a standard result in the literature on environmental regulation (Buchanan, 1969).

**Proposition 2** In a complete information framework, if all players are dirty then environmental regulation in a precommitment policy game yields the following social welfare:

\[ W^*_\left(\frac{z}{d}\right) = \frac{d(2h-1) + h}{2h(1+d)^2} \]  

(11)

**Proof.** Straightforward using Lemma 1. ■

Note that the social welfare function is a positive and increasing function in \( h \) which is economically intuitive:

\[ \frac{\partial W^*_\left(\frac{z}{d}\right)}{\partial h} (\cdot) > 0 \quad \text{and} \quad \frac{\partial^2 W^*_\left(\frac{z}{d}\right)}{\partial h^2} (\cdot) = \frac{-d}{h^3(1+d)^2} < 0 \]  

(12)

Furthermore the social welfare function is a decreasing function with the regulator’s ecological conscience:

\[ \frac{\partial W^*_\left(\frac{z}{d}\right)}{\partial d} (\cdot) = \frac{-d(2h-1) + 1}{2h(1+d)^3} < 0 \]  

(13)

The rationale behind this finding lies on the fact that the regulator is maximizing an un-weighted social welfare function. Since the tax is affecting the oligopolistic competition in the market and polluters’ strategic behavior, the industry production level is heavily affected\(^{11}\). Thus the environmental taxes heavily affect consumers’ surplus.

From figure 1 it can be seen how welfare changes as a function of the regulator’s ecological conscience and the number of dirty players in the marketplace. The impact of changes in \( d \) and \( h \) on social welfare corresponds to the economic intuition. In figure 1 we can see that welfare decreases as \( d \) increases. Furthermore, the social welfare in an

\(^{11}\text{Recall that } Q^*_\left(\frac{z}{d}\right) \text{ is a decreasing function with respect to } d : \text{ an increase in environmental stringency causes producers to reduce the industry output level in order to reduce emissions.} \)
oligopolistic market strictly increases as the number of dirty players in the marketplace increases. Therefore, the curve representing $W(z)$ becomes flatter as $h$ increases. This can be explained by the fact that the regulator has an active role in the determination of industry output. The social planner uses taxes not only to curb emissions but also to correct the underproduction that emerges in concentrated market structures. Thus, the regulator has to be more careful with the market structure as this may have important implications for the outcome of environmental regulation in terms of output level, even if environmental taxes unambiguously reduces global emissions levels. This result is consistent with the existing literature.

Figure 1: Social Welfare when all players are dirty for $2 \leq h \leq 50, 0.6 \leq d \leq 5$.

Much more, under the optimal environmental tax defined in (7), as $h \to \infty$, the social welfare function approaches to the sum of consumers’ surplus and the regulator’ revenues:

$$\lim_{h \to \infty} W^*_z = \lim_{h \to \infty} \left( \frac{d(2h - 1) + h}{2h(1+d)^2} \right) = \lim_{h \to \infty} \left( CS^*_z + R^*_z \right) \Rightarrow W^*_z \to \frac{2d + 1}{2(d+1)^2} \quad (14)$$

This result is quite intuitive and lies on the fact that, as $h \to \infty$, the equilibrium converges to the perfect competitive market equilibrium. All players are facing a "tough" competition and are mainly guided by efficiency criteria and their survival in the market. Thus, for a sufficiently large number of players, profits are equal to zero.
2.2. The mixed case: $n$ dirty players and $m$ clean players

The previous discussion considers only homogenous dirty players. Agents heterogeneity leads to important and interesting additional results. In this section, we extend the benchmark case to allow the presence of clean players.

Intuitively, in the context of environmental regulation, the presence of clean players in the marketplace is welfare improving. Furthermore, the tax rate should be smaller in the mixed case than in the benchmark case where all players are dirty.

To prove our claim, we consider the partition of the market into $n$ dirty players and $m$ clean players with $h = n + m$. The regulator’s program will be identical to the benchmark case except that $Z = \{1, \ldots, h\}$ in the first case will be replaced by $Z = N \cup M$ where $N = \{1, \ldots, n\}$ and $M = \{1, \ldots, m\}$. In the following, to ensure tractability of our model, we assume that all players within the two subgroups are symmetric\(^\text{12}\): our goal is to isolate the pure effect of the presence of a group of $m$ clean players while $n$ players are dirty, on the regulator’s tax setting mission, holding all other factors unchanged.

This market fragmentation procedure provides a pure test of the multiplicity problem in the sense that it captures the effect of mixed oligopoly on policy design without changing the scale of the industry. The partition process of the industry clearly imposes a very specific relationship, on the one hand between dirty and clean players, and on the other hand between the social planner and all players in the market.

In fact, environmental taxation influences polluters’ optimal strategies in two opposite ways: the tax makes them internalize their pollutant emissions, thus reducing their output level and the environmental damage that they cause; at the same time, the tax set by the regulator decreases competition in the market, which reduces the production of the industry. As a result, a player in the market is facing the following dilemma: in the presence of emissions taxes, it has a strategic incentive to convince the regulator that it is using a clean technology (having high level costs), and at the same time it has an incentive to be perceived as a dirty player by its rival but productively efficient since this enables the latter to decrease its production while the former increases it. The regulator believing that the player is clean will set a relatively low environmental tax. But, overabatement is expensive and the problem facing the players requires them to balance the desire to appear clean (high costs) against the desire to be efficient in the market and to minimize

\(^{12}\)The analysis of the symmetric case provides a basis for comparison. However, the restriction to symmetry can be relaxed by considering the case where $c_1 \leq \ldots \leq c_z \leq \ldots \leq c_h$. In this case, one may assume that the marginal cost of firm $z$ is given by $c_z = (z-1)c$, where $c$ is a direct measure of cost asymmetry in the industry (Barros, 1998; Straum, 2006), i.e. the cost gap between producing a clean and dirty output. Firms’ asymmetry might result from different environment-friendly production technologies. In order to make sure that the least efficient firm is always active in any possible market structure, it is necessary to introduce an upper bound on $c_z$. 

actual costs (low costs).

Since there exist parameter values such that the optimal strategies are negative, it is then important to define the sufficient magnitude of our parameters in order to ensure that the probability of an equilibrium solution having negative quantities is negligible and all players are going to be active in equilibrium\(^\text{13}\). To this end we make use of the following assumptions about the values of \(c\) and \(d\).

**Assumption 1** The marginal production and abatement cost \(c\) carried by the clean technology is defined by: \(0 < c < \frac{1}{2}\).

**Assumption 2** The regulator’s ecological conscience \(d\) must satisfy: 
\[
d > \frac{1 + mc(n + m + 2)}{n(m + 1)(1 + mc)}.
\]

Under complete information, assumptions (1) and (2) ensure that each dirty player has to pay a non-negative emissions tax regardless of its rivals’ type. Otherwise, if a dirty player is facing a clean producer, then it will pay a negative tax (i.e. a subsidy) and a non-negative tax will only hold in the case of dirty rivals. Assumption (2) also ensures that all players produce positive quantities in equilibrium especially the less productively efficient players (i.e. clean competitors) when they face dirty rivals which are considered as the more productively efficient players. Finally, these assumptions define the admissible values of the regulator’ conscience.

**Proposition 3** If \(d > \overline{d}\), then the optimal per unit of emissions environmental tax imposed on each dirty player in the presence of \(m\) clean players is given by:

\[
\tau^*_i = \frac{nd(m + 1)(1 + mc) - 1 - mc(n + m + 2)}{n + nd(m + 1)^2} \quad \forall i \in N, \forall j \in M
\]  

**Proof.** See Appendix B\(^\text{14}\). ■

Under our assumptions one may verify that the tax on dirty players is positive and clean players remain in the market. If \(d \leq \overline{d}\), then the emissions tax collapses to zero. Otherwise, the social planner will face market failure arising from underproduction unless it uses production subsidies. In addition, we can show that \(\frac{\partial \tau^*_i}{\partial d} (\cdot) > 0\) and \(\frac{\partial^2 \tau^*_i}{\partial d^2} (\cdot) < 0\). This means that the environmental tax is an increasing function with respect to \(d\) which

\(^{13}\)Nonparticipation could be interpreted as bankruptcy. Thus, the regulator can be politically held responsible for forcing firms into bankruptcy. It may even be optimal for the regulator not to induce bankruptcy, as bankruptcy will result in a lower total contribution by firms toward remediation costs, leaving the regulator a larger "orphan share" of the costs to fund itself. Therefore, for very high values of \(d\), one can not disregard the fact that the regulator may find optimal to shut down the market.

\(^{14}\)We can show that \(\forall i \in N, \forall j \in M, \tau^*_i = (nd - 1)(q^*_i) + \left(\frac{m}{n + 1}\right)(q^*_j)\). \(q^*_{i \in N}\) and \(q^*_{j \in M}\) are defined in Lemma 2.
is economically intuitive since the parameter represents the marginal social damage of emissions.

Figure 2 shows the optimal tax in the case of mixed oligopoly for admissible values of parameters $d$ and $c$. From this simulation result, we can observe how the tax changes as a function of the number of clean and dirty players in the marketplace. More specifically, as $m$ increases, the emission tax decreases. The presence of clean players implies an additional product market inefficiency resulting from underproduction which suggests that the regulator decreases the tax rate.

![Figure 2: The optimal tax rule for admissible values of $d$ and $c.$](image)

**Proposition 4** When the regulator’s ecological conscience is sufficiently large, i.e. $d \geq \frac{(n+m+2)}{n(m+1)}$, the optimal emissions tax is increasing with respect to the technology used by clean competitors. Otherwise, the tax rate is decreasing with $c$.

**Proof.** Define the critical value $\tilde{d} \equiv \frac{(n+m+2)}{n(m+1)}, \tilde{d} > d$. Differentiating (15) with respect to $c$ yields:

$$\frac{\partial \tau^*_i}{\partial c} (\cdot) = \frac{m(nd(m+1) - (n + m + 2))}{n + nd (m + 1)^2}$$

(16)

Consider $g(n, m) = nd(m+1) - (n + m + 2)$. For any given $n$ and $m$, the sign of equation (16) is the sign of $g(n, m)$. $g(n, m) \geq 0 \Rightarrow d \geq \tilde{d}$. Thus,

$$\frac{\partial \tau^*_i}{\partial c} (\cdot) = \begin{cases} > 0 & \text{if } d \geq \tilde{d} \\ < 0 & \text{if } \tilde{d} < d < \tilde{d} \end{cases}$$

(17)
This means that, in the presence of \( m \) clean players and \( n \) dirty players, the tax rate paid by dirty players heavily depends on the cost of the technology used by clean rivals.

Consider the first case when the regulator’s ecological conscience is sufficiently large. For \( d \geq \tilde{d} \), environmental taxes increase with \( c \). Under mixed oligopoly clean competitors are less productively efficient in the marketplace and produce less than their dirty rivals. Thus, as a reaction, polluters behave strategically and substantially overproduce, i.e. dirty players are more aggressive in the product market than clean players and competition between players is exacerbated. Such overproduction, however, entails an increase in pollution, thereby inducing the regulator to respond with tougher regulation. In this context, the only effect the regulator considers is the trade-off between production and emissions levels. In order to reduce emissions, the social planner with higher environmental conscience values significantly environmental quality and sets taxes accordingly making dirty players’ overproduction efforts more costly. In this case the government enjoys tax revenue and a reduction in environmental damage.

Now let us look at the case when the marginal damage from pollution is in the range of \( \tilde{d} < d < \tilde{d} \). For intermediate values of \( d \), the tax magnitude decreases with \( c \). Since the efficiency gap between clean and dirty players is small, this case does not require a very large trade-off between the industry output and the emissions levels. Thus, until the degree of damage reaches the critical value, the regulator who is more concerned with the market failure arising from underproduction, avoids overtaxation which entails welfare loss, and sets emissions taxes accordingly.

**Lemma 2** In the context of Cournot mixed oligopoly, if \( d > \tilde{d} \) then equilibrium values in an ex ante environmental policy game are:

| Total Output | \( Q^*_{(i,j)} = \frac{1 + m((m+1)(1-c) + c)}{1 + d(m+1)^2} \) |
| Player i Output | \( q^*_i = \frac{1 + m(c(m+2))}{n + nd(m+1)^2} = e^*_i \) |
| Player j Output | \( q^*_j = \frac{(m+1)((1-c) + c)}{1 + d(m+1)^2} \) |
| Price | \( p^*_{(i,j)} = \frac{(m+1)(1+mc)(d-mc)}{1 + d(m+1)^2} \) |
| Player i Profits | \( \pi^*_i = \frac{(1 + m((m+2)))}{n + nd(m+1)^2} \) |
| Player j Profits | \( \pi^*_j = \frac{(m+1)((1-c) + c)}{1 + d(m+1)^2} \) |
| Consumers’ Surplus | \( CS^*_{(i,j)} = \frac{1}{2} \frac{1 + m((m+1)(1-c) + c)}{1 + d(m+1)^2} \) |
| Revenue | \( R^*_{(i,j)} = \frac{nnd(m+1)(1+mc) - 1 - mc(h+2)}{n + nd(m+1)^2} \) |
| Environmental Damages | \( D^*_{(i,j)} = \frac{d}{2} \frac{1 + mc(m+2)}{1 + d(m+1)^2} \) |
Proof. Straightforward using equation (15).

Note that the industry production level in equilibrium is independent of the number of dirty players. It depends on the marginal production and abatement cost, the regulator’s ecological conscience, and the number of active clean players in the industry. Furthermore, the socially optimal output level is decreasing in the marginal and abatement cost of clean players, decreasing in $d$, and increasing in $m$, under our assumptions, which is economically intuitive.

\[
\frac{\partial Q^*_{(i,j)}}{\partial c} (\cdot) = -\frac{m(d(m + 1) - 1)}{1 + d(m + 1)^2} < 0
\]  

\[
\frac{\partial Q^*_{(i,j)}}{\partial d} (\cdot) = -\frac{(1 + m)(1 + mc(2 + m))}{(1 + d(m + 1)^2)^2} < 0
\]  

\[
\frac{\partial Q^*_{(i,j)}}{\partial m} (\cdot) = \frac{(d(1 - c) - c) \left( d(1 + m)^2 - 1 \right)}{(1 + d(m + 1)^2)^2} > 0
\]  

Figure 3: Industry output level for admissible parameter values $d$ and $c$.

Figure 3 depicts the industry output level for admissible parameter values in the $(m, Q^*_{(i,j)})$-space in the presence of $m$ clean players in the marketplace. It shows that the curve representing the industry output level increases when the number of clean players $m$ increases for different $c$ and $d$ values. It also shows that the curve shifts downward as $c$ increases and $d$ decreases which confirms our results presented below. The increase in the industry output in the marketplace has two opposing effects on economic welfare.
That is, an increase in the output level has a positive impact on the consumer surplus and the regulator’s revenue, but results in environmental damage. In this case, the social planner has to consider these effects and their implications on social welfare. As we will show later, under our assumptions, the positive dominates the negative effect, entailing an overall positive effect on welfare.

We also can show that the individual output levels, $q_i^*$ and $q_j^*$, are decreasing with respect to the number of clean players $m$. Differentiating $q_i^*$ and $q_j^*$ with respect to $m$ yields:

\[
\frac{\partial q_i^*}{\partial m}(\cdot) = -\frac{2\left(q_j^*(\cdot)\right)}{n + nd(m+1)^2} < 0
\]

and,

\[
\frac{\partial q_j^*}{\partial m}(\cdot) = -\frac{\left(d(1+m)^2 - 1\right)\left(d(1-c) - c\right)}{\left(1 + d(m+1)^2\right)^2} < 0
\]

In the absence of environmental regulation, clean players are productively inefficient and they produce less than productively efficient dirty players in the marketplace. In this case, if the number of clean players increases in the industry, then the impact of dirty players’ aggressiveness is very large and the optimal non-cooperative reaction of dirty players is to produce more. Therefore, setting an emission tax reduces polluters’ production levels because environmental tax makes dirty players internalize the damage caused by their emissions, which yields a reduction in their output levels, and hence in the emissions levels. Thus, when the number of clean players increases, the impact of the dirty players’ aggressive behavior decreases: in front of the tax, if dirty players react by increasing their output, they will be heavily taxed depending on their emissions. Hence, to preserve their market power, dirty players strategic reaction is to reduce their production level if the number of clean players increases in the industry.

Figure 4 shows dirty players output and emissions levels for admissible parameters values in the $(m, Q_i^*)$-space. The curve representing $Q_i^*$ decreases with the number of clean players in the marketplace $m$. It shifts upwards when $c$ increases and $d$ decreases which is economically intuitive.

Finally, note that one can verify that, in the market fragmentation case, the regulator
also sets a tax below the marginal damage, \( MD_{(i,j)}^{*} = \frac{n^2d(1+mc(m+2))}{n+nd(m+1)^2} \):

\[
\tau_{i}^{*} - MD_{(i,j)}^{*} = -\frac{d [mc(n(m+2) - (m+1)) + (n - m - 1)]}{1 + d(m+1)^2} - \frac{1 + mc(h+2)}{n + nd(m+1)^2} < 0 \tag{23}
\]

Since market power distorts the industry output level, environmental taxes are set below the marginal damage in order to prevent any further reduction in the output.

![Figure 4: Dirty players output and emissions for admissible parameters values \( d \) and \( c \).](image)

\textbf{Proposition 5} If \( d > \tilde{d} \), under a precommitment policy game, a dirty player is willing to switch to an environment-friendly technology for the following cutoff values:

\[
n \geq n^{*} = \frac{1 + mc(m+2)}{(m+1)(d(1-c) - c)} \tag{24}
\]

\[
c > c^{*} = \frac{nd(m+1) - 1}{n(m+1)(1+d) + m(m+2)} \tag{25}
\]

\textbf{Proof.} A typical dirty player is indifferent about adopting a clean technology if and only if \( \pi_{j}^{*} = \pi_{j}^{*} \). Thus,

\[
\left( \frac{(m+1)((1-c)d-c)}{1 + d(m+1)^2} \right)^2 - \left( \frac{1 + mc(m+2)}{n + nd(m+1)^2} \right)^2 = 0 \tag{26}
\]
Solving the last relation for $n$ and $c$, we obtain the particular threshold values. ■

In the marketplace, any dirty player has a vertical incentive to adopt a pollution-reducing technology in order to avoid the emission tax. At the same time, it also has a horizontal incentive to be a productive-efficient dirty player since this leads a clean player to reduce its output and, consequently, to increase its own output and profit. This proposition states that the vertical incentive outweighs the horizontal one for the given cutoff values.

**Proposition 6** Within the interval of admissible values of the regulator’s ecological conscience, there exists a relatively large cut-off value $d^*$, namely

\[
d^* \equiv \frac{1 + mc(m + 2) + nc(m + 1)}{n(1 - c)(m + 1)} > \bar{d}
\]

for which a dirty player has an incentive to adopt a clean technology.

**Proof.** A typical dirty player is indifferent between adopting the clean technology and staying dirty if and only if the following condition is satisfied: $\pi^*_j = \pi^*_i$. Since $d^* > \bar{d}$ under our assumptions\(^{15}\), solving this condition yields $d^*$. ■

Thus, a dirty player fully adopts the clean technology for any $d \geq d^*$. Therefore, the decision to be environmentally clean depends on:

1. the willingness to pay for the new technology, which is in turn determined by the adoption costs $(c)$;

2. the number of clean players $(m)$ which is in turn determined by the profits on the output market those firms accrue from adopting the clean technology;

3. and finally on the values of the parameter $d$ because an increase in environmental stringency may encourage producers to use environmentally friendly technologies.

**Proposition 7** If $d > \bar{d}$, under an ex ante environmental policy game, dirty players are taxed more when their rivals are also dirty than when they are clean, i.e. $\tau^*_i(z) > \tau^*_i$.

**Proof.** This proposition states that:

\[
\frac{(n + m)d - 1}{(n + m)(d + 1)} > \frac{nd(m + 1)(1 + mc) - 1 - mc(n + m + 2)}{n + nd(m + 1)^2}
\]

Under Assumptions (1) and (2), using (7) and (15) in terms of the equilibrium output levels one can verify that $\Delta \tau = \tau^*_i(z) - \tau^*_i = dQ^*_i(z) - ndq^*_i - \frac{m}{m+1}q^*_j + q^*_i - \frac{Q^*_i(z)}{n} > 0$. The

\(^{15}\)Since the upper bound on $c$ is $\frac{1}{2}$, one can verify that $d^* > \bar{d}$ because $d^* - \bar{d} > 0$ and $\bar{d} > \bar{d}$. \]
sum of the first three terms is positive and the sum of the last two terms is also positive. This completes our proof. ■

This proposition means that a polluter is taxed more when rivals are also dirty than when they are clean. The intuition behind this proposition lies on the fact that, if all players are dirty and are using the same technology, then Cournot competition is “tough” in the marketplace and the impact of dirty players’ aggressiveness is very large, yielding a larger industry production level. As a result of the over-pollution due to quantities competition, the social planner sets higher environmental taxes to reduce environmental damage.

The under-taxation in the mix of clean and dirty player with respect to the benchmark case increases, other things being equal, as the level concern for environmental issues increases, i.e. $\frac{\partial \Delta \tau}{\partial d} > 0$. Intuitively, $\Delta \tau$ increases with $d$ because higher environmental quality significantly increases environmental taxes in order to reduce emissions. Likewise, the under-taxation increases with the cost gap between clean and dirty players for intermediate values of the regulator’s ecological conscience, i.e. $\frac{\partial \Delta \tau}{\partial c} > 0$ if $\overline{d} < d < \tilde{d}$. In other words, if the cost difference between clean and dirty players is high, the regulator sets environmental taxes accordingly, i.e. reduces emissions taxes on dirty players in the mix case because clean players are acquiring costly equipment, and as a consequence, output of clean player is too small. Thus, in order not to harm social welfare any further, the regulator is forced to set lower taxes, so that dirty players do not produce an outcome below their optimal level. Therefore, if $d \geq \tilde{d}$ then $\frac{\partial \Delta \tau}{\partial c} < 0$. This means that, for higher values of $d$, the under-taxation decreases as parameter $c$ increases. This leads to higher taxes on dirty players in the mix case. The regulator is forced to set higher taxes in order to reduce the environmental damage since $d$ is too high.

From figure 5, we can observe that the difference between environmental taxes increases with the parameter $d$ and decreases with $c$. 

In addition, we can show that, in the presence of $m$ clean players facing $n$ dirty players in the marketplace, the output of a clean player is reduced with respect to the case in which it is dirty due to its productive inefficiency. Furthermore, the impact of the dirty players’ aggressive behavior decreases with the magnitude of the emission tax. This yields a reduction in the output of a dirty player in the industry. Finally, the reduction in the aggregate output level implies unambiguously a decrease in the aggregate levels of emissions, $\Delta E$, given by the following relation:

$$\Delta E = \frac{1 + mc(n + m + 2)}{1 + d(m + 1)^2} - \frac{1}{(d + 1)} = -\frac{m(m + 2)(d(1 - c) - c)}{(d + 1)(1 + d(m + 1)^2)} < 0$$ (29)

This leads the social planner to strategically set a lower environmental tax on dirty players in the fragmentation case in order to avoid further distortions and inefficiencies in the marketplace.

As we can see in figure 6, from $\Delta E$, it follows that total output is higher than the industry production level when the used technology by all players is dirty. As a result, consumers’ surplus is larger in the market fragmentation case.

$$\Delta Q^* = Q^*_{[z]} - Q^*_{[i,j]} = \frac{-m(d(1 - c) - c)(d(m + 1) - 1)}{(d + 1)(1 + d(m + 1)^2)} < 0$$ (30)
3. Welfare analysis

To complete our analysis, we need now to figure out the impact of clean players on welfare. To this end, we first characterize the resulting welfare function in the next proposition. Then we examine how the presence of clean players affects welfare.

Proposition 8  In the context of mixed Cournot oligopoly, if $d > \bar{d}$ then ex ante environmental regulation yields welfare benefits of

$$W^*_{(i,j)} = \frac{n(n(1 + 2d) - d)}{2} (q_i^*)^2 + \frac{m(m + 2)}{2(m + 1)} q_j^* \left(2nq_i^* + (m + 1) q_j^* \right)$$

(31)

Proof. Straightforward using Lemma 2 and the definition of $W^*_{(i,j)}$. ■

We expressed the welfare function in term of output levels for ease of presentation and to avoid mathematical complications. Furthermore, $W^*_{(i,j)}$ can be maximized by the direct choice of the equilibrium outputs under some mathematical conditions. $W^*_{(i,j)}$ depends on the magnitude of the admissible parameter values of the model, and on the manner in which clean players interact with dirty players. It also depends on whether the industry output ameliorates or exacerbates the environmental problems.
We can show that $W_{(i,j)}$ is an increasing function with respect to $m$. From figure 7, it can be seen how welfare changes as a function of $m$ clean players and for admissible parameter values $c$ and $d$. As the number of clean players increases, $W_{(i,j)}^*$ increases and becomes flatter for higher values of $m$. Further, an increase in $c$ and a decrease in $d$ shift the curve downward which yields a lower welfare benefits. In fact, when the marginal abatement and production costs increase, clean players are less productively efficient in the marketplace. As a result, the competition in the marketplace is hard and dirty players overproduce which yields higher pollution level above the social optimum level, thereby generating more environmental damage. In the presence of environmental regulation, taxation reduces dirty players’ production thus decreasing environmental damage. The net benefit is a reduction in social welfare. Figure 7 also describes how the welfare benefits from an increase in environmental stringency: an increase in $d$ causes dirty producers to shift to a more environmentally friendly method of production and produces an upward shift in the welfare benefits.

**Proposition 9** *In the presence of $m$ clean players in the marketplace, social welfare is higher than that under the benchmark case, i.e. $\Delta W = W_{(i,j)}^* - W_{(z)}^* > 0$.***
Proof. Recall that

\[
\Delta W = \frac{(2d + 1) \left( \left( nQ_{(z)}^* q_i^* \right)^2 - 1 \right) - nd \left( Q_{(z)}^* q_i^* \right)^2}{2 \left( Q_{(z)}^* \right)^2} + \frac{m(m+2)}{2(m+1)} q_j^* (2nq_i^* + (m+1) q_j^*) + \frac{d}{2(n+m) \left( Q_{(z)}^* \right)^2} > 0. \tag{32}
\]

A simple comparison of welfare under both cases yields the results of Proposition 9. ■

Figure 8 gives a graphical representation of \( \Delta W \). It depicts welfare benefits in the \((m, \Delta W)\)-space for \( n = 30 \) and for admissible parameter values \( d \) and \( c \). From the numerical simulations result, we observe that \( \Delta W > 0 \). Under emission taxes, the presence of \( m \) clean players in the industry implies highest social welfare, since environmental regulation induces the socially optimal output. Further, as \( c \) increases and \( d \) decreases, the curve representing \( \Delta W \) shifts downward which is economically intuitive. Not only that but, as the number of clean players increases in the marketplace and approaches the number of dirty players in the industry, the curve becomes flatter for a given value of \( c \) and \( d \). This result suggests that, even if players are symmetric in their cost structure within each subgroup, a regulator can not underestimate the welfare benefit of regulation.

Figure 8: \( \Delta W \) for \( 1 \leq m \leq 30 \), \( n = 30 \), and for admissible parameters values \( d \) and \( c \).
Conclusion and Policy Implications

If we accept that we inhabit a world with serious and severe environmental problems, then changes that affect those problems have to be undertaken. The point of environmental regulation and of designing an environmental tax system is to accomplish deep and structural changes in the economic and ecological behavior of individuals, households, firms, and institutions in order to curtail environmentally and ecologically undesirable effects. Hence, it is reasonable that policy makers view environmental related issues through the lens of environmental economists. That conclusion reinforces the argument in favor of the use of market-based instruments in environmental regulation. The environmental effectiveness and economic efficiency of emissions taxes could be improved further if they are well designed and implemented.

Choosing the appropriate environmental policy is a key part of successful regulation. Environmental taxation has been broadly analyzed in the literature on environmental economics. Regulators often face imperfect competition and strategic behaviors. Although many authors analyzed emission taxes in mixed duopolistic markets, there are few works analyzing environmental policy in large oligopoly market structure and the impact of the regulator’s level of concern for environmental issues on the tax setting process.

Our paper deals with market power and strategic behavior in a Cournot-type environmental policy game. We kept the formalism down to a minimum focusing on simple ideas and concepts. Obviously, our results are in part specific to our setting but they do raise the issue that is not evident how the regulator’s environmental concern affect the tax setting process in a mix of clean and dirty strategic players. An interesting application of the model is to an international carbon-energy market with countries as players. The paper shows that emission taxes strongly depend on the market structure and are a useful instrument for improving economic welfare. It also shows that higher taxes lower outputs and impact welfare through two channels, pollution reduction and strategic behaviors. The optimal taxation reduces the pollution harm and enhances social welfare.

In the benchmark case, our results are closely related to those in the literature on environmental taxation. Therefore, important changes appear in the mixed oligopoly case. We show that the presence of clean players in the market is welfare improving. Furthermore, the tax design issues can have significant effects on the strategic decision of dirty players and rely on a wide range of considerations in the presence of clean players. We also highlight the incentives created by the use of emissions taxes to adopt environmentally friendly technology by dirty players. Comparative statics have been performed in order to illustrate our findings. The results obtained and the conclusions drawn for this model are valid for the entire range of the parameters defining the ex ante policy game.
We are aware that our results are defined within the context of a simplified model that is general in some respects, but they obviously depend on other less general assumptions. For example, one of our simplifications comprised the normalization of production costs to zero. Although this variable does not affect the optimal tax rates in this setting of the model, it can easily be included. Our analysis can also be refined to include the choice of the technology before taking any production decision. It can be extended to include asymmetric information between the social planner and both, clean and dirty players in the market. Interestingly, our analysis can easily be adapted to deal with other cases which can shed more light on the optimal environmental taxes in large industrial markets (such as mixed Bertrand oligopoly with differentiated products, Hotelling spatial competition, and even Stackelberg competition). It would be interesting to explore these extensions in the future.
Appendix A: The benchmark case

We study a two-stage game where, in the first stage, the social planner selects an emission fee \( \tau_z \) and then all players respond by choosing an output level \( q_z, \forall z \in Z \). We assume that all players are risk neutral and are profit-maximizers. At the second period, each player \( z, \forall z = 1, \ldots, h \), chooses \( q_z \) in order to maximize its profits. Specifically, a \( z \) player solves:

$$
\max_{q_z} \pi_{z, z \in Z} \equiv (1 - c_z - q_z - q_{-z}) q_z - \tau_z e_z
$$

(A1)

where \( q_{-z} = \sum_{k \neq z} q_k \) is the combined output of all players except firm \( z \).

In the case all players are supposed to be dirty, i.e. the cost gap between players is \( c_z = 0 \), the profit function for any \( z \) becomes \( \pi_{z, z \in Z} \equiv (1 - q_z - q_{-z}) q_z - \tau_z e_z \). Since we simplified the problem for expositional purposes by assuming that \( q_z = e_z \), then any \( z \) player has to:

$$
\max_{q_z} \pi_z \equiv (1 - q_z - q_{-z} - \tau_z) q_z
$$

(A2)

which leads to the following first-order conditions defining the equilibrium levels of outputs:

$$
\frac{\partial \pi_z}{\partial q_z}(\cdot) = 1 - 2q_z - q_{-z} - \tau_z = 0
$$

(A3)

The second-order conditions are satisfied:

$$
\frac{\partial^2 \pi_z}{\partial q_z^2}(\cdot) = -2 < 0
$$

(A4)

The first-order conditions give the optimal reaction functions for all players. \( \forall z = 1, \ldots, h \):

$$
q_z = \frac{1 - q_{-z} - \tau_z}{2} \Rightarrow q_1 + \cdots + 2q_z + \cdots + q_h = 1 - \tau_z
$$

(A5)

We are interested in the equilibrium in which the outcomes of all dirty players are symmetric. In this case, \( \forall z = 1, \ldots, h, (c_1, \ldots, c_z, \ldots, c_h) = (0, \ldots, 0, \ldots, 0) \), then \( q_1 = \cdots = q_z = \cdots = q_h \). Adding-up the first-order conditions, we obtain an expression for equilibrium total output:

$$
Q(z) = \frac{h(1 - \tau_z)}{h + 1}
$$

(A6)
and \( \forall z = 1, \ldots, h, \)

\[ q_z = \frac{(1 - \tau_z)}{h + 1} \quad (A7) \]

The last two equations show that raising environmental tax reduces players’ individual production levels and the output of the industry, which causes their emissions to decrease with the tax. It is easy now to calculate the equilibrium price:

\[ p(z) = \frac{1 + h\tau_z}{h + 1} \quad (A8) \]

Finally, \( \forall z = 1, \ldots, h, \)

\[ \pi(z) = (q_z)^2 = \left( \frac{1 - \tau_z}{h + 1} \right)^2 \quad (A9) \]

Since the product is homogeneous, it is straightforward to compute equilibrium consumers’ surplus:

\[ CS(z) = \frac{1}{2}Q^2 = \frac{1}{2} \left( \frac{h(1 - \tau_z)}{h + 1} \right)^2 \quad (A10) \]

Adding-up consumers’ surplus and environmental damages generated by the production activity, we obtain:

\[ CS(z) - D(z) = \frac{1}{2} (1 - d) \left( \frac{h(1 - \tau_z)}{h + 1} \right)^2 \quad (A11) \]

Let \( \Pi = \sum_{z, z \in Z} \pi(z). \) Since all players are dirty, then,

\[ \Pi(z) = h \left( \frac{1 - \tau_z}{h + 1} \right)^2 \quad (A12) \]

Finally, the regulator total expected revenue generated by pollution taxes can be written:

\[ R(z) = \sum_{z, z \in Z} \tau_z \left( \frac{1 - \tau_z}{h + 1} \right) \quad (A13) \]
At the first stage, the regulator sets the tax rule that maximizes the social welfare function given by:

$$\max_{\tau_z} W(\tau_z) \equiv \frac{1}{2} (1-d) \left( \frac{h(1-\tau_z)}{h+1} \right)^2 + h \left( \frac{1-\tau_z}{h+1} \right)^2 + h\tau_z \left( \frac{1-\tau_z}{h+1} \right)$$ (A14)

Differentiating the social welfare function given in (A14) with respect to the tax gives the solution of this optimization problem which yields the result presented in Proposition 1:

$$\frac{\partial W(\tau_z)}{\partial \tau_z}(\cdot) = -\frac{h(1+hd(-1+\tau_z)+h\tau_z)}{(h+1)^2} = 0 \Rightarrow \tau_z^* = \frac{hd-1}{h(d+1)}$$

It is easy to verify that the second-order conditions are satisfied, i.e. $\frac{\partial^2 W(\tau_z)}{\partial \tau_z^2}(\cdot) = -\frac{h^2(d+1)}{(h+1)^2} < 0$.

Appendix B: The market fragmentation case

Under the market fragmentation adopted in the text, $n$ dirty players and $m$ clean players with $h = n + m$, as previously defined, $\forall i = 1, \ldots, n, c_{i,i\neq j} = 0$ and $\forall j = 1, \ldots, m, c_{j,j\neq j} = c$. Since we assumed that $e_i = q_i$ then a player $i, i \in N$, has to:

$$\max_{q_i} \pi_i, i \in N \equiv \left( 1 - q_i - q_{-i} - \sum_{j=1}^{m} q_j - \tau_i \right) q_i$$ (B1)

where $q_{-i} = \sum_{l \neq i, l \in N} q_l$ is the combined output of all dirty players except player $i$. This leads to the following first-order conditions defining the equilibrium levels of outputs for any $i \in N$:

$$\frac{\partial \pi_i}{\partial q_i}(\cdot) = 0 \Rightarrow 1 - 2q_i - q_{-i} - \sum_{j=1}^{m} q_j - \tau_i = 0$$ (B2)

The second-order conditions are satisfied:

$$\frac{\partial^2 \pi_i}{\partial q_i^2}(\cdot) = -2 < 0$$ (B3)

The first-order conditions give the optimal reaction function for each $i$ player:

$$q_1 + \cdots + 2q_i + \cdots + q_j + \cdots + q_h = 1 - \tau_i, \forall i = 1, \ldots, n$$ (B4)

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16It is convenient to solve the welfare maximization problem by using the equilibrium output $q_z$ as choice variables, and afterward infer the optimal tax. The two methods yield the same solution.
Now we consider players using clean technology. \( \forall j = 1, \ldots, m, i \neq j \) each player \( j \) has to:

\[
\max_{q_j} \pi_{j, j \in M} \equiv \left( 1 - q_j - q_{-j} - \sum_{i=1}^{n} q_i - c \right) q_j
\]

(B5)

where \( q_{-j} = \sum_{f \neq j} q_f \) is the combined output of all clean players except player \( j \). The first-order conditions to maximize this profit function are:

\[
\frac{\partial \pi_j}{\partial q_j} (\cdot) = 0 \Rightarrow 1 - 2q_j - q_{-j} - \sum_{i=1}^{n} q_i - c = 0
\]

(B6)

Again, the second-order conditions for a maximum is satisfied, i.e. \( \frac{\partial^2 \pi_j}{\partial q_j^2} (\cdot) = -2 < 0 \). The first-order conditions give the optimal reaction function for each \( j \) firm. \( \forall j = 1, \ldots, m \):

\[
q_1 + \cdots + q_i + \cdots + 2q_j + \cdots + q_h = 1 - c
\]

(B7)

Adding-up the first-order conditions for \( h = n + m \), we obtain an expression for equilibrium total output:

\[
(h + 1) (q_1 + \cdots + q_z + \cdots q_h) = \left( n - \sum_{i=1}^{n} \tau_i \right) + \left( m - \sum_{j=1}^{m} c \right)
\]

(B8)

Since we are interested in the equilibrium in which the outcomes of all players in each subset are symmetric, then the last relation gives the industry output level:

\[
Q_{(i, j)} = \sum_{i=1}^{n} q_i + \sum_{j=1}^{m} q_j = \frac{h - n \tau_i - mc}{h + 1}
\]

(B9)

It is easy now to calculate equilibrium price:

\[
p_{(i, j)} = 1 - Q_{(i, j)} = \frac{1 + n \tau_i + mc}{h + 1}
\]

(B10)

Using the first order conditions for any \( i \in N \) and \( j \in M, i \neq j \), individual output levels in equilibrium are given by:

\[
q_i = \frac{1 - (m + 1) \tau_i + mc}{h + 1}
\]

(B11)
and,  
\[ q_j = \frac{1 - (n + 1) c + n \tau_i}{h + 1} \]  

Finally, \( \forall i \in N, j \in M \), profits are given by the following expressions:

\[ \pi_i = \left( \frac{1 - (m + 1) \tau_i + mc}{h + 1} \right)^2 \]

(B13)

and,

\[ \pi_j = \left( \frac{1 - (n + 1) c + n \tau_i}{h + 1} \right)^2 \]

(B14)

Since the product is homogeneous, it is straightforward to compute equilibrium consumers’ surplus,

\[ CS_{(i,j)} = \frac{1}{2} \left( \frac{h - n \tau_i - mc}{h + 1} \right)^2 \]

(B15)

Let \( \Pi_i = \sum_{i=1}^{n} \pi_i \). Then, the aggregate profits for all dirty players are:

\[ \Pi_i = n \left( \frac{1 - (m + 1) \tau_i + mc}{h + 1} \right)^2 \]

(B16)

And, if \( \Pi_j = \sum_{j=1}^{m} \pi_j \) then the aggregate profits for all clean players are:

\[ \Pi_j = m \left( \frac{1 - (n + 1) c + n \tau_i}{h + 1} \right)^2 \]

(B17)

The environmental damage generated by the production activity of dirty players is given by:

\[ D_{(i,j)} = \frac{n^2 d}{2} \left( \frac{1 - (m + 1) \tau_i + mc}{h + 1} \right)^2 \]

(B18)

Finally, the government total expected revenue generated by pollution taxes, \( R_{(i,j)} \), can be written:

\[ R_{(i,j)} = n \tau_i \left( \frac{1 - (m + 1) \tau_i + mc}{h + 1} \right) \]

(B19)
At the first stage, the regulator has to determine the pollution taxes that maximize the social welfare function defined in (5).

\[
\max_{\tau_i} W_{(i,j)} \equiv \frac{1}{2} \left( \frac{h - n\tau_i - mc}{h + 1} \right)^2 + n \left( \frac{1 - (m + 1) \tau_i + mc}{h + 1} \right)^2 + m \left( \frac{1 - (n + 1) c + n\tau_i}{h + 1} \right)^2 \\
- \frac{n^2}{2} d \left( \frac{1 - (m + 1) \tau_i + mc}{h + 1} \right)^2 + n\tau_i \left( \frac{1 - (m + 1) \tau_i + mc}{h + 1} \right) \\
\text{(B20)}
\]

Differentiating the social welfare function with respect to the tax yields the optimal environmental tax given in Proposition 3:

\[
\frac{\partial W_{(i,j)}}{\partial \tau_i} = \frac{n}{(h + 1)^2} \left[ -n\tau_i (1 + d (1 + m)^2) + nd (1 + m) (1 + mc) - 1 - mc (m + n + 2) \right] = 0
\]

\[
\Rightarrow \tau_i^* = \frac{nd (m + 1) (1 + mc) - 1 - mc (n + m + 2)}{n + nd (m + 1)^2}
\]

Again, the second-order conditions are satisfied, i.e. \( \frac{\partial^2 W_{(i,j)}}{\partial \tau_i^2} = -\frac{n^2 (1+d(1+m)^2)}{(h+1)^2} < 0. \)
References


