The Derivation of the Fine Structure Constant

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Abstract

With an exponential approach for the mass scales of the observable universe including the gravitational constant as a time dependent parameter, the Planck mass can be generalized yielding a mass model where elementary particle masses are related and accurately calculated [1]. As a premise the observable universe evolves as a black hole, i.e. from the initial singularity it fulfiled the conditions of the Schwarzschild mass radius relation. With these assumptions and a logarithmic potential for elementary particle constituents the fine structure constant α is derived as α=0,007297359 (0,007297353, $\Delta\alpha/\alpha=8\cdot 10^{-7}$), the measured value and the relative deviation of both are in brackets. The result is independent of the fundamental physical constants defining α.

Keywords: Hypothetical particle physics models, composite models, cosmology

1. Introduction

With the approach of constructing an exponential model that covers the observed mass scales of the universe, the proton, neutron and tau masses were calculated with a precision of up to 8 decimal digits [1]. The results for the neutron and tau masses are within the estimated standard deviation of the experimental values. As a premise the gravitational constant G is assumed to be a function of time G(t) proportional to 1/t and a set of mass dependend integers were observed and introduced. The calculation of the proton mass includes a mass contribution that enables the derivation of α without the elementary charge e, Planck´s constant ħ and the speed of light c by introducing a logarithmic potential for elementary particle´s confined constituent masses. In the following the basic approaches and results of [1] needed to derive α are presented.

Within a space of a given radius the smallest rest mass greater zero that can be observed corresponds to a compton wavelength that is of the order of that radius. In this model the observable mass of the universe m_U and the such defined smallest mass m_γ inside are fitted by an exponential approach. The largest possible wavelenght of a particle with mass m_γ thus is of the order of the radius of the universe, which is proportional to m_U. Assuming that the reduced compton wavelenght r_C is equal to the gravitational radius r_G of the universe, then

$$r_c = \frac{\hbar}{m_\gamma c} = \frac{G m_u}{c^2} = r_G\ ,\ \ m_{pl} = \sqrt{\frac{\hbar c}{G}} \quad \rightarrow \quad m_\gamma \cdot m_u = m_{pl}^2 \tag{1.0}$$

where G is the gravitational constant and m_{pl} is the Planck mass. The observable horizon of the universe then is the Schwarzschild radius $r_S=2r_G$. The results for m_γ and m_U are:

$$m_\gamma(t) = \frac{m_e m_p^2}{m_{pl}^2\big(G(t)\big)} \tag{1.1a}$$

$$m_u(t) = \frac{m_{pl}^4\big(G(t)\big)}{m_e m_p^2} \tag{1.1b}$$

where the gravitational constant G is a time dependent parameter that was larger in the past and m_e, m_p are the electron and proton masses. Inserting the present value for G [2] in Eq. (1.1b) results in accurate values based on latest measurements of the mass and age of the observable universe [1]. Since m_U is supposed to fulfil Eq. (1.0), the model universe evolves as a black hole. When m_U is reduced, presenting earlier stages of the universe, there is eventually a condition reached defined by $m_\gamma=m_U$, which is referred to as the state of equilibrium. Since the smallest observable mass m_γ cannot exceed the mass of the universe m_U, it is the initial state of the model. At equilibrium the gravitational and electromagnetic interaction converge to unification [1]. This state is given by:

$$m_E = m_{pl}(G_E) = m_u = m_\gamma = \left(m_e m_p^2\right)^{\frac{1}{3}} \tag{1.2}$$

with

$$m_E = 1{,}365929 \cdot 10^{-28}\,kg$$

The equilibrium mass m_E is smaller than the muon, between the electron and proton masses. To calculate elementary particle masses the equilibrium mass i.e. Planck mass m_E at equilibrium is generalized:

$$m'(i,j) = \left(m_i m_j^2\right)^{\frac{1}{3}} \tag{1.3}$$

where every particle combination i, j is assigned a mass m´(i,j). Then in the probed mass range, from electron to proton and the tau, particle masses are a combination of other particle masses, for example:

$$m_{K0} = (m_{\pi+} m_n^2)^{\frac{1}{3}} = 8{,}871 \cdot 10^{-28}\,kg \quad (8{,}871 \cdot 10^{-28}\,kg)$$

with the measured value for the kaon K^0 in brackets and $i=\pi^+$ (pion), j=n (neutron). Thus they are generalized Planck masses. For particular equilibrium masses which are a function of the electron mass, quantum structures i.e. integer mass numbers can be observed which allow to describe particle masses and mass relations between particles. For example:

$$\frac{\left(m_e m_p^2\right)^{\frac{1}{3}}}{m_e} = \left(\frac{m_p}{m_e}\right)^{\frac{2}{3}} = 149{,}947 \approx 150$$

where the equilibrium mass is expressed in units of the electron mass. The proton, neutron and tau masses then can be derived as a function of α and the electron mass m_e. Thus they are also a function of α and m´(i,j) since m_e=m´(e,e). The formular derived and solved for the proton mass then is:

$$m_b = m_c\left(1 + \frac{\alpha}{2} + n_3 \xi' \alpha^2\right) = m_c\left(1 + \frac{\alpha}{2} + \frac{3}{5n_3}\frac{m_e}{m_c}\right), \quad m_b = 112 m_e\,, \quad n_3 = 10 \tag{1.4}$$

where ξ' is a constant and m_b is a constituent mass of the equilibrium mass m_E in Eq. (1.2) with

$$m_c = \left(\frac{100}{243} m_e m_p^2\right)^{\frac{1}{3}} \tag{1.5a}$$

yielding

$$m_p = c_m (2+\alpha)^{-\frac{3}{2}} m_e = 1{,}672621 \cdot 10^{-27} kg \quad (1{,}672622 \cdot 10^{-27} kg) \tag{1.5b}$$

with

$$c_m = \left(\frac{243}{100}\right)^{\frac{1}{2}} \left(2\left(112 - \frac{3}{50}\right)\right)^{\frac{3}{2}} = 5{,}2218703 \cdot 10^3$$

where the measured value is in brackets [2]. Thus m_b and m_c both are a function of the equilibrium mass m_E and therefore of a mass $m'(i,j)$ and α, since the proton mass is a function of α. The last term for m_b in Eq. (1.4) can be written as

$$m_c\, \xi' n_3 \alpha^2 = \frac{3}{5n_3} m_e \tag{1.6}$$

with

$$\xi' = 1{,}0102\,, \quad \alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

where ε_0 is the electric constant, e the elementary charge, ħ Planck´s constant and c the speed of light. The terms in Eq. (1.6) are a second order mass contribution to m_b and the clue for deriving α.

2. Derivation of the Fine Structure Constant

From Eq. (1.6) it follows

$$m_e = \frac{5}{3} m_c\, \xi' n_3^2 \alpha^2 \tag{2.0a}$$

Since the proton, neutron and tau masses resp. are a function of the smaller mass m_e, it could be considered as a constituent mass itsself. This is generalized by assuming that Eq. (2.0a) in principle is true for any mass contribution of a constituent mass m_0 that like m_b and m_c in Eqs. (1.4) and (1.5a) resp. can be expressed as a function F_1 of α and $m'(i_1,j_1)$ as defined in Eq. (1.3). Then Eq. (2.0a) can be written as

$$m_0 = b m_c n_3^2 \alpha^2 \quad \rightarrow \quad m_0 = n_3^2 F_1\big(\alpha, m'(i_1, j_1)\big) \tag{2.0b}$$

were b is a constant. With Eq. (1.5a) and since the proton mass is a function of α it is assumed that m_c is a function F_2 of α and $m'(i_2,j_2)$:

$$m_c = n_3^2 F_2\big(\alpha, m'(i_2, j_2)\big) \tag{2.0c}$$

Inserting this into Eq. (2.0b) yields

$$m_0 = b n_3^4 F_2\big(\alpha, m'(i_2, j_2)\big)\alpha^2 \tag{2.1}$$

But the constituent mass m_0 can be expressed as a Planck mass for a specific G(t):

$$m_0 = m_{pl}\big(G(t)\big) = \sqrt{\frac{\hbar c}{G(t)}}$$

Then inserting m_0 from Eq. (2.1) as $m_{pl}(G)$ into Eq. (1.1a) results in the smallest possible rest mass greater zero that can be observed inside the universe, which here corresponds to an early stage where it´s mass m_U was in the mass range of elementary particles, yielding

$$2m_\gamma = \frac{2 m_e m_p^2}{b^2 n_3^8 F_2^2 \alpha^4} \tag{2.2}$$

where the factor 2 is applied because two charged particles with masses m_γ each have to be considered to enable electromagnetic interaction. The particle with mass m_γ is assumed to have an internal structure itsself consisting of a two dimensional harmonic oscillator i.e. confined constituent with zero rest mass moving with constant energy pc and angular frequency ω in the plane of motion within a sphere of radius R. The force F needed to counteract the centrifugal force $F_c \propto c^2/R$ on the constituent is assumed to act in the plane of motion only and to be proportional or equal to pc/R, then

$$F = F_c = \frac{s_1}{R}$$

where s_1 is a parameter independent of R. The energy needed to confine the constituent is

$$E = \int \frac{s_1}{R} dR = s_1 \ln \frac{R}{R_a} \tag{2.3}$$

where R_a is the integration constant. With a ground state of radius R_0 and

$$\omega \propto \frac{c}{R_0}$$

then for the energy of an harmonic oscillator it follows

$$E = (n+1)\hbar\omega = s_2(n+1)\frac{\hbar c}{R_0}\,, \quad n = 0,1,2\,... \tag{2.4}$$

where s_2 is the proportional constant and independent of R. Choosing the integration constant by setting Eq. (2.3) equal to Eq. (2.4) results in

$$\ln \frac{R_a}{R(n)} = -(n+1)\frac{R_a}{R_0}\,, \qquad R_a = \frac{s_2}{s_1}\hbar c \tag{2.5}$$

For n=0 the value for R(n) is set to R_0, allowing to determine the ratio

$$lnf = -f\,, \qquad f = \frac{R_a}{R_0} = 0{,}5671433 \tag{2.6}$$

Then with Eqs. (2.5) and (2.6)

$$R(n) = R_a e^{(n+1)f} = R_a f^{-(n+1)}$$

Thus successive R(n) are seperated by a factor *f*. R(n) is assumed to be proportional or inverse proportional to it´s mass content m(n) as in Eq. (1.0), thus successive m(n) are seperated by a factor *f* resp. 1/*f*. Now Eq. (2.2) can be solved for α

$$\alpha^4 = \frac{1}{n_3^8}\frac{m_h}{2m_\gamma}\,, \qquad m_h = \frac{2m_e m_p^2}{b^2 F_2^2} \tag{2.7}$$

For m_h/m_γ corresponding to the ratio of two successive masses m(n) seperated by a factor *f* it follows

$$\alpha = \frac{1}{n_3^2}\left(\frac{f}{2}\right)^{\frac{1}{4}}\,, \qquad f = \frac{m_h}{m_\gamma} \tag{2.8}$$

Inserting *f* from Eq. (2.6) and n_3 from Eq. (1.4) yields

$$\alpha = \frac{1}{100}\left(\frac{f}{2}\right)^{\frac{1}{4}} = 0{,}007297359 \quad (0{,}007297353) \tag{2.9}$$

where the measured value is in brackets [2]. This derivation with an accuracy of nine decimal digits is based on exponential scaling and generalized equilibrium masses introduced in [1]. The result is independent of the fundamental physical constants ε_0, e, ħ and c.

Conclusion

A model assuming the observable universe is an evolving black hole with a gravitational constant G(t) inverse proportional to it´s age enables the calculation of particle masses and the derivation of the fine structure constant α. It also provides a unification approach for gravitational and electromagnetic interaction. The significantly larger G(t) in the early universe supports the assumption that in the evolution of galaxies black holes formed before stars. The fine structure constant α can be derived with an exponential approach for the mass scales of the observable universe and a logarithmic potential for elementary particle´s constituent masses. The result depends solely on an integer from the proton mass derivation´s quantization approach and on the solution of the equation $\ln x=-x$. Since it is independent of the constants ε_0, e, ħ and c, supposably at least one of these parameters is not a fundamental physical constant and depends on the other three.

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