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On the Calculation of Elementary Particle Masses

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Abstract

With an exponential model including the gravitational constant as a time dependent parameter a mass function is
derived where elementary particle masses are combined and related. The proton mass \( m_p \) is derived as a function
of the electron mass \( m_e \) and fine structure constant \( \alpha \) as \( m_p = 1.672621 \times 10^{-27} \) kg (1.672622 \( \times 10^{-27} \) kg, \( \Delta m/m = 6 \times 10^{-7} \)),
the measured mass and the relative deviation of both are in brackets. The neutron mass then is calculated as a
function of \( m_p \) and \( \alpha \) as \( m_n = 1.67492745 \times 10^{-27} \) kg (1.67492747 \( \times 10^{-27} \) kg, \( \Delta m/m \approx 10^{-8} \)). The tau mass is expressed
as a function of \( m_p \) and \( m_e \) resulting in \( m_\tau = \sqrt[1]{\frac{m_p}{m_e}} \cdot m_p = m_{pl}^2 \) (\( \times 10^{-27} \) kg, \( \Delta m/m \approx 10^{-5} \)). The results for the
neutron and tau are within the estimated standard deviation of the experimental values.

Keywords: Hypothetical particle physics models , Composite models, Cosmology

1. Introduction

The precise prediction and calculation of elementary particle masses still is not covered by any theory, e.g. the
standard model. Even though the mechanisms that provide the specific particle masses are thought to be under-
stood and confirmed by the discovery of the Higgs boson, a clue to why elementary particles have their specific
mass values would be of great importance for our understanding of quantum objects and matter. A correlated
unresolved issue is why and how nature offers such a wide scale of masses from neutrinos to galaxies and the
observable universe itself. This paper attempts to provide an approach to elementary particle masses by
constructing an exponential model that covers the mass scale of the observable universe and allows to calculate
the proton and neutron masses with an accuracy of 6 resp. 8 decimal digits. As a premise the gravitational
constant \( G \) is assumed to be a function of time \( G(t) \) proportional to \( 1/t \), compatible with Dirac´s conclusions
from the Large Number Hypothesis LNH [1]. As a result a set of mass dependent integers is observed and int-
roduced. The relation of particle masses and integer values has already been pointed out by the formula of Koide
[2] for lepton masses. It is compared with the result for the tau mass calculated with the approach of this paper.

2. The Exponential Model

Estimations based on measurements of the mass and radius of the observable universe result in a ratio of these
values that about matches the conditions of a black hole. Here the mass relates to the so far observable ordinary
baryonic matter, thus not including dark matter nor dark energy. Within a space of a given radius the smallest
rest mass greater zero that can be observed corresponds to a compton wavelength that is of the order of that ra-
dius. In this model the observable mass \( m_U \) of the universe and the such defined smallest mass \( m_\gamma \) inside are
fitted by an exponential approach. The largest possible wavelength of a particle with mass \( m_\gamma \) thus is of the order of that ra-
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fitted by an exponential approach. The largest possible wavelength of a particle with mass \( m_\gamma \) thus is of the order of that ra-

\[ r_c = \frac{h}{m_p c} = \frac{G m_U}{c^2} = r_G \]  
\[ m_{pl} = \frac{h}{G c} \]  
\[ r_{pl} = \frac{h G}{c^3} \rightarrow m_\gamma \cdot m_u = m_{pl}^2 \]  

where \( m_{pl}, r_{pl} \) are the Planck mass and length resp. with the gravitational constant \( G \), Planck’s constant \( h \) and the
speed of light \( c \) [3]. The observable horizon of the universe is the Schwarzschild radius \( r_S = 2r_G \). The initial mass
of the exponential model is the Planck mass. To obtain an exponential scaling the Planck mass is multiplied by successive factors to obtain first the masses for the stable particles proton resp. electron.

The first step results in the proton mass

\[ m_p = m_p \cdot \frac{m_p}{m_{pl}} \]

followed by the electron mass

\[ m_e = m_{pl} \cdot \frac{m_p \cdot m_e}{m_{pl} \cdot m_p} \]

and finally the smallest mass

\[ m_\gamma = m_{pl} \cdot \frac{m_p \cdot m_e \cdot (m_x \cdot m_{pl})^n}{m_{pl} \cdot m_p} \] (2.1a)

where \( n \) is an integer and \( m_x \) a mass to be determined. For \( m_U \), it follows with Eq. (2.0b)

\[ m_u = \frac{m_{pl}^2}{m_\gamma} \rightarrow m_u = \frac{m_{pl}^2}{m_e \cdot (m_x)} \] (2.1b)

For \( m_\gamma = m_p \) and \( n=2 \) the result for \( m_U \) and the approximate age \( t_0 = r_0/c \) of the observable universe is

\[ m_u = 8,805 \cdot 10^{52} kg \ (\approx 10^{53} kg), \quad t_0 = 13,822 \cdot 10^9 \text{ a} \ (13,799 \pm 0,021 \cdot 10^9 \text{ a}) \]

This is in agreement with measurements of the mass and radius resp. the age [4] of the observable universe, where the measured values are in brackets. Now \( m_U \) and \( m_\xi \) are defined as

\[ m_\gamma = \frac{m_e m_p^2}{m_{pl}(G(t))} \] (2.2a)

\[ m_u = \frac{m_{pl}^2(G(t))}{m_e m_p^2} \] (2.2b)

The model mass \( m_U \) is assumed to always fulfil the mass radius relation of Eq. (2.0a). Provided the radius of the universe increases with time, \( m_U \) is less for a smaller radius in the past and there is no singularity at an initial radius \( r_0 \) that exceeds the mass radius relation of Eq. (2.0a). But with Eq. (2.2b) this implies that the Planck mass cannot be a time independent constant, when assuming that the particle masses \( m_e \) and \( m_p \) are constant.

Here it is assumed that the gravitational constant \( G \) is a parameter \( G(t) \) that was larger in the past. Thus the Planck mass \( m_{pl}(G(t)) \) and \( m_{\xi}(m_{pl}) \) were smaller. The time dependencies of \( G(t) \) and \( m_{\xi}(t) \) are compatible with the conclusions from the LNH, see Appendix C. For \( m_{\xi}(t) \) being smaller in the past, there is eventually a condition reached when it is equal to the smallest observable mass \( m_\gamma(t) \), which is referred to as the state of equilibrium. This state is derived and analysed.

3. State of Equilibrium

The state of equilibrium is defined by \( m_e = m_\gamma = m_U \). Since the smallest observable mass \( m_\gamma \) cannot exceed the mass of the universe \( m_U \), it is the initial state of the model. In the following \( m_{pl} \), \( r_{pl} \) and \( G_0 \) are the present values of Planck mass, Planck length and gravitational constant, whereas \( m_{pl}(G_\xi) \) and \( r_{pl}(G_\xi) \) are the values for a specific \( G_\xi \). With Eq. (2.0b) it follows \( m_{pl}(G_\xi) = m_{pl} \) and then for this state the gravitational radius is equal to the Planck length. Setting Eqs. (2.2a) and (2.2b) equal results in:

\[ m_u = m_\gamma \rightarrow m_{pl}^2(G_E) = (m_e m_p^2)^2. \]
then

\[ m_E = m_{pl}(G_E) = (m_e m_p^2)^{\frac{1}{3}} \] (3.0)

This result is independent of the definition of the Planck mass in Eq. (2.0b), which may vary depending on its derivation. Inserting \( m_E \) for \( m_{pl} \) into Eqs. (2.2a) and (2.2b) yields

\[ m_E = m_{pl}(G_E) = m_u = m_p = (m_e m_p^2)^{\frac{1}{3}} \] (3.1)

Then with Eq. (2.0b)

\[ G_E = \frac{\hbar c}{m_{pl}^2(G_E)} = \frac{\hbar c}{(m_e m_p^2)^{\frac{2}{3}}} = G_0 \frac{m_{pl}^2}{(m_e m_p^2)^{\frac{2}{3}}} \] (3.2)

and

\[ r_E = \frac{\hbar G_E}{\sqrt{c^3}} \frac{\hbar}{c(m_e m_p^2)^{\frac{1}{2}}} = r_{pl} \frac{m_{pl}}{(m_e m_p^2)^{\frac{1}{3}}} \]

The equilibrium values are:

\[ G_E = 1.694 \cdot 10^{30} \frac{m^3}{kg \cdot s^2}, \quad r_E = 2.575 \cdot 10^{-15} m, \quad m_E = 1.365929 \cdot 10^{-28} kg \] (3.3)

The equilibrium mass \( m_E \) is smaller than the muon, between the electron and proton masses. To roughly estimate the ratio of the strength of gravitational to electromagnetic interaction at equilibrium, the interaction of two masses \( m_U/2 \) is compared with the interaction of two elementary charges \( e \):

\[ \frac{e^2}{4 \pi \varepsilon_0 G_E \frac{m_E^2}{4}} = \frac{4e^2}{4 \pi \varepsilon_0 \hbar c} = 4\alpha \approx 0.03 \]

At equilibrium the gravitational and electromagnetic interaction converge and the spatial size of the model mass \( m_U \) is with \( r_S=2r_E \) of the order of a proton.

### 4. Composition of the Equilibrium Mass

When \( m_U \) is of the order of \( m_E \) and thus exhibits the mass and size properties of an elementary particle, the value for \( G(t) \) is too large to neglect binding energy effects. It is assumed that within the space of \( m_U \) there is a constituent mass \( m_b \) as well as its self resp. binding energy \( m_b c^2 \). Further it is assumed that there is an energy resp. mass contribution \( m_0 \) from an elementary charge \( e \) distributed over the sphere of \( m_U \). Thus a general approach for the energy resp. mass content of \( m_U \) is made. With Eq. (2.2b) it follows:

\[ m_u = \frac{m_{pl}^4(G_u)}{m_e m_p^2} = m_b + m_s + m_q + m_\alpha \] (4.0a)

where \( m_\alpha \) resembles a second order correction term. The binding energy \( m_b c^2 \) is

\[ m_b c^2 = \frac{3}{5} G_u \frac{m_b^2}{r_c(m_b)} \quad r_c(m_b) = \frac{\hbar}{m_b c} \]

then with Eqs. (2.0a) and (2.0b)

\[ m_e = \frac{3}{5} \frac{m_b^3}{m_{pl}^2(G_u)} \] (4.0b)

and
\[
m_q c^2 = \frac{e^2}{8\pi \varepsilon_0 r_c(m_b)} = \alpha G_u \frac{m_p^2}{2r_c(m_b)} \quad \rightarrow \quad m_q = \frac{\alpha}{2} m_b, \quad (4.0c)
\]

and
\[
m_a = m_b \xi \alpha^2, \quad \alpha = \frac{e^2}{4\pi \varepsilon_0 \hbar c} \quad (4.0d)
\]

where \(\xi\) is a constant to be determined later. Then \(m_u\) is

\[
m_u = \frac{m_p^4(G_u)}{m_p^2 m_p^2} = m_b \xi + \frac{3}{5} \frac{m_b^2}{m_p^4(G_u)} \quad (4.1)
\]

with
\[
c_u = \left(1 + \frac{\alpha}{2} + \xi \alpha^2\right)
\]

Rearranging Eq. (4.1) yields

\[
m_p^4(G_u) - m_p^2(G_u) m_e m_b^2 c_u = \frac{3}{5} m_e m_p^2 m_b^3 \quad (4.2)
\]

The solution of this equation [5] is

\[
m_{pl}(G_u) = 3^{-1} 10^{-1.5} \left(Y + \frac{10^2 m_e m_b^2 m_b c_u}{Y}\right)^{\frac{1}{2}} \quad (4.3a)
\]

where

\[
Y = \left(m_e m_b^2 m_b \left(3^5 m_b^5 + (243 m_b^3 - 100 m_e m_b^2 m_b c_u^3)^{\frac{1}{2}}\right)^{\frac{1}{2}} \right)^{\frac{1}{3}} \quad (4.3b)
\]

A lower limit for \(m_b\) is defined by the square root within \(Y\), hence

\[
243 m_b^3 - 100 m_e m_p^2 m_b c_u^3 = 0
\]

resulting in

\[
m_b = \left(\frac{100}{243} m_e m_p^2\right)^{\frac{1}{3}} c_u \quad (4.4)
\]

Inserting \(m_b\) into Eq. (4.3b) yields

\[
Y = \left(\frac{5}{3} \frac{100}{243} \left(m_e m_p^2\right)^{\frac{1}{3}} c_u \right)^{\frac{1}{3}} c_u
\]

Inserting \(Y\) into Eq. (4.3a) is a straightforward calculation and yields

\[
m_{pl}(G_u) = \left(2 \left(\frac{10}{81}\right)^{\frac{1}{3}} c_u \right)^{\frac{1}{2}} \left(m_e m_b^2\right)^{\frac{1}{3}} \quad (4.5)
\]

For the condition of equilibrium (Eq. (3.1)) the Planck mass in Eq. (4.5) is set equal to the equilibrium mass \(m_e\) in Eq. (3.0):
yielding

\[ c_\alpha = \frac{1}{2} \left( \frac{10}{81} \right)^{\frac{1}{3}} = 1.004149 \]  

Inserting this into Eq. (4.4) results in

\[ m_b = \left( \frac{5}{12} \right)^{\frac{1}{3}} (m_e m_p^2)^{\frac{1}{3}} = 1.020214 \cdot 10^{-28} \text{ kg} \]  

which is about half the muon mass. Eq. (4.4) is rewritten by defining

\[ m_c = m_b(c_\alpha = 1) = \left( \frac{100}{243} m_e m_p^2 \right)^{\frac{1}{3}} = 1.015998 \cdot 10^{-28} \text{ kg} \]  

resulting in

\[ m_b = m_c \left( 1 + \frac{\alpha}{2} + \xi \alpha^2 \right) \]  

The general approach to particle masses is the equilibrium mass in Eq. (3.0). To solve Eq. (4.7c) for the proton mass \( m_p \), the equilibrium mass will be generalized.

5. The Generalized Equilibrium Mass

The equilibrium mass in Eq. (3.0) is utilized for finding dependencies between elementary particle masses. Expressing it in units of the electron mass \( m_e \) results in

\[ \left( \frac{m_e m_p^2}{m_p} \right)^{\frac{1}{3}} m_e = \left( \frac{m_p^2}{m_e} \right)^{\frac{3}{2}} = 149,947 \]  

Replacing the proton by the neutron mass results in

\[ \left( \frac{m_e m_n^2}{m_p} \right)^{\frac{1}{3}} m_e = \left( \frac{m_n^2}{m_e} \right)^{\frac{3}{2}} = 150.085 \]  

Thus both values group around the integer value 150. The relation of particle masses and integer values have already been pointed out by other authors such as Koide [2] with his pure empirical though astonishing precise and profound formula for lepton masses (see Appendix B). To evaluate whether the integer value for the proton resp. neutron mass is a random result, the electron resp. proton mass in Eq. (3.0) are replaced by other elementary particle masses. Here the lepton and meson masses smaller than the proton, the proton \( p \), neutron \( n \) and the tau \( \tau \) are applied, since this will be sufficient for the derivation of the proton mass. The elementary particles smaller than the proton are the electron \( e \), muon \( \mu \), pions \( \pi^0 + \), kaons \( k^+0 \), the eta \( \eta \), rho \( \rho \), omega \( \omega \) and \( K^+0 \), see Appendix A and [3,6]. The equilibrium mass is generalized as

\[ m'(i, j) = \left( \frac{m_i m_j}{m_p} \right)^{\frac{1}{3}} \]  

(5.0a)

Every particle combination \( i, j \) is assigned a mass \( m' \). The results for setting \( j=n \) and \( j=p \) resp. then relate the proton, neutron, pion, kaon, \( \omega \)and \( \eta \) masses, where the measured values are in brackets:

\[ m_{K^+} = \left( \frac{m_\pi m_\eta}{m_p} \right)^{\frac{1}{3}} = 8.772 \cdot 10^{-28} \text{ kg} \]  

(5.0b)

\[ (8.801 \cdot 10^{-28} \text{ kg}) \]
\[ m_{K^0} = (m_\pi m_\pi^2)^{\frac{1}{3}} = 8.871 \cdot 10^{-28} \text{ kg} \quad (8.871 \cdot 10^{-28} \text{ kg}) \]
\[ m_\omega = (m_\eta m_\pi^2)^{\frac{1}{3}} = 1.398 \cdot 10^{-28} \text{ kg} \quad (1.395 \cdot 10^{-28} \text{ kg}) \]

It is notable that the kaons and the adjacent pions are related pairwise and that \( \omega \) is the equilibrium mass of the adjacent \( \eta \) with \( j=p \). This approach is now investigated for \( i=e \). Every elementary particle \( j \) is assigned a mass \( m'(e,j) \) and a mass number \( N(j) \), which is the ratio of \( m'(e,j) \) and the electron mass.

\[
N(j) = \left( \frac{m_{e}}{m'} \right)^{\frac{1}{3}} = \frac{m'}{m_e}^{\frac{1}{3}} \quad (5.1)
\]

The results are shown in Fig. 1, where the \( \mu \), pion \( \pi^+ \), the symmetric grouping of the kaons, \( K^+ \) and \( p, n \) build up an integer scheme. Thus the particles values \( N(j) \) are supposed to group around a scheme of integers and are assigned these integers:

\[
\begin{align*}
N(\mu) &= 35 \quad (34,967) \\
N(\pi^0) &= 42 \quad (41,168 \text{ and } 42,097) \\
N(K^+) &= 98 \quad (97,727 \text{ and } 98,246) \\
N(\eta) &= 105 \quad (104,753) \\
N(\rho,\omega) &= 133 \quad (132,374 \text{ and } 132,871) \\
N(K^{*^0}) &= 145 \quad (144,939 \text{ and } 145,389) \\
N(p,n) &= 150 \quad (149,947 \text{ and } 150,085)
\end{align*}
\]

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Fig. 1. \( N(j) \) for elementary particles from electron to neutron

Every mass \( m'(e,j) \) in the range \( N(j)=35...133 \) is located at an integer value \( N(j)=7n \).

\[
N(j) = 7n \quad , \quad n = 5,6,14,15,19 \quad (5.2)
\]

The proton is assigned \( N(p)=150 \) i.e. a mass \( m'(e,p) \) which is equal to the equilibrium mass \( m_E \) in Eq. (3.0). In addition there is an exponential scaling for particle masses by an factor \( f_N \) to be applied in the calculation of the neutron mass. For the range of particles from pion to tau:

\[
f_N = (m'(e,\pi)/m'(e,\pi^+))^\frac{1}{2} = 1.528065 \quad (5.3a)
\]

Then within an error of \( \approx 10^{-3} \):

\[
\begin{align*}
N(\pi^0) f_N &= 64,18 = N(g), \text{ which is within the gap between } \pi^-'s \text{ and } K^-'s \\
N(g) f_N &= N(K^+) \\
N(K^+) f_N &= N(p,n) \\
N(p,n) f_N &= N(\tau) = 229,52
\end{align*}
\]

The reason for this exponential behavior can be approached by assigning each particle a generalized equilibrium mass \( m'(i,e) \) with the properties

\[
M(i) = \left( \frac{m_{e} m_{i}^{2}}{m_{e}^{3}} \right)^{\frac{1}{3}} = \left( \frac{m_{i}}{m_{e}} \right)^{\frac{1}{3}} = N(i)^{\frac{1}{2}} \quad (5.4)
\]
M(π0+)  = 6.48074  
M(g) = 8.01116  
M(K+0) = 9.89949  
M(p,n)   = 12.2474  
M(τ)   = 15.1498  

Then the following observation can be made:

\[
\begin{array}{c|c|c|c|c}
\text{phi} & M(g) & M(K+0) & M(p,n) & M(τ) \\
\hline
2M(\pi0+) & 2M(g) & 2M(K+0) & 2M(p,n) \\
\end{array}
\]

Successive ratios of an M-value and twice the preceding M-value approach with values between 0.61786 and 0.61859 the golden ratio \( \text{phi} \approx 0.618 \) as shown in Fig. 2, which is the perfect ratio of resonances. These perfect proportions already have been found in experiments observing other quantum mechanical systems e.g. quantum phase transitions in atomic chains [7]. Then the ratio of M(i)’s is \( f \approx 2\text{phi} \) and with Eq. (5.4)

\[
f_M = f_M^2 \approx (2\text{phi})^2 = 1.527864
\]

which is consistent with Eq. (5.3a). With Eq. (5.1) particles m´ resp. N ratios are expressed as integer ratios which can be related to each other:

\[
\frac{N(i_1)}{N(j_1)} = u \frac{N(i_2)}{N(j_2)}
\]

where \( u \) is a rational resp. integer number. An obvious example for the \( \mu \), the \( \pi \)’s and p,n is

\[
\frac{150}{35} = u \frac{42}{98} = u = 10
\]

With these approaches and results the proton mass can be calculated as a function of the electron mass.

### 6. Calculation of the Proton Mass

In Chapter 4 the mass \( m_b \) in Eq. (4.7c) has been derived, which is a component of the equilibrium mass and now defined as a mass \( m \) (e,j). Then according to Eqs. (5.1) and (5.2) \( m_b \) is assigned a mass number \( N(b)=7n \).

Thus Eq. (4.7c) becomes

\[
N(b)m_e = m_e \left( 1 + \frac{\alpha}{2} + \xi a^2 \right)
\]

The closest value \( m' \) of a particle to \( m_b \) in Eq. (4.7a) is the \( \eta \) with \( n=15 \):

\[
m_\eta = 9,767 \cdot 10^{-28} \text{kg}
\]

and

\[
m'(e,\eta) = 9,543 \cdot 10^{-29} \text{kg} , \ N(\eta) = 105
\]

The next possible mass number is \( N(b)=7n, n=16, \) thus
The second order contribution $m_c \xi \alpha^2$ is solved in a separate approach independently of Eq. (5.0). Considering Eq. (5.1), then Eq. (5.6) is rewritten as

$$\frac{n_1}{n_2} = u \frac{m_c \xi \alpha^2}{m_e} \quad (6.2a)$$

where $n_1$ and $n_2$ are integers. Since $m_c \xi \alpha^2$ also is assumed to be a mass $m'(e,j)$, the approach is

$$\frac{n_1}{n_2} m_e = um_c \xi \alpha^2 = m_c \xi'(u\alpha)^2, \quad \xi' = \frac{\xi}{u} \quad (6.2b)$$

and

$$\frac{n_1}{n_2} m_e = um_c \xi \alpha^2 = m_c \xi'(u\alpha)^2, \quad \xi' = \frac{\xi}{u} \quad (6.2c)$$

where the right side of the equation is a second order correction term with a constant $\xi'$ to be determined. The number $u$ is assumed to be an integer similar to common fine structure corrections which are of the order $(u\alpha)^2$. With the results of Eq. (5.1), $m_e$ is considered a constituent mass of the generalized equilibrium masses $m'(e,j)$. The left side of Eq. (6.2c) is assumed to be proportional to a self energy contribution of the electron, which is proportional to $3/5$ and to $m_e$. This is evident when replacing $m_b$ by $m_e$ in Eq. (4.0b) and since the Planck mass in the self energy term is proportional to $m_e$.

Then from Eq. (6.2c) it follows

$$u\xi \alpha^2 = \xi' \alpha^2 = \frac{3 m_e}{5 m_e} \quad (6.3)$$

Solving Eq. (6.3) by inserting $m_c$ from Eq. (4.7b) results in $\xi' \alpha^2 = 101.02$. In a first approach the combination with the best fitting integer

$$u = n_3 = 10, \quad \xi' = 1.0102 \quad (6.4)$$

is utilized to solve Eq. (6.3). The second order term in Eq. (6.0) now is

$$\xi \alpha^2 = \frac{3 m_e}{5 n_3 m_c} \quad (6.5)$$

which yields

$$N(m_e) m_e = m_c \left(1 + \frac{\alpha}{2} + \frac{3}{5 n_3 m_c} \right) \quad (6.6)$$

Inserting $N(b)$ and $n_3$ yields

$$112 m_e = m_c \left(1 + \frac{\alpha}{2} + \frac{3}{50 m_c} \right)$$

Besides the integers in Eq. (4.4) resulting from the solution of Eq. (4.2) the proton mass depends on two integers $N(b)=112$ and $n_3=10$ as a result of the quantisation approach in chapter 5. Solving Eq. (6.6) for $m_p$ with Eq. (4.7b) yields

$$\left(112 - \frac{3}{50}\right) m_e = \left(\frac{100}{243} m_e m_p^\frac{1}{3}\right) \left(1 + \frac{\alpha}{2}\right)$$

or

$$m_p = \left(\frac{243}{100}\right)^\frac{1}{3} \left(2 \left(112 - \frac{3}{50}\right) \left(2 + \alpha\right)^\frac{3}{5} m_e \right)$$

(6.7)
and with defining
\[
c_m = \left( \frac{243}{100} \right)^{\frac{1}{2}} \left( 2 \left( 112 - \frac{3}{50} \right) \right)^{\frac{3}{2}} = 5.2218703 \cdot 10^3
\]
then
\[
m_p = c_m (2 + \alpha)^{\frac{3}{3}} m_e = 1.672621 \cdot 10^{-27} \text{kg} \quad (1.672622 \cdot 10^{-27} \text{kg}) \quad (6.8)
\]
where the measured mass is in brackets, see Appendix A.

7. Calculation of the Neutron Mass

The factor \( f_N \) in chapter 5 provides an exponential scaling of particle masses but no precise results as for the proton mass in chapter 6. But the assumption is that this is a function of the mass difference involved, i.e. that for small scales resp. mass differences the precision increases. To verify this the splitting of the neutron and proton mass is now approached with the principles previously deployed. With Eqs. (4.0c) and (6.0) it is assumed that the splitting is proportional to \( \alpha \) and \( m_p \). Then with Eq. (6.2a) a related approach is
\[
\frac{n_1}{n_2} (m_n - m_p) \propto \alpha m_p = f_N^{\frac{3}{2}} \alpha m_p \quad (7.0)
\]
where according to Eqs. (5.2) and (5.6) \( n_1 \) and \( n_2 \) are assumed to be from the set of mass numbers and integers in the previous chapters, thus the number of possible combinations is limited. The factor \( f_N \) from Eq. (5.3a) has been adjusted with the correct exponent, since it relates to ratios of \( m' \) and thus of \( N(j) \), but masses \( m \) are applied here. Then with Eq. (5.1) it follows
\[
\frac{m_i}{m_j} = \left( \frac{N(i)}{N(j)} \right)^{\frac{3}{2}} = f_N^{\frac{3}{2}} = 1.8889163
\]
Considering the previous results, then a supposable combination from the set of integers resp. mass numbers is \( n_1=n_3 \) from the solution for the proton mass in Eq. (6.4) and \( n_2=N(e)=1 \), resulting in
\[
n_3 (m_n - m_p) = f_N^{\frac{3}{2}} \alpha m_p \quad (7.1a)
\]
yielding a mass formula that with \( f_N \) relates the pion, proton, neutron and tau masses
\[
n_3 = \frac{f_N^{\frac{3}{2}} \alpha m_p}{(m_n - m_p)} = 9.99993 \approx 10 \quad (7.1b)
\]
Then solving for the neutron mass and inserting \( f_N \) and \( n_3=10 \) yields
\[
m_n = \left( \frac{f_N^{\frac{3}{2}} \alpha}{n_3} + 1 \right) m_p = 1.67492745 \cdot 10^{-27} \text{kg} \quad (1.67492747 \cdot 10^{-27} \text{kg}) \quad (7.2)
\]
The factor \( f_N \) from Eq. (5.3a) is the best fit for Eq. (5.3b). Replacing it by the theoretical value from the observation presented in Fig. 2 and Eq. (5.5) results in
\[
m_n = \left( \frac{(2 \pi h)^2 \alpha}{n_3} + 1 \right) m_p = 1.6749270 \cdot 10^{-27} \text{kg} \quad (7.3)
\]
with an accuracy of seven decimal digits. Solving Eq. (7.2) for \( m_p \) yields
\[ m_p = \left( \frac{f_n^2 \alpha}{n^2} + 1 \right)^{-1} \]
\[ m_n = 1.67262191 \cdot 10^{-27} \text{kg} \quad (1.67262190 \cdot 10^{-27} \text{kg}) \]  

(7.4)

The results of Eqs. (7.2) and (7.4) are within the standard deviation of the measured masses, see Appendix A.

8. Calculation of the Tau Mass

The results of Eq. (5.1) shown in Fig. 1. can be written as:

\[ m'(e,j) = \left( m_em_\mu \right)^{\frac{1}{3}} = N(j)m_e \]  

(8.0)

Here \( N(J) \) is the ratio of an equilibrium mass and the electron mass. The ratio of an equilibrium mass and the proton mass then is approached with

\[ m'(e,j) = \left( m_em_\mu \right)^{\frac{1}{3}} = \frac{m_p}{I(j)} \]

(8.1)

since \( m'(e,j) \) is smaller than \( m_p \) in the considered mass range, where \( I(j) \) are integers. For \( j=\tau \) an accurate result for the tau mass \( m_\tau \) is obtained:

\[ I(\tau) = \frac{m_p}{(m_e m_\mu)^{\frac{1}{3}}} = 8.00006 = 8 \]  

(8.2a)

or

\[ (m_e m_\mu)^{\frac{1}{3}} = \frac{1}{8} m_p \]  

(8.2b)

yielding

\[ m_\tau = \left( \frac{m_p}{E} \right)^{\frac{1}{2}} = 3.16750 \cdot 10^{-27} \text{kg} \quad (3.16747 \cdot 10^{-27} \text{kg}) \]  

(8.2c)

Inserting \( m_p \) from Eq. (6.8) yields

\[ m_\tau = \left( \frac{c m_\mu}{E} \right)^{\frac{1}{2}} \left( 2 + \alpha \right)^{\frac{9}{2}} m_e = 3.16750 \cdot 10^{-27} \text{kg} \]  

(8.3)

which is within the standard deviation of the measured mass in brackets, see Appendix A. For a comparison the formula of Koide in Appendix B is used to calculate the tau mass [5] as a function of the muon and electron mass with the result \( m_\tau =3.16773 \cdot 10^{-27} \text{kg} \). The accuracy of both results in principle allows to equate the tau mass in Eq. (8.2c) and in the Koide formula to relate the proton, electron and muon masses.

Discussion

The proton, neutron and tau as well as meson masses in the range probed can be calculated by defining a generalized equilibrium mass as a combination of two particle masses. For particular equilibrium masses which are a function of the electron mass, quantum structures i.e. mass numbers can be observed which allow to relate elementary particle masses. The proton and neutron masses then are a function of the finestructure constant, the electron mass and integers resp. an exponential factor, which does not seem to be accidentally due to the accurate results of six resp. eight decimal digits. The exponential dependencies of mass numbers can be traced back to the golden ratio, but their origin remains unresolved. This result is similar to the empirical formula of Koide (Appendix B), which relates the \( e, \mu, \text{and } \tau \) masses with two integers only with the accuracy of four to five decimal digits. Zenczykowski [8] is pointing out that this does not seem to be an ‘accident’ and thus strongly suggests an algebraic origin of mass. For the exponential model of mass scales which is needed for the calculations,
the gravitational constant becomes a parameter \( G(t) \) that was larger in the past and results in a strength converging with electromagnetic interaction at the proton sized equilibrium state. The results suggest that the origin of the observed elementary particle masses is linked to fundamental constants, an inherent algebraic structure and that the constancy of the gravitational constant has to be questioned.

**Appendix A**

Elementary particle masses in kg from [3,6]

\[
egin{align*}
    m_e &= 9,10938356 \cdot 10^{-31} \\
    m_p &= 1,8835316 \cdot 10^{-28} \\
    m_{\mu} &= 2,40618 \cdot 10^{-28} \\
    m_{\pi^+} &= 2,48807 \cdot 10^{-28} \\
    m_{K^+} &= 8,8006 \cdot 10^{-28} \\
    m_{K^0} &= 8,87078 \cdot 10^{-28} \\
    m_{\eta} &= 9,76653 \cdot 10^{-28} \\
    m_{\rho} &= 1,38203 \cdot 10^{-27} \\
    m_{\pi^0} &= 1,39520 \cdot 10^{-27} \\
    m_p &= 1,67427471(214) \cdot 10^{-27} \\
    m_n &= 1,67261898(21) \cdot 10^{-27} \\
    m_{\tau} &= 3,16747(29) \cdot 10^{-27}
\end{align*}
\]

The values in brackets are the estimated standard deviations of the last digits of the measured masses.

**Appendix B**

The formula of Koide [2]:

\[
Q = \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = 0.666659(10) \approx \frac{2}{3}
\]

The value of \( Q \) should be a random number, but it is exactly halfway between 1/3 when all lepton masses would be equal, and 1 when two masses would be negligible, which suggests an unresolved physical meaning of the formula.

**Appendix C**

The time dependencies of \( G \) and \( m_u \) derived by Dirac from the LNH [1], resulting in \( G \) proportional \( 1/t \) and \( m_u \) proportional \( t^2 \), are solutions of \( G(t) \) and \( m_u(G) \) in Eqs. (2.0b) and (2.2b). In the following \( m_{pl}(G_0) \) and \( G_0 \) are the present values of the Planck mass and gravitational constant, whereas \( m_{pl}(G_x) \) is the value for a specific \( G_x \).

When \( m_u \) was smaller by a factor \( x \), then with Eq. (2.2b)

\[
xm_u = x \cdot \frac{m_{pl}(G_0)}{m_e m_p^2} = \left(\frac{hc}{G_0 \sqrt{x}}\right)^2, \quad x = \frac{m_x}{m_u}
\]

Then with Eqs. (2.0a) and (2.0b) it follows

\[
m_{pl}(G_x) = \left(\frac{hc}{G_0 \sqrt{x}}\right)^2 \rightarrow G_x = \frac{G_0}{\sqrt{x}} \rightarrow \frac{G_x}{G_0} = \sqrt{m_u m_x} = \sqrt{\frac{r}{G_0 r_x}}
\]

and
\[ \left( \frac{G_x}{G_0} \right)^2 = \frac{r_x}{r_0} \frac{G_x}{G_0} \rightarrow \frac{G_x}{G_0} = \frac{r_x}{r_0} \]  
(A1)

The solutions for \( G(t) \) thus depend on \( r(t) \). With Eq. (A1) and the approximation for the ages of the observable universe \( t_0 = 2r_0/c \) and \( t_x = 2r_x/c \) resp. we get

\[ G_x = G_0 \frac{t_0}{t_x} \rightarrow G \propto \frac{1}{t} \]  
(A2)

Inserting \( G \) from Eq. (A2) into Eqs. (2.0b) and (2.2b), then the mass of the universe is:

\[ m_u \propto \left( \frac{\hbar c}{t^2} \right)^2 \propto \frac{m^2}{m_e m_p} \propto t^2 \]

These are the time dependencies of the gravitational constant and the mass of the universe concluded from LNH.

References

3. CODATA Recommended Values
6. PDG Particle Data Group
7. Science 08 Jan 2010: Vol. 327, Issue 5962, pp. 177-180