Linear and nonlinear buckling analysis of a locally stretched plate

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Uniformly stretched thin plates do not buckle unless they are in special boundary conditions. However, buckling commonly occurs around discontinuities, such as cracks, cuts, narrow slits, holes, and different openings, of such plates. This study aims to show that buckling can also occur in thin plates that contain no defect or singularity when the stretching is local. This specific stability problem is analyzed with the finite element method. A brief literature review on stretched plates is presented. Linear and nonlinear buckling stress analyses are conducted for a partially stretched rectangular plate, and various load cases are considered to investigate the influence of the partial loading expanse on the critical tensile buckling load. Results are summarized in iso-stress areas, tables and graphs. Local stretching on one end of the plate induces buckling in the thin plate even without geometrical imperfection.

Keywords: Buckling; FEM; Linear and nonlinear analysis; Local stretching; Thin plate

1. Introduction

Thin plates are structural elements that are widely used in many engineering fields, such as aerospace, shipbuilding and civil engineering. Ribbed and stiffened ship plate, offshore, and aerospace panels are commonly utilized and always subjected to partial edge traction on their own plane. This type of loading can induce buckling, which negatively affects the function of the structural elements concerned.

Plate instability is generated globally or locally through in-plane compression and shear loading. The corrugations that result from plate instability may cause permanent deformation and reduce the efficiency of the entire structure. Thin plate buckling under compression and shear was studied by Timoshenko [1] and Bulson [2], and analytical solutions were presented. Chen and Kim [3] utilized a combined equivalent energy method and proposed a stretched corrugated plate model. Numerical methods, such as finite element method, can solve particular structures (e.g., crack propagation in a bearing ring [4], blast loaded corroded plates [5] and stretched corrugated beam elements [6]) and have provided solutions to highly complex problems encountered in plate buckling, such as laminated composite plates under compression [7], partially compressed plate [8], and defective plate under shear [9], uniform compression [10, 11] and local compression [12].

The buckling phenomenon appears as a result of the appli-

cation of compression or shear load; it occurs in less usual situations, in which tensile loading induces buckling instability and the formation of buckling configurations. Despite the fact that no compressive component exists in the applied tension, local compressive stresses resulting from the presence of a cut, crack, or hole induce stability loss.

Datta and Carlson [13] presented the results of an analytical and experimental study on the buckling of a thin sheet with a hole in the tensile field. The analytical method used in stress distribution around the opening was based on the elastic solution introduced by Timoshenko and Goodier [14].

Shaw [15] and Brighenti [16-18] utilized the finite element method to investigate the buckling of a defective uniformly tensioned plate. A few authors studied the effect of point loading. Tomita et al. [19], Woo and Jenkins [20], and Kremer [21] examined the case of concentrated tensile load applied to defective thin plates.

The experimental studies of Gilabert et al. [22] and Seifi [23] also revealed that local buckling occurs around defects on a stretched plate. The waves of the buckling undulation are perpendicular to the applied tensile stress.

These studies have shown that perfectly flat plates subjected to tensile load may buckle locally in the presence of defects. However, buckling occurs even in the absence of any geometric discontinuity. Such a phenomenon was well described by Bryan [24], who compared a tensioned plate in one direction to a stretched sheet of paper wet in the middle. Moisture, which illustrates paper elasticity rather than weakness, causes the surface of the paper to expand and wrinkle. The wrinkles

become fine and close as the tensile forces increase. This issue was discussed by Bryan [24] through an analytic method. Friedl et al. [25] presented a numerical solution to this problem for special boundary conditions. However, only a few studies have been conducted on partially tensioned plates that exhibit no geometric discontinuity or imperfection. Moreover, only a few authors have addressed the problem of nonlinear buckling of thin stretched plates; even when an analysis on such plates had been conducted, the partially tensioned plate was not considered.

In the present study, the linear and nonlinear buckling of a thin, partially stretched, non-defective plate at its one end in the lateral direction, as depicted in Fig. 1, was investigated with the Finite element method (FEM). The aim of this study is to provide representative linear and nonlinear buckling load results for a simply supported plate. The effect of applied load length and plate length (l/b) ratio on the critical buckling load was investigated. The obtained numerical results in both linear and nonlinear fields were graphically summarized, and several interesting conclusions were obtained. To the authors' best knowledge, no experimental study is currently available to validate the results presented in this numerical investigation.

2. Stability problem formulation

Analytical estimations should be performed prior to numerical buckling analyses. However, no analytical solution exists for the present extremely complex buckling problem. The governing differential equation for the buckled plate is [1]

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(N_x \frac{\partial^2 w}{\partial x^2} \right), \quad (1)$$

where w is the plate deflection and D is the plate flexural rigidity. An analogous problem with a uniformly distributed compressive load applied over the width b of a simply supported thin plate was treated analytically in a previous study and has led to the well-known expression of the compressive load critical value as follows [1]:

$$N_x = k \frac{\pi^2 D}{b^2} \quad \text{for } a/b \geq 1, \quad (2)$$

where k is the plate buckling coefficient. However, Eq. (2) is inapplicable to the case of patch tensile loading, and numerical methods, such as FEM, should be utilized. The linear tensile buckling problem is formulated as an eigenvalue problem leading to the equation

$$([K] + \lambda_i [S]) \{\Psi\}_i = \{0\}, \quad (3)$$

where

$$\begin{aligned} [K] &: \text{Stiffness matrix,} \\ [S] &: \text{Stress stiffness matrix,} \end{aligned}$$

Fig. 1. Partially stretched thin plate.

λ_i : i th eigenvalue used to multiply the loads that are generated $[S]$, and

Ψ_i : i th eigenvector of displacements.

Hook's law is used to relate the strain vector $\{\epsilon\}$ to the resultant stress vector $\{\sigma\}$.

$$\{\sigma\} = [D] \times \{\epsilon\}, \quad (4)$$

with: $\{\sigma\} = [\sigma_x \ \sigma_y \ \sigma_z \ \sigma_{xy} \ \sigma_{yz} \ \sigma_{xz}]^T$;

$$\{\epsilon\} = [\epsilon_x \ \epsilon_y \ \epsilon_z \ \epsilon_{xy} \ \epsilon_{yz} \ \epsilon_{xz}]^T.$$

The Von-Mises' stress is computed as

$$\sigma_{\text{Von-Mises}} = \left(\frac{1}{2} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\sigma_{xy}^2 + \sigma_{yz}^2 + \sigma_{xz}^2) \right] \right)^{\frac{1}{2}}. \quad (5)$$

3. Results and discussions

3.1 Description and modelling of the analyzed plate

3.1.1 Description of the analyzed plate

The present investigation aims to examine the buckling behavior of a mild steel plate subjected to localized lateral tension (see Fig. 1). The plate is simply supported at its short sides and free at its lateral loaded edges. Only the displacement in the z direction is prevented on the AB side (Fig. 1), whereas the prevented displacements on the opposite side, CD, are along the z and y axes. A middle node is also restrained in the x direction on this last edge. Aspect ratio a/b , where a and b are the width and length of the examined plate, respectively, is equal to $1/3$. Aspect ratio l/b , where l is the distance over which the localized load extends on the plate length, provides different load cases to be analyzed. Given the importance of the plate thickness effect on the buckling load, the a/h ratio, where h is the plate thickness, is equal to 100. Poisson's ratio $\nu = 0.3$, Young modulus $E = 2.09 \times 10^5 \text{ MPa}$,

Table 1. Critical linear buckling loads obtained in the present convergence analysis using (a), (b) and (c) meshes for different loading cases.

l/b	$P_{cr} [MN / ml]$			Error rate [%]	
	(a)	(b)	(c)	(b)	(c)
0.1667	7.7810	7.7820	7.7815	10^{-2}	10^{-2}
0.3333	8.4427	8.4460	8.4407	4×10^{-2}	2×10^{-2}
0.5000	8.4793	8.4783	8.4781	10^{-2}	10^{-2}
0.6667	8.8732	8.8724	8.8740	10^{-2}	10^{-2}
0.8333	9.9793	9.9791	9.9789	0	0

Fig. 2. Isoparametric quadrilateral element with eight nodes.

Fig. 3. Meshes studied in the present convergence analysis.

and yielding strength $\sigma_{yield} = 400 MPa$ were also considered.

3.1.2 Finite element mesh density

The plate in the numerical tests performed with ANSYS 14.5 software academic version was modeled by an eight-noded element (depicted in Fig. 2). This element with six degrees of freedom at each node is well-suitable for linear and nonlinear applications.

To obtain reasonable results, several successive linear analyses were conducted to determine the optimum mesh density to be used. Three different types of meshes were designed for the plate described in Sec. 3.1.1 to perform a convergence analysis. The grids are depicted in Fig. 3 by their quarters. The Fig. 3(a) type mesh is uniformly gridded with 50×150 elements. The Fig. 3(b) type mesh that contains 70×150 elements is refined only at the neighborhood of the loaded edge. The Fig. 3(c) type mesh comprising 100×300 elements is uniformly and tightly refined. The subdivisions refer to the entire plate in the x and y directions. Table 1 presents a summary of the results obtained in the conducted numerical experiments. The figures in Table 1 show that the use of a mesh particularly refined in the vicinity of the local stretching (b) or entirely and tightly gridded (c) does not cause important changes in the results. Similar results were obtained by Deolassi [26]. Consequently, the mapped discretization of

type (a) depicted in Fig. 3(a) was adopted in the buckling analysis.

3.2 Linear and nonlinear plate buckling analyses

The numerical experiments show that the examined plates in their different mode shapes always buckle in the direction perpendicular to the applied tensile load. Thus, only stresses in this direction were analyzed. Several quantitative results were recorded for the plate described in Sec. 3.1.1 by adopting the ratio $l/b = 1/6$; these results are presented in Table 2.

The finite element results suggest that a limited number of mode shapes is sufficient to describe the plate buckling behavior correctly [8]. The first 50 mode shapes were derived, and the first 5 mode shapes in the linear and nonlinear fields were recorded and are shown in Fig. 4.

3.2.1 Linear longitudinal stress analysis

Fig. 4 shows the iso-stress areas for the linear longitudinal stress component on the entire plate. The drawing in the figure shows that negative stresses, which are compressive stresses perpendicular to the direction of the applied tensile loading, occurred in different parts of the plate. These stresses are evidently responsible for the occurrence of plate buckling because of the Poisson's effect, which is associated with the present boundary conditions. The maximum longitudinal stresses, both compressive and tensile, are located on the plate edges where in-plane loading was directly applied. The stress value decreased toward the longitudinal center line region and eventually reached its minimum. The maximum compressive stress showed a significant value of 20% higher than the maximum tensile stress although the loading was tensile (see Table 2). However, the minimum compressive stress value was almost nine times greater than the minimum tensile stress value. Fig. 5 shows that compressive stress covers the plate surface more than tensile stress does. The compressive zone is about five times larger than the tensile zone.

3.2.2 Nonlinear longitudinal stress analysis

The described plate was also analyzed in the nonlinear field. Fig. 6 shows the iso-stress areas for the nonlinear longitudinal stress component on the entire plate. The iso-stress areas show that compressive stresses perpendicular to the direction of the applied tensile loading occurred but over a much smaller surface area. The obtained nonlinear values show that the compressive stresses are exclusively located on the plate edge and in its nearby neighborhood, whereas the tensile stress spreads all over the plate and reaches its maximal value in the center-line region in the applied load vicinity.

The maximum compressive stress shows a significant value of 5.5 times the maximum tensile stress. Moreover, when the maximum compressive stress value in the linear analysis is smaller than that obtained in the nonlinear analysis, the maximum tensile stress value is greater than that obtained in the nonlinear field. The maximum stress values in both analyses

Table 2. Maximal and minimal buckling stress values (σ_c = compressive stress value, σ_t = tensile stress value) in the present work.

---	$\sigma_{c \max}$ [MPa]	$\sigma_{c \min}$ [MPa]	$\sigma_{t \max}$ [MPa]	$\sigma_{t \min}$ [MPa]	$\sigma_{c \max} / \sigma_{t \max}$	$\sigma_{c \min} / \sigma_{t \min}$
Linear analysis	-210.6000	-38.6382	176.3140	4.3522	1.194	8.878
Nonlinear analysis	-351.9490	-28.4486	63.9802	17.7658	5.501	1.601

Fig. 4. Buckling mode shape configurations obtained in the present investigation for a plate with $a/b=1/3$ and $l/b=1/6$.

Fig. 5. Linear iso-stress areas (in MPa) for y direction stresses on a plate with $a/b=1/3$ and $l/b=1/6$.

Fig. 6. Nonlinear iso-stress areas (in MPa) for y direction stresses on a plate with $a/b=1/3$ and $l/b=1/6$.

are less than the yielding strength. Therefore, plate instability occurred in the elastic field.

3.2.3 Von-Mises' stress analysis

A Von-Mises' stress analysis in both linear and nonlinear fields was performed to provide further insight into the elastoplastic behavior of the plate. The obtained results are presented in Figs. 7 and 8.

Comparison of the two stress distributions reveals that in the nonlinear analysis, plasticization occurred in the area bounded by the loading application points and the width of the simply supported plate. However, in the linear analysis where the obtained failure area is much smaller, this phenomenon appeared partially in the loaded zone. Furthermore, in the linear field, failure was reached with the stress value varying between 6% and 20% of the yielding strength, whereas in the

nonlinear field, failure was obtained with a stress value equal to the yielding stress. Yielding may not occur in this last plate configuration. Thus, the strength of the whole material is utilized to the best advantage.

3.3 Critical buckling load for different load cases

The previous described plate was studied for different load cases to investigate the influence of partial loading expanse on critical tensile buckling load P_{cr} . The l/b ratio value, which is the ratio between the distance over which the tensile loading extends and the plate length, was varied from 0.033 to 1.000 with an increase of one-thirtieth in each numerical test. Fig. 9 shows critical tensile buckling load P_{cr} versus the l/b ratio. Given that it is used in plate stability, the critical buckling load resulting from numerous localized tensile load

Table 3. Critical resultant buckling load R_{cr} values reached in the present investigation for $0 \leq l/b \leq 0.20$.

l/b	$P_{cr} [MN/ml]$		$R_{cr} [MN]$		R_{cr} relative increase	
	Linear analysis	Nonlinear analysis	Linear analysis	Nonlinear analysis	Linear analysis	Nonlinear analysis
0	0.41		0.41		---	
0.03	7.82		0.78		$1.90 \cong 2P_{cr}(l/b=0)$	
0.07	6.34	6.00	1.27	1.20	$3.10 \cong 3P_{cr}(l/b=0)$	$2.93 \cong 3P_{cr}(l/b=0)$
0.20	8.14	4.69	4.88	2.81	---	$6.85 \cong 7P_{cr}(l/b=0)$

Fig. 7. Linear Von-Mises' iso-stress areas (in MPa) on a plate with $a/b=1/3$ and $l/b=1/6$.

Fig. 8. Nonlinear Von-Mises' iso-stress areas (in MPa) on a plate with $a/b=1/3$ and $l/b=1/6$.

cases may be derived from this graph for a simply supported plate with $a/b=1/3$. A change of scale for l/b values up to 0.200 is shown in Fig. 9. The continuous line represents the results of linear buckling analysis, and the dashed line represents those of nonlinear analysis.

Fig. 9 shows that in the linear and nonlinear analyses, the

Fig. 9. Critical buckling load P_{cr} versus l/b ratio for a plate with $a/b=1/3$.

graphs show the same global tendency, but linear analysis generally has a larger P_{cr} value. Furthermore, in the numerical tests conducted for $l/b=1$, the plate shows no buckling when the loading is uniform. The obtained critical buckling load value is $P_{cr}(l/b=0)=0.41MN$ in both linear and nonlinear fields when loading is concentrated on one unique point.

Comparison of the two curves obtained in the linear and nonlinear buckling analyses shows three common significant sections as follows.

(1) The first section is delimited by $0.033 \leq l/b \leq 0.200$ values. The nonlinear curve reduces the overlap in the linear one up to about $l/b=0.067$. The linear curve increases when $0.067 \leq l/b \leq 0.200$. Given that l/b is small in this section, critical loading P_{cr} can be likened to concentrated load, the value of which is the resultant loading $R_{cr}=l \times P_{cr}$. Table 3 shows several R_{cr} derived values obtained in the linear and nonlinear analyses. The figures in Table 3 show that when $l/b \leq 0.067$, R_{cr} increases almost proportionally to l/b , and the nonlinear R_{cr} value maintains this behavior up to $l/b=0.200$. The resultant linear buckling load increase is important for the last l/b ratio. It is about 12 times as large as $P_{cr}(l/b=0)$. Moreover, the last linear R_{cr} value reached is 74% greater than the corresponding nonlinear critical resultant buckling load. These results reveal the significant effect of material elasticity on plate behavior for small l/b values.

(2) The second section, which is delimited by $0.200 \leq l/b \leq 0.900$ values, shows a constant curve behavior. Buckling occurs under the same critical load value regardless of the l/b increase. Linear analysis in this section leads to a

Table 4. Nonlinear to linear P_{cr} rate values reached in the present investigation for $0.20 \leq l/b \leq 0.97$.

l/b	$P_{cr} [MN/ml]$		Nonlinear to linear P_{cr} rate [%]
	Linear analysis	Nonlinear analysis	
0.20	8.14	4.69	58
0.90	12.50	7.50	60
0.97	40.13	28.80	72

critical buckling load of about $8.14 MN/ml$, whereas nonlinear analysis results in P_{cr} with an average value of $4.69 MN/ml$, which is the minimum critical nonlinear buckling load.

(3) With $0.900 \leq l/b \leq 0.967$, both curves in the last section exhibit a dramatic increase in tensile buckling load with a small increase in the l/b ratio. For $l/b = 0.967$ in nonlinear analysis, $P_{cr} = 28.80 MN/ml$; in linear analysis, P_{cr} reaches the value of $40.13 MN/ml$.

The obtained nonlinear values are always lower than the linear results and are closer to those obtained from linear analysis because the loading tends to cover the entire plate length. This tendency is illustrated in Table 4, which contains the P_{cr} rates reached in the present investigation. Nonlinearities tend to prevent "ideal" structural elements from achieving their theoretical elastic buckling strength. Thus, the eigenvalue buckling load is somewhat overestimated. The numerical tests conducted in this study show that when the analyzed plates tended to be uniformly stretched, they developed material nonlinearities.

4. Conclusions

The present paper considered the buckling phenomenon of thin, simply supported, partially stretched plates. Numerical FEM analyses were performed to examine the buckled plate compressive and tensile stresses and determine the critical buckling loads for various load cases from uniform to point loading tension in linear and nonlinear domains.

The main purpose of the present study was achieved. Local tensile loading induces buckling instability in thin plates with no defects. Owing to the Poisson's effect, which is associated with the present boundary conditions, compressive stresses are obviously responsible for the occurrence of flat plate buckling.

In determining the plate buckling shapes, the direction of the buckling shape half waves is always perpendicular to that of the applied tensile load, and no buckling exists under uniform tension.

The linear and nonlinear numerical analyses showed that plate instability in specific boundary conditions occurs in the elastic field. Furthermore, the Von-Mises' nonlinear analysis showed that yielding may not occur, and the strength of the entire material is used to the best advantage, which is not the case in the linear analysis.

Finally, diagrams with linear and nonlinear critical buckling loads as a function of the ratio l/b were introduced. The diagrams allow for the direct reading of the tensile critical load values for a plate with a ratio of $a/b = 1/3$ for conventional plate buckling problems. These diagrams show the same global tendency; generally, in the linear analysis, a greater P_{cr} value is obtained, and no buckling occurs when the loading is uniform.

Further research is required to investigate the experimental buckling load of partially stretched plates so that the present numerical results can be compared with those from experimental work.

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