Multi-Spherical MRI: Breaking the Boundaries of Diffusion Time
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Abstract: Effective representation of the diffusion signal’s dependence on diffusion time is a sought-after, yet still unsolved challenge in diffusion MRI. We propose a functional basis approach that is specifically designed to represent the dMRI signal in this four-dimensional space - that we call the multi-spherical space. We provide regularization tools to drastically reduce the number of measurements we need to probe the properties of this multi-spherical space.

1 The Multi-Spherical Space

Diffusion restriction occurs when water diffusion is obstructed by tissue boundaries. The amount of restriction is time-dependent, meaning that the observed diffusion coefficient will change for varying diffusion times [1].

Multi-Spherical MRI [2] describes diffusion restriction by fitting the diffusion signals over varying:
- **Gradient strength (G)**
- **Gradient direction (g)**
- **Diffusion time (τ)**

We call this four-dimensional space the Multi-Spherical Space.

We sampled this space on 35 different “shells”, varying only **G**, for different **G** ranging from [50-490] mT/m and **τ** ranging from [9.1-18.3] ms.

2 Modeling the Multi-Spherical Space

Multi-Spherical MRI uses a separable Fourier Basis to reconstruct diffusion propagator \( P(\mathbf{r}, \tau; c) \) from signal attenuation \( E(\mathbf{q}, \tau; c) \), represented in coefficients \( c \).

\[
\hat{E}(\mathbf{q}, \tau; c) = \sum_{i} c_{i} \Phi_{i}(\mathbf{q}) T_{i}(\tau) \quad \Rightarrow \quad \hat{P}(\mathbf{r}, \tau; c) = \sum_{i} c_{i} \Psi_{i}(\mathbf{r}) T_{i}(\tau)
\]

\( \Psi_{i}(\mathbf{r}) = F(T(\Phi_{i}(\mathbf{q})): 3D Fourier basis over \mathbf{q} and displacement \mathbf{r} [3]. \)

\( T_{m}(\tau): \) Exponential diffusion time basis over \( \tau [4]. \)

We constrain the fitting of \( c \) to respect boundary conditions of the signal and impose signal smoothness and sparsity:

\[
\min_{c} \int \left[ E(\mathbf{q}, \tau; c) - \hat{E}(\mathbf{q}, \tau; c) \right]^{2} \, d\mathbf{q} \, d\tau + \int \nabla^{2} \hat{E}(\mathbf{q}, \tau; c)^{2} \, d\mathbf{q} \, d\tau + \left\| c \right\|_{1}
\]

Where smoothness is imposed using closed-form Laplacian regularization.

Once fitted, all q-space indices [3] can be estimated for any \( \tau \). As examples we show:
- Mean Squared Displacement (MSD), related to restriction
- Return-To-Origin Probability (RTOP), related to cellularity.

3 In-Silico results

We studied fitting performance under random subsampling by simulating the multi-spherical diffusion signal from gamma-distributed axons using Camino [5].

- Combined sparsity and Laplacian regularization produces the lowest fitting error (left).
- Time-dependent MSD and RTOP follow expected trends - MSD increasing and RTOP decreasing over time - down to about 200 DWIs (right two)

4 Application In-vivo Mouse Data

After eddy current correction, we chose an ROI of 173 voxels in Corpus Callosum. After subsampling we find

- Stable fitting errors from 400 down to 200 DWIs
- Expected trends for time-dependent MSD and RTOP

5 Discussion and Conclusions

- Multi-Spherical MRI allows for the characterization of diffusion restriction through time-dependent q-space indices.
- Through signal sparsity and smoothness, our approach can represent the multi-spherical signal with less samples, allowing more realistic acquisition schemes.
- Additional signal or propagator constraints can be conveniently included in the optimization.
- Through resampling, our approach could be used as a pre-processing for other methods studying properties of the multi-spherical space, e.g. axon packing [6].