Multi-Spherical MRI: Breaking the Boundaries of Diffusion Time
Rutger Fick, Alexandra Petiet, Mathieu Santin, Anne-Charlotte Philippe,
Stephane Lehericy, Rachid Deriche, Demian Wassermann

To cite this version:

HAL Id: hal-01360440
https://hal.archives-ouvertes.fr/hal-01360440
Submitted on 5 Sep 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
1 The Multi-Spherical Space

**Diffusion restriction** occurs when water diffusion is obstructed by tissue boundaries. The amount of restriction is **time-dependent**, meaning that the observed diffusion coefficient will change for varying diffusion times [1].

Multi-Spherical MRI [2] describes diffusion restriction by fitting the diffusion signals over varying:
- **Gradient strength (G)**
- **Gradient direction (g)**
- **Diffusion time (τ)**

We call this four-dimensional space the **Multi-Spherical Space**.

We sampled this space on 35 different "shells", varying only \( g \), for different \( G \) ranging from [50-490] mT/m and \( \tau \) ranging from [9.1-18.3] ms.

2 Modeling the Multi-Spherical Space

Multi-Spherical MRI uses a separable **Fourier Basis** to reconstruct diffusion propagator \( P(r, \tau, c) \) from signal attenuation \( E(q, \tau, c) \), represented in coefficients \( c \).

\[
E(q, \tau, c) = \sum_{i, k} c_i \Phi_i(q) T_k(\tau) \quad \rightarrow \quad P(r, \tau, c) = \sum_{i, k} c_i \Psi_i(r) T_k(\tau)
\]

\( \Psi_i(r) = F^T(\Phi_i(q)) \): 3D Fourier basis over \( q \) and displacement \( r \) [3].

\( T_m(\tau) \): Exponential diffusion time basis over \( \tau \) [4].

We constrain the fitting of \( c \) to respect boundary conditions of the signal and impose **signal smoothness and sparsity**:

\[
\arg\min_c \int \left[ E(q, \tau) - E(q, \tau; c) \right]^2 dq d\tau + \int \left[ \nabla^2 E(q, \tau; c) \right]^2 dq d\tau + \alpha \left\| c \right\|_1
\]

Where smoothness is imposed using **closed-form Laplacian regularization**.

Once fitted, all \( q \)-space indices [3] can be estimated for any \( \tau \). As examples we show:
- Mean Squared Displacement (MSD), related to restriction
- Return-To-Origin Probability (RTOP), related to cellularity

3 In-Silico results

We study fitting performance under random subsampling by simulating the multi-spherical diffusion signal from gamma-distributed axons using Camino [5].

- Combined sparsity and Laplacian regularization produces the lowest fitting error (left).
- Time-dependent MSD and RTOP follow expected trends - MSD increasing and RTOP decreasing over time - down to about 200 DWIs (right two)

4 Application In-vivo Mouse Data

After eddy current correction, we chose an ROI of 173 voxels in Corpus Callosum. After subsampling we find:
- Stable fitting errors from 400 down to 200 DWIs
- Expected trends for time-dependent MSD and RTOP

5 Discussion and Conclusions

- Multi-Spherical MRI allows for the characterization of diffusion restriction through time-dependent \( q \)-space indices.
- Through signal sparsity and smoothness, our approach can represent the multi-spherical signal with less samples, allowing more realistic acquisition schemes.
- Additional signal or propagator constraints can be conveniently included in the optimization.
- Through resampling, our approach could be used as a pre-processing for other methods studying properties of the multi-spherical space, e.g. axon packing [6].

Acknowledgements: This work has received funding from the European Research Council (ERC) under the Horizon 2020 research and innovation program (ERC Advanced Grant agreement No 694665 : CoBoCoM)

References