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Global Position-Feedback Tracking Control of Flexible-joint Robots

Sofia Avila-Becerril Antonio Loría Elena Panteley

Abstract—We solve the open problem of *global tracking control of 2nd-degree under-actuated lossless (without friction) Lagrangian systems via position measurements only*. For flexible-joint robots, we design a dynamic controller which is based on measurements of link and joint positions only. Then, approximate differentiation is used for link velocities and a simple Luenberger observer for rotor velocities. The main results constitute a significant extension of recent work on observerless output-feedback control of Lagrangian systems. Strictly speaking, we establish uniform global asymptotic stability for the closed loop system.

I. INTRODUCTION

The tracking control problem by output-feedback for Euler-Lagrange systems has been widely discussed in the literature –[18], [12], [4], [6], [15], [14]. For the system

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = u \quad (1)$$

where $q \in \mathbb{R}^n$ and \dot{q} denotes the generalized positions and velocities (respectively), the maps $D : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ and $C : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ corresponds to the inertia matrix and the Coriolis and centrifugal forces matrix, respectively, while $g : \mathbb{R}^n \rightarrow \mathbb{R}^n$ stands for the vector of forces derived from the potential energy function and $u \in \mathbb{R}^n$ is the vector of control inputs. Assuming that only generalized positions are available for measurement, the problem consist in design an internally stable control law that ensures that

$$\lim_{t \rightarrow \infty} q(t) \rightarrow q_d(t), \quad \lim_{t \rightarrow \infty} \dot{q}(t) \rightarrow \dot{q}_d(t).$$

One of the first results on this line was reported in [10] where, in the context of robot tracking, the authors consider state feedback controllers with a nonlinear observer that reproduces the whole dynamic ensuring local asymptotic stability. A large number of articles were published (including those cited above) in which different stability properties are established: semiglobal asymptotic stability –[9], [1], global asymptotic stability in part of the coordinates [18], global asymptotic stability –[12], [6], global exponential stability –[15], [14], *etc.* In [18], [12] however, it is assumed that the system has inherent dissipative forces (friction); for lossless systems, only [4], [6], [15], [14] establish uniform global asymptotic stability. In [4], however, the result applies to one-degree-of-freedom systems. It is only in [6], [15], [14]

that the long-standing open problem of establishing uniform global asymptotic stability for the closed-loop system, was fully solved, independently. See also [5].

For flexible-joint robots, the output-feedback tracking control is even more complex since the relative degree with respect to link positions is augmented by two relative to (1). In [11] the authors presented a result based on a semi-global nonlinear observer for the unmeasured variables, which needs only the link positions. In [19] the authors propose a robust output-feedback link position tracking control ensuring semi-global uniformly ultimately bounded link position tracking; and in the same spirit, in [3] based on a set of filters, removes the need of measuring link and actuator velocities, however the result relies on the restrictive assumption that the system is internally damped by viscous friction. In [17] a nonlinear observer-based certainty-equivalence tracking controller for Euler Lagrange systems is presented; global conditions are obtained under the assumption that one disposes of a controller that can be bounded by an affine function in the position times a polynomial of the velocities.

In [8] we presented a controller for flexible-joint robots which ensures uniform global asymptotic stability for the closed-loop system under the assumption that joint, but not link velocities, are measured. The controller in the latter is based on the more general results recently published in [7]. In the latter we established uniform global asymptotic stability for n degrees of freedom systems with arbitrarily-high-relative-degree (with respect to generalized positions), without measurements of link velocities, the last includes under-actuated systems as the flexible joint robots. The control law is implemented through a chain of integrators in which the derivatives of virtual control inputs are replaced by approximate differentiation filters so that the result states a theoretical foundation for the use of the “dirty” derivatives.

In this paper, we follow the control method of [7], [8] and derive a certainty-equivalence controller which employs a reduced-order observer, in order to relax the assumption that joint velocities are measured. That is, under the assumption that only positions are available, we establish, for lossless flexible-joint robots, uniform global asymptotic stability in closed loop. As far as we know, there exists no other article in the literature establishing all this property under similar assumptions.

The rest of the paper is organized as follows: in Section II we formulate the global tracking problem and present some preliminary recent results. In Section III we present our main result. In Section IV we illustrate our findings in simulation,

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before concluding with some final remarks, in Section V.

II. PROBLEM FORMULATION AND PRELIMINARIES

According to [16], [2] joint flexibility in robot manipulators may be modelled using linear torsional springs; hence, if we denote by K the joint stiffness ($K = \infty$ implying no elasticity) the Lagrangian model is given by the equations

$$D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + G(q_1) = K(q_2 - q_1) \quad (2a)$$

$$J\ddot{q}_2 + K(q_2 - q_1) = \tau \quad (2b)$$

where $q_1 \in \mathbb{R}^n$ denote the under-actuated coordinates related to the links positions while $q_2 \in \mathbb{R}^n$ represents the vector of motor shafts angles. The matrix $D(q_1) = D^\top(q_1) > 0$ corresponds to the link-inertia matrix, J stands for the diagonal rotor-inertia matrix, $C(q_1, \dot{q}_1)\dot{q}_1$ contains the terms due to the Coriolis and centrifugal forces, G represents the gravitational forces vectors and τ corresponds to the physical control input. As it is customary, we make the following hypothesis on the model (2).

Assumption 1:

- 1) The inertia matrix $D(q_1)$ is positive definite and uniformly bounded; there exist positive real numbers d_m and d_M such that

$$d_m \leq |D(q_1)| \leq d_M$$

- 2) The matrix $C(x, y)$ is bounded in x and linear in y . More precisely, there exists a saturation function $\text{sat} : \mathbb{R} \rightarrow \mathbb{R}$ such that, for all $x, y, z \in \mathbb{R}^n$,

$$|C(x, y) - C(z, y)| \leq k_c \text{sat}(|x - z|)|y|, \quad |\text{sat}(s)| \leq 1,$$

$$C(x, y)z = C(x, z)y,$$

- 3) The matrix $\dot{D}(q_1) - 2C(q_1, \dot{q}_1)$ is skew symmetric for all $q_1 \in \mathbb{R}^n$.
- 4) There exists k_v such that function that represents the potential-energy forces, G , satisfies

$$\exists k_v > 0 : \left| \frac{\partial G}{\partial q_1} \right| \leq k_v \quad (3)$$

Under these conditions, we solve the global tracking control problem via position measurements only. More precisely, for any given reference trajectory $t \mapsto q_d$, twice continuously differentiable and bounded, *i.e.*, such that $k_\delta > 0$ such that

$$\max \left\{ \sup_{t \geq 0} |q_d(t)|, \sup_{t \geq 0} |\dot{q}_d(t)|, \sup_{t \geq 0} |\ddot{q}_d(t)| \right\} \leq k_\delta \quad (4)$$

for some $k_\delta > 0$, it is required to construct an output-feedback dynamic controller

$$\dot{x}_c = f(t, q_1, q_2, x_c) \quad (5)$$

$$u = u(t, q_1, q_2, x_c) \quad (6)$$

(that is, $x_c \in \mathbb{R}^m$ corresponds to the state of the controller) such that the origin of the closed-loop system is uniformly globally asymptotically stable. In order to solve this problem

we follow the method of [7] which relies on the use of approximate differentiators to replace unavailable derivatives, such as velocity measurements.

For the sake of clarity, we first recall the main streamlines of the control method proposed in [7]. It concerns Lagrangian systems augmented by a chain of integrators; for the purposes of this paper we consider the case of two additional states, *i.e.*,

$$D(q_1)\ddot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_1 + g(q_1) = \xi_1 \quad (7a)$$

$$\dot{\xi}_1 = \xi_2 \quad (7b)$$

$$\dot{\xi}_2 = u. \quad (7c)$$

We remark that the model (2) may be transformed into (7) via a preliminary feedback and a change of variable. Indeed, this follows after a direct calculation, defining $g(q_1) = G(q_1) + Kq_1$, $\xi_1 = Kq_2$, $\xi_2 = K\dot{q}_2$ and setting the input torque to

$$\tau = \xi_1 - Kq_1 + JK^{-1}u. \quad (8)$$

Thus, the output-feedback tracking control problem boils down to stabilizing $(q_{1d}, \dot{q}_{1d}, 0, 0)$ without measurement of \dot{q}_1 and ξ_2 .

In [7] a controller that depends on all motor variables (q_2 and \dot{q}_2) but not on link velocity measurements, \dot{q}_1 is proposed. In this paper, we relax the assumption that \dot{q}_2 is measurable. The output-feedback control design method proposed in [7] follows the rationale of backstepping control, in which unavailable derivatives are replaced by approximate differentiation. Hence, the starting point is to consider ξ_1 as a virtual control input in (7) and to design a virtual control law ξ_1^* which is set as reference for the integrator (7b) variable ξ_i , *i.e.*,

$$\xi_1^* = -k_{p_0}\tilde{q}_1 - k_{d_0}\vartheta_0 + D(q_1)\ddot{q}_{1d} + C(q_1, \dot{q}_{1d})\dot{q}_{1d} + g(q_1), \quad (9)$$

with $\tilde{q}_1 := q_1 - q_{1d}(t)$, whereas ϑ_0 corresponds to the approximate differentiation of \tilde{q}_1 that is,

$$\dot{q}_{c0} = -a_0\vartheta_0 \quad (10a)$$

$$\vartheta_0 = q_{c0} + b_0\tilde{q}_1. \quad (10b)$$

Then, considering ξ_2 as a control input in (7b) we define

$$\left. \begin{aligned} \xi_2^* &= -k_{p_1}\tilde{\xi}_1 + k_{d_1}\vartheta_1 + \dot{\xi}_{10}^*, \\ \vartheta_i &= q_{c_i} + b_i\xi_i^* + \zeta_i \\ \dot{q}_{c_i} &= -a_i\vartheta_i \\ \zeta_i &= -(k_{d_i} - \sigma_i)\tilde{\xi}_i - b_i\dot{\xi}_{i0}^* \end{aligned} \right\} \forall i \in [1, 2] \quad (11a)$$

$$u = -k_{p_2}\tilde{\xi}_2 + k_{d_2}\vartheta_2 + \dot{\xi}_{20}^* \quad (11b)$$

where $\tilde{\xi}_i := \xi_i - \xi_i^*$, with $\sigma_1 = 0$ and $\sigma_2 = b_2k_{p_1}$. It is proved in [7] that the origin $\{z = 0\}$, with $z = [\tilde{q}_1^\top \ \tilde{\dot{q}}_1^\top \ \vartheta_0^\top \ \tilde{\xi}_1^\top \ \tilde{\xi}_2^\top \ \vartheta_1^\top \ \vartheta_2^\top]^\top$ and $\tilde{q}_1 := \dot{q}_1 - \dot{q}_{1d}(t)$, is uniformly globally asymptotically stable. Nevertheless, the controller (11) is based on the measurement of ξ_1 and ξ_2 , which involves the measurement of motor shaft angular velocity, \dot{q}_2 , since $\xi_2 = K\dot{q}_2$.

We remark that the dynamic controller (11) is designed on the basis of a backstepping design however, notice that the derivatives of ξ_1^* and ξ_2^* depend on unmeasurable variables such as velocities and accelerations hence, in place of $\dot{\xi}_1^*$ and $\dot{\xi}_2^*$, we employ approximate differentiation. This is implemented via the filters defined by (11a). To see this more clearly notice that these equations are equivalent to the Laplace representations:

$$\vartheta_i = \frac{b_i}{s + a_i} \dot{\psi}_i \Leftrightarrow \vartheta_i = \frac{b_i s}{s + a_i} \psi_i \quad (12)$$

with input $\dot{\psi}_i = \dot{\xi}_i^* + \dot{\zeta}_i/b_i$ and $\psi_0 = \tilde{q}$. This filter defines an output-strictly proper passive map $\vartheta_i \mapsto \dot{\psi}_i$ that is internally stable and input to state stable with respect to the input $\dot{\psi}_i$. However, it does not constitute an observer in the sense that one does not have $\vartheta_i \rightarrow \dot{\psi}_i$, in particular, $\vartheta_0 \rightarrow \dot{\tilde{q}}$

III. MAIN RESULT

Our main statement (Proposition 1 below) is to establish a separation principle via position-feedback control of robot manipulators with flexible joints. We use a certainty-equivalence controller based on (11) and a simple reduced-order Luenberger observer. The latter is possible since the observer is designed only to estimate the unavailable state ξ_2 , while other unmeasured variables such as link velocities, are “estimated” via dirty derivatives. While the design may be considered as naive, we remark that, as far as we know, there does not exist in the literature, a statement of uniform *global* asymptotic stability.

We define the reduced-order observer

$$\dot{\hat{\xi}}_1 = \hat{\xi}_2 + \lambda_1(\xi_1 - \hat{\xi}_1), \quad (13a)$$

$$\dot{\hat{\xi}}_2 = u + \lambda_2(\xi_1 - \hat{\xi}_1) \quad (13b)$$

where λ_1 and λ_2 are positive constant observer gains. Now, let the observer error $e = [e_1^\top \ e_2^\top]^\top \in \mathbb{R}^{2n}$ be defined as $e = \xi - \hat{\xi}$ which, along (7) and (13), satisfies

$$\dot{e}_1 = e_2 - \lambda_1 e_1 \quad (14a)$$

$$\dot{e}_2 = -\lambda_2 e_1 \quad (14b)$$

Then, defining $\bar{\xi} = \hat{\xi} - \xi^*$, consider the certainty equivalent controller

$$\xi_2^* = -k_{p1} \bar{\xi}_1 + k_{d1} \vartheta_1 + \dot{\xi}_{10}^*, \quad (15a)$$

$$\left. \begin{aligned} \vartheta_i &= q_{ci} + b_i \xi_i^* + \zeta_i \\ \dot{q}_{ci} &= -a_i \vartheta_i \\ \dot{\zeta}_i &= -(k_{di} - \sigma_i) \bar{\xi}_i - b_i \dot{\xi}_{i0}^* \end{aligned} \right\} \forall i \in \{1, 2\} \quad (15b)$$

$$u = -k_{p2} \bar{\xi}_2 + k_{d2} \vartheta_2 + \dot{\xi}_{20}^* \quad (15c)$$

where k_{pi} and k_{di} , which denote “proportional” and “derivative” control gains respectively, are positive constants. Then, we have the following.

Proposition 1: Let q_{1d} be given as in (4). Consider the system (2) under Assumption 1 in closed loop with the controller defined by Equations (8), (9) and (15). Then,

the origin of the closed-loop system is uniformly globally asymptotically stable for sufficiently large control gains. Moreover, this holds for any positive observer gains λ_1, λ_2 , independently of the control gains.

The proof is constructive, we derive explicit conditions on the control gains that imply uniform global asymptotic stability. It is organized in the following logical steps: first, we derive the closed-loop equations; then, we recognize that the closed-loop dynamics has a cascaded form which stems from the application of certainty equivalence principle. The latter allows to invoke a cascades argument. Namely, that for a nonlinear time-varying system

$$\dot{x}_1 = f_1(t, x_1) + g(t, x) x_2 \quad (16a)$$

$$\dot{x}_2 = f_2(t, x_2) \quad (16b)$$

where $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$, $x := \text{col}[x_1, x_2]$ with f_1, f_2 and g continuous and locally Lipschitz in x , uniformly in t , and f_1 continuously differentiable, it holds that if the respective origins of the subsystems (16b) and $\dot{x}_1 = f_1(t, x_1)$ are uniformly globally asymptotically stable, it is sufficient and necessary for the origin of the overall system to possess the same property, that the solutions of (16) be uniformly globally bounded. This property is established along the proof-lines of [7, Theorem 5].

A. The closed-loop equations

The first error equation is obtained using the identity $\xi_1 = \tilde{\xi}_1 + \xi_1^*$, by replacing ξ_1^* from (9) in (7a) and adding $-C(q_1, \dot{q}_{1d})\dot{q}_1 + C(q_1, \dot{q}_1)\dot{q}_{1d} = 0$ to the right-hand side of (15c). Then, we differentiate on both sides of (10b) and we use (10a) to obtain

$$D(q_1)\ddot{\tilde{q}}_1 + [C(q_1, \dot{q}_1) + C(q_1, \dot{q}_{1d})]\dot{\tilde{q}}_1 + k_{p0}\tilde{q}_1 + k_{d0}\vartheta_0 = \tilde{\xi}_1 \quad (17a)$$

$$\dot{\vartheta}_0 = -a_0\vartheta_0 + b_0\dot{\tilde{q}}_1. \quad (17b)$$

On the other hand, note that the equations (7b)–(7c) are equivalent to

$$\dot{\tilde{\xi}}_1 = \xi_2^* - \dot{\tilde{\xi}}_1 + \tilde{\xi}_2 \quad (18)$$

$$\dot{\tilde{\xi}}_2 = u - \dot{\tilde{\xi}}_2 \quad (19)$$

so, using the identity $\bar{\xi}_i = \tilde{\xi}_i - e_i$ for $i \in \{1, 2\}$ as well as (15), we obtain

$$\dot{\tilde{\xi}}_1 = -k_{p1}\tilde{\xi}_1 + k_{d1}\vartheta_1 + \tilde{\xi}_2 + k_{p1}e_1 - (\dot{\tilde{\xi}}_1 - \dot{\xi}_{10}^*) \quad (20a)$$

$$\dot{\tilde{\xi}}_2 = -k_{p2}\tilde{\xi}_2 + k_{d2}\vartheta_2 + k_{p2}e_2 - (\dot{\tilde{\xi}}_2 - \dot{\xi}_{20}^*) \quad (20b)$$

$$\dot{\vartheta}_i = -a_i\vartheta_i - (k_{di} - \sigma_i)\tilde{\xi}_i + (k_{di} - \sigma_i)e_i + b_i(\dot{\xi}_i^* - \dot{\xi}_{i0}^*) \quad (20c)$$

where $\sigma_1 = 0$.

Next, to compact the notation, we define $A := \text{diag}\{a_1, a_2\} \otimes I_n$, $B := \text{diag}\{b_1, b_2\} \otimes I_n$, $K_d := \text{diag}\{k_{d1}, k_{d2}\} \otimes I_n$, $K_p := \text{diag}\{k_{p1}, k_{p2}\} \otimes I_n$, $K'_d := \text{diag}\{k_{d1}, k_{d2} - \sigma_2\} \otimes I_n$,

$$K'_p = \begin{bmatrix} k_{p1} & -1 \\ 0 & k_{p2} \end{bmatrix}, \quad L = \begin{bmatrix} \lambda_1 & -1 \\ \lambda_2 & 0 \end{bmatrix}$$

and defining $\vartheta^\top = [\vartheta_1^\top \quad \vartheta_2^\top]$, then Equations (20) take the form

$$\dot{\xi} = -K'_p \tilde{\xi} + K_d \vartheta + K_p e - [\dot{\xi}^* - \dot{\xi}_0^*] \quad (21a)$$

$$\dot{\vartheta} = -A\vartheta - K'_d \tilde{\xi} + K'_d e + B[\dot{\xi}^* - \dot{\xi}_0^*] \quad (21b)$$

$$\dot{e} = -Le \quad (21c)$$

where the matrix $-L$ is Hurwitz.

These equations have a convenient structure. Firstly, the terms in brackets, $[\dot{\xi}^* - \dot{\xi}_0^*]$, vanish at the origin. This follows, on one hand, from the identity

$$\dot{\xi}_2^* - \dot{\xi}_{20}^* = -k_{p_1} \bar{\xi}_2 + \eta \bar{\xi}_1 - \mu \vartheta_1 + \beta [\dot{\xi}_1^* - \dot{\xi}_{10}^*] - \lambda_1 e_1 \quad (22)$$

where $\mu = k_{d_1}(k_{p_1} + a_1)$, $\eta = k_{p_1}^2 - k_{d_1}^2$, and $\beta = k_{p_1} + b_1 k_{d_1}$ and $\bar{\xi} = \xi - e$. On the other hand, Assumption 1, together with (4), guarantees that $\xi_1^*(t, q, \dot{q}, \vartheta_0)$ is globally Lipschitz in the last two arguments, uniformly in t and that is bounded in the first two arguments, so there exist non-negative real numbers η_1 , η_2 and η_3 as well as continuous saturation function $\text{sat} : \mathbb{R} \rightarrow \mathbb{R}$ such that $\text{ysat}(y) > 0$ for all $y \neq 0$ and $|\text{sat}(y)| \leq 1$, and

$$|[\dot{\xi}_1^* - \dot{\xi}_{10}^*]| \leq \eta_1 \text{sat}(|\bar{q}|) + \eta_2 |\dot{\bar{q}}_2| + \eta_3 |\vartheta_0|. \quad (23)$$

Furthermore, the variable $\dot{\xi}_2^*$ can be written as a function of $\tilde{\xi}$, ϑ , e and $\dot{\xi}_1^*$; therefore, in view of (15a) and (20) we have

$$\begin{aligned} \dot{\xi}_2^* &= \eta \tilde{\xi}_1 - \mu \vartheta_1 - (\eta + k_{p_1} \lambda_1) e_1 \\ &\quad - k_{p_1} \tilde{\xi}_2 + k_{p_1} e_2 + \beta [\dot{\xi}_1^* - \dot{\xi}_{10}^*] + \dot{\xi}_{10}^* \end{aligned} \quad (24)$$

So that equations (21) take the form

$$\begin{aligned} \dot{\xi} &= -(K'_p + \Gamma_1) \tilde{\xi} + (K_d - \Gamma_2) \vartheta + (K_p + \Gamma_4) e \\ &\quad - \Gamma_3 [\dot{\xi}_1^* - \dot{\xi}_{10}^*] \end{aligned} \quad (25a)$$

$$\begin{aligned} \dot{\vartheta} &= -(K_2 - B\Gamma_1) \tilde{\xi} - (A - B\Gamma_2) \vartheta + (K_2 - B\Gamma_4) e \\ &\quad + (B\Gamma_3) [\dot{\xi}_1^* - \dot{\xi}_{10}^*] \end{aligned} \quad (25b)$$

$$\dot{e} = -Le \quad (25c)$$

where we defined

$$\begin{aligned} \Gamma_1 &= \begin{bmatrix} 0 & 0 \\ \eta & -k_{p_1} \end{bmatrix} \otimes I_n, \\ \Gamma_2 &= \begin{bmatrix} 0 & 0 \\ -\mu & 0 \end{bmatrix} \otimes I_n, \\ \Gamma_3 &= \begin{bmatrix} 1 \\ \beta \end{bmatrix} \otimes I_n, \\ \Gamma_4 &= \begin{bmatrix} 0 & 0 \\ k_{p_1} \lambda_1 + \eta & -k_{p_1} \end{bmatrix} \otimes I_n, \\ K_2 &= \begin{bmatrix} k_{d_1} & 0 \\ 0 & k_{d_2} - \sigma_2 \end{bmatrix} \otimes I_n \end{aligned}$$

Now, if we define $x_1 := [\tilde{\xi}^\top \quad \vartheta^\top]^\top$ and $x_2 := e$ the system (25) becomes

$$\dot{x}_1 = \mathcal{A}x_1 + \mathcal{B} [\dot{\xi}_1^* - \dot{\xi}_{10}^*] + K'x_2 \quad (26a)$$

$$\dot{x}_2 = -Lx_2 \quad (26b)$$

where

$$\mathcal{A} = \begin{bmatrix} -(K'_p + \Gamma_1) & K_d - \Gamma_2 \\ -(K_2 - B\Gamma_1) & -(A - B\Gamma_2) \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} -\Gamma_3 \\ B\Gamma_3 \end{bmatrix}$$

$$K' = \begin{bmatrix} K_p + \Gamma_4 \\ K_2 - B\Gamma_1 \end{bmatrix}.$$

This system has a convenient cascaded form, as (16). Moreover, since L is Hurwitz by design, the origin of system (26b) is uniformly globally exponentially stable. Furthermore, after [7], the origin of

$$\dot{x}_1 = \mathcal{A}x_1 + \mathcal{B} [\dot{\xi}_1^* - \dot{\xi}_{10}^*]$$

is uniformly globally asymptotically stable for appropriate values of the control gains. In order to invoke the cascades argument, more precisely, [13, Lemma 2], it is left to prove that the solutions of (26a) are uniformly globally bounded.

B. Boundedness of solutions

Proposition 2: Let L be Hurwitz, $k_{p_1} = k_{d_1}$ and assume that

$$\min \left\{ k_{p_1}, \frac{[k_{p_2} - k_{p_1}]}{\beta^2}, \frac{a_1}{b_1^2}, \frac{a_2}{b_2^2 \beta^2} \right\} > [\eta_2^2 + \eta_3^2] \quad (27)$$

$$[k_{p_2} - k_{p_1}] \geq \max \left\{ \frac{2}{k_{p_1}}, \frac{4\mu^2}{a_1} \right\}, \quad a_1 a_2 \geq 2b_2^2 \mu^2. \quad (28)$$

Then, the solutions of the closed-loop system (17), (26) are uniformly globally bounded.

Proof. Let the Hurwitz property of L generate positive definite matrices P_L, Q_L , satisfying $-Q_L := (L^\top P_L + P_L L)$ and let $\kappa > 0$ be a real constant. Then, consider the function $W : \mathbb{R}^{10n} \rightarrow \mathbb{R}_{\geq 0}$, defined as

$$W(x) = \kappa \|x_1\|^2 + x_2^\top P_L x_2; \quad (29)$$

a direct computation shows that its total derivative along the trajectories of (26) yields

$$\begin{aligned} \dot{W}(x) &= \kappa [x_1^\top (\mathcal{A} + \mathcal{A}^\top) x_1 + 2x_1^\top \mathcal{B} [\dot{\xi}_1^* - \dot{\xi}_{10}^*] + \\ &\quad + 2x^\top P K' x_2] - x_2^\top Q_L x_2. \end{aligned}$$

Now, let $Q = -(\mathcal{A}^\top + \mathcal{A})$; we see that, since $k_{p_1} = k_{d_1}$,

$$Q = \begin{bmatrix} 2k_{p_1} & -1 & 0 & 0 \\ -1 & 2[k_{p_2} - k_{p_1}] & -\mu & 0 \\ 0 & -\mu & 2a_1 & -b_2 \mu \\ 0 & 0 & -b_2 \mu & 2a_2 \end{bmatrix} \otimes I_n. \quad (30)$$

Hence, defining $c := |K'|$, $\lambda := |L|$, using the Cauchy-Schwartz and the triangle inequalities, we obtain

$$\begin{aligned} \dot{W} &\leq -\kappa x_1^\top Q x_1 + 2\kappa x_1^\top \mathcal{B} [\dot{\xi}_1^* - \dot{\xi}_{10}^*] + 2\kappa c |x_1| |x_2| - \lambda |x_2|^2 \\ &\leq -\kappa x_1^\top Q x_1 + 2\kappa x_1^\top \mathcal{B} [\dot{\xi}_1^* - \dot{\xi}_{10}^*] + \frac{\kappa c}{\epsilon} |x_1|^2 \\ &\quad - (\lambda - \kappa c \epsilon) |x_2|^2 \end{aligned}$$

Now, notice that the matrix $Q - \frac{1}{2} \text{diag}\{Q\}$ is positive semidefinite if so are

$$\begin{bmatrix} k_{p_1} & -1 \\ -1 & \frac{1}{2}[k_{p_2} - k_{p_1}] \end{bmatrix},$$

$$\begin{bmatrix} \frac{1}{2}[k_{p2} - k_{p1}] & -\mu \\ -\mu & \frac{1}{2}a_1 \end{bmatrix}, \begin{bmatrix} \frac{1}{2}a_1 & -b_2\mu \\ -b_2\mu & a_2 \end{bmatrix},$$

which hold in view of (28). Therefore, under these conditions, we see that

$$\begin{aligned} \dot{W} \leq & -\kappa x_1^\top \left(\frac{1}{2} \text{diag} \{Q\} - \frac{c}{\epsilon} I \right) x_1 + 2\kappa x_1^\top \mathcal{B} \left[\dot{\xi}_1^* - \dot{\xi}_{10}^* \right] \\ & - (\lambda - \kappa c \epsilon) |x_2|^2 \end{aligned} \quad (31)$$

Notice that the factors of $|x_1|^2$ and $|x_2|^2$ are negative for any $\lambda > 0$, sufficiently large values of ϵ and $\kappa := 1/\epsilon^2$. On the other hand, under Assumption 1, from Inequality (23) and the triangle inequality, it follows that

$$\begin{aligned} x_1^\top \mathcal{B} \left[\dot{\xi}_1^* - \dot{\xi}_{10}^* \right] \leq & \eta_1 (|\tilde{\xi}_1| + \beta_1 |\tilde{\xi}_2| + b_1 |\vartheta_1| + b_2 \beta_1 |\vartheta_2|) \\ & + \frac{1}{2} (\eta_2^2 + \eta_3^2) (|\tilde{\xi}_1|^2 + \beta_1^2 |\tilde{\xi}_2|^2 + b_1^2 |\vartheta_1|^2 + b_2^2 \beta_1^2 |\vartheta_2|^2) \\ & + 2|\dot{\hat{q}}_1|^2 + 2|\vartheta_0|^2. \end{aligned} \quad (32)$$

Next, let $\mathcal{V} : \mathbb{R}_{\geq 0} \times \mathbb{R}^{13n} \rightarrow \mathbb{R}_{\geq 0}$ be defined as $V(t, \tilde{q}_1, \dot{\hat{q}}_1, \vartheta_0) + W(x)$, where

$$V = \frac{\kappa}{2} \left(\dot{\hat{q}}_1^\top D(\tilde{q}_1 + q_{1d}(t)) \dot{\hat{q}}_1 + k_{p0} |\tilde{q}_1|^2 + \frac{k_{d0}}{b_0} |\vartheta_0|^2 \right) \quad (33)$$

The total derivative of $\mathcal{V} := V + W$ along the trajectories of (17) and (26) yields

$$\begin{aligned} \dot{\mathcal{V}} \leq & -\frac{a_0 k_{d0} \kappa}{b_0} |\vartheta_0|^2 + k_c k_\delta \kappa |\dot{\hat{q}}_1|^2 + \kappa \dot{\hat{q}}_1^\top \tilde{\xi}_1 - (\lambda - \kappa c \epsilon) |x_2|^2 \\ & - \kappa \dot{\hat{q}}_1 e_1 - \kappa x_1^\top \left(\frac{1}{2} \text{diag} \{Q\} - \frac{c}{\epsilon} I \right) x_1 + 2\kappa x_1^\top \mathcal{B} \left[\dot{\xi}_1^* - \dot{\xi}_{10}^* \right] \end{aligned}$$

that in view of (32) can be expressed as

$$\begin{aligned} \dot{\mathcal{V}} \leq & -\kappa x_1^\top \left(\frac{1}{2} \text{diag} \{Q\} - \frac{c}{\epsilon} I \right) x_1 - \left(\lambda - \frac{\kappa}{2} - \kappa c \epsilon \right) |x_2|^2 \\ & + \frac{\kappa}{2} |\tilde{\xi}_1|^2 + 2\kappa \eta_1 (|\tilde{\xi}_1| + \beta_1 |\tilde{\xi}_2| + b_1 |\vartheta_1| + b_2 \beta_1 |\vartheta_2|) \\ & + \kappa (\eta_2^2 + \eta_3^2) (|\tilde{\xi}_1|^2 + \beta_1^2 |\tilde{\xi}_2|^2 + b_1^2 |\vartheta_1|^2 + b_2^2 \beta_1^2 |\vartheta_2|^2) \\ & + \kappa \left(\frac{7}{2} + k_c k_\delta \right) |\dot{\hat{q}}_1|^2 - \kappa \left(\frac{a_0 k_{d0}}{b_0} - 2 \right) |\vartheta_0|^2 \end{aligned} \quad (34)$$

By assumption, the quadratic term in x_1 is negative definite and dominates over all positive terms, except on that involving $|\dot{\hat{q}}_1|^2$.

The rest of the proof of boundedness follows as for Theorems 5 and 6 of [7]. Roughly speaking in the latter it is established that, even though $\vartheta_0 \not\rightarrow \dot{\hat{q}}_1$, the *output-input* gain of the filter (12) is finite. More precisely, to any monotonically increasing sequence $\{[|\dot{\hat{q}}_1|^2]_i\}$ where $[|\dot{\hat{q}}_1|^2]_i = |\dot{\hat{q}}_1(t_i)|^2$, corresponds a sequence $\{[|\vartheta_0|^2]_i\}$ which increases at the *same* rate. Therefore, if $\{|\dot{\hat{q}}_1(t)|^2\}$ grows unboundedly so does $\{|\vartheta_0(t)|^2\}$ hence, sequentially, $\dot{\mathcal{V}}(t_i)$ becomes non-positive and $\{\mathcal{V}(t_i)\}$ is bounded. Rigorous proof along these arguments is provided in [7].

Finally, the statement of Proposition 1 follows invoking [13, Lemma 2].

IV. SIMULATIONS RESULTS

In order to evaluate the global characteristic of the controller (15) we present some numerical simulations. To this end, we consider a two-degrees-of-freedom planar flexible joint robot whose model is given by

$$D(q_1) = \begin{bmatrix} 8.77 + 1.02c_{12} & 0.76 + 0.51c_{12} \\ 0.76 + 0.51c_{12} & 0.62 \end{bmatrix}$$

$$C(q_1, \dot{q}_1) = \begin{bmatrix} -0.51s_{12}\dot{q}_{12} & -0.51s_{12}(\dot{q}_{11} + \dot{q}_{12}) \\ 0.51s_{12}\dot{q}_{11} & 0 \end{bmatrix}$$

where c_{12} and s_{12} denotes the $\cos(q_{12})$ and $\sin(q_{12})$ respectively. Furthermore, the joint stiffness matrix is $K = \text{diag}\{10000, 10000\}$ while the rotor inertia matrix is $J = \text{diag}\{0.1, 0.1\}$. The gravitational forces vector $g(q_1)^\top = [9.81(7.6s_{11} + 0.63s_{11,12}) \quad 9.81(0.63s_{11,12})]$ with $s_{11,12} = \sin(q_{11} + q_{12})$. And we have set the desired value of the under-actuated coordinates to $q_{1d}(t) := \sin(\omega t)$ rad, with $\omega_1 = 2$ rad/s.

With the aim at illustrating the *global* character of the stability enhanced by our controller, we purposefully performed a simulation test with an unrealistic 5000% of initial error. The control parameters for the filters were set to $a_0 = 500$, $b_0 = 500$, $a_1 = 2500$, $b_1 = 5$, $a_2 = 5000$, $b_2 = 5$, the control gains were fixed in $k_{p0} = 400$, $k_{p1} = 1$, $k_{p2} = 8000$ and $k_{d0} = 35$, $k_{d1} = 500$, $k_{d2} = 10$, while the observer parameters $\lambda_1 = 10$ and $\lambda_2 = 20$. In Figures 1 and 2 we show the position and the velocity of the under-actuated coordinates q_1 and \dot{q}_1 respectively. These graphs show the proper functioning of the controller and its global property. In Figure 3 we depict the observer performance.

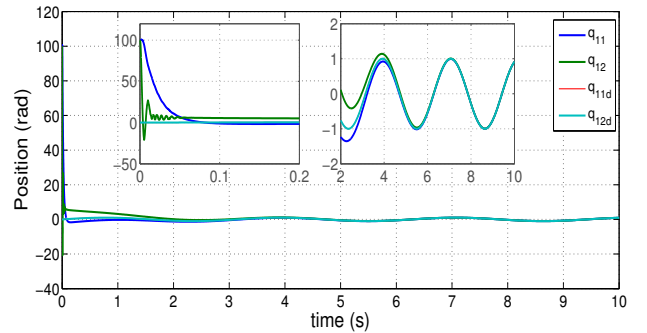


Fig. 1. Link angular positions and reference

V. CONCLUSIONS

In this paper we make an extension of the results in [7] in a way that the output-feedback problem can be solved based only in the measurement of the link position preserving the uniformly globally asymptotic stability property for the closed-loop system. The result is based on the use of a Luenberger observer and in particular, solves the problem for the flexible joint robots under the conditions: the system is lossless and motor velocities are unmeasured.

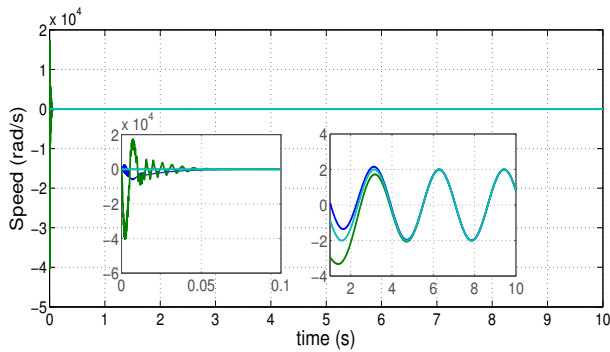


Fig. 2. Link angular velocities and reference

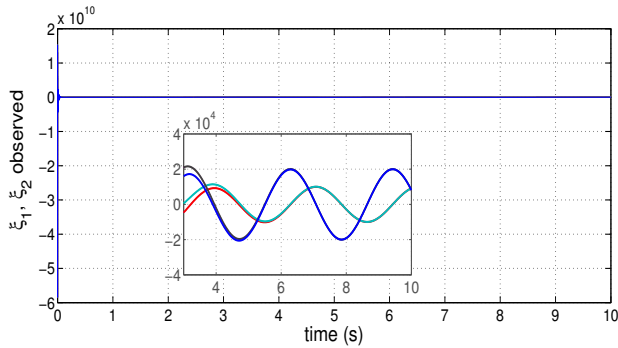


Fig. 3. Observed variables $\hat{\xi}_1$, $\hat{\xi}_2$ and references.

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