Optimal Transportation for Data Assimilation
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Data assimilation

Given
- a physical system and its state \(x(t,x,y)\);
- partial observations of the system \(y^i\);
- a (numerical) model \(M\) simulating the evolution of \(x\);

\(\nabla \cdot g \cdot \nabla h\).

Can we estimate the initial condition \(x_0\) of the system?

\(
\begin{align*}
J(x_0) := & \sum_i d\left(H(M(x_0), \gamma^i) \right)^2 + \omega d(\rho(x_0), \rho)^2. \\
\end{align*}
\)

(1)

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Results on a shallow-water equation

Let the model \(M\) be a shallow-water equation, with initial condition \((h_0, u_0)\).

\(M:\)
\[
\begin{align*}
\frac{\partial h}{\partial t} + \text{div}(hu) = 0, \\
\frac{\partial u}{\partial t} + u \cdot \nabla u = -g \nabla h.
\end{align*}
\]

We control the initial condition \(h_0\) only, thanks to the Wasserstein cost function \(J_W\): We set \(u_0 = 0\).

\(\begin{array}{cccc}
t_0 & t_2 & t_4 & t_6 \\
\end{array}\)

Results:

Values of \(h\) and \(u\) for the background and true states, as well as analysis for Euclidean and Wasserstein distances, at time \(t = t_6\)

### Specificities on using the Wasserstein distance

- The Wasserstein distance is only defined for probability measures, i.e. \(\rho\) s.t.
  \[\rho \geq 0 \quad \text{and} \quad \int \rho = 1\]

- Relaxations of the latter constraint are possible, however complex:
  - the \(W_2\) interpolation works well if \(\rho_0\) and \(\rho_1\) are of distinct support;

- when \(J(\rho) \rightarrow \min_{\rho \in \mathcal{P}} J(\rho)\), then there is only weak convergence of \(\rho^n\) to \(\rho^{opt}\): oscillations or diracs can occur!

- Computing the Wasserstein distance is expensive [Peyré, Papadakis, Oudet, 2013].