The Qubit in de Broglie-Bohm Interpretation

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How to explain spin's quantification ?

- Either by the measurement postulates of quantum theory
- Either by Pauli equation with spatial extension of the spinor

Representation of the particle with spin

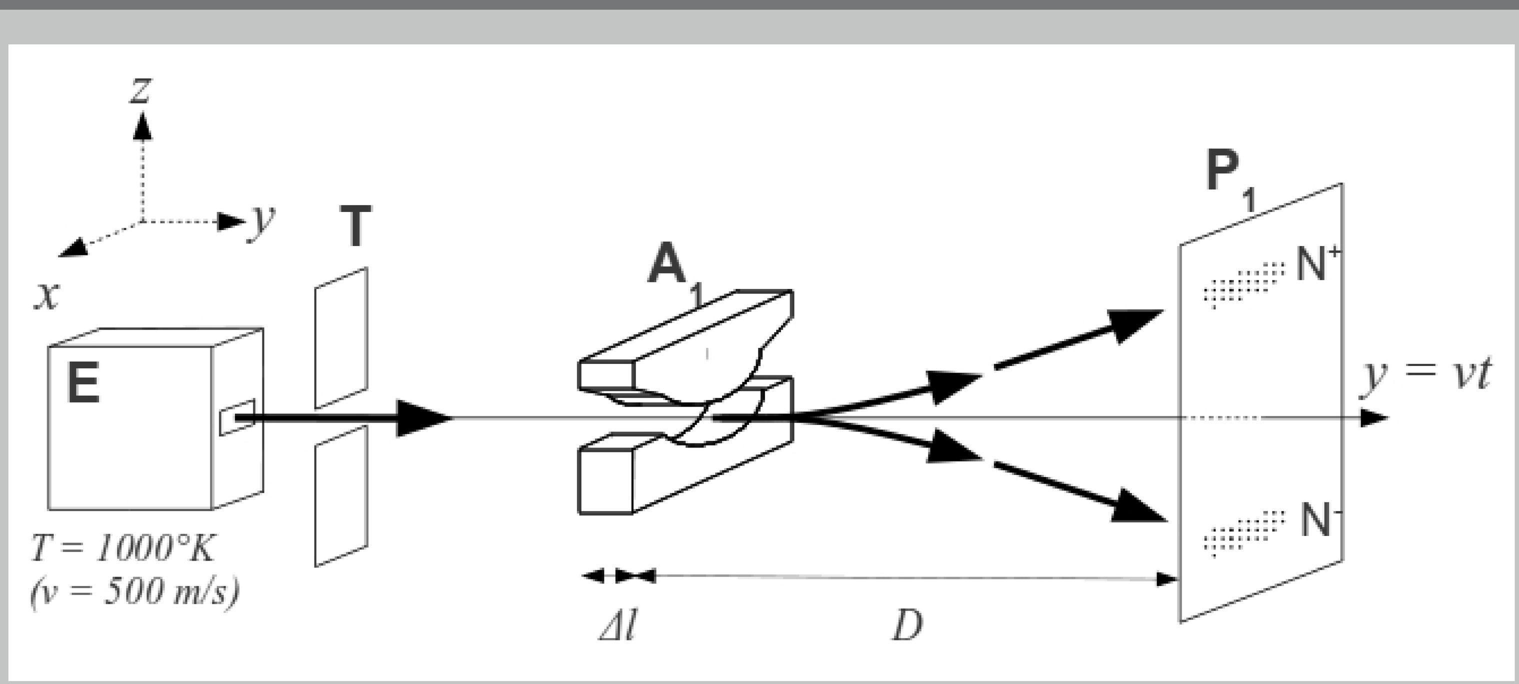
- Complete spinor with **spatial extension**

$$\Psi^0(z) = (2\pi\sigma_0^2)^{-\frac{1}{4}} e^{-\frac{z^2}{4\sigma_0^2}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2} e^{i\frac{\varphi_0}{2}} \end{pmatrix}$$

- Simplified spinor used in quantum information (qubit)

$$\Psi^0 = \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2} e^{i\frac{\varphi_0}{2}} \end{pmatrix}.$$

Stern-Gerlach experiment



$$\Psi^0(z) = (2\pi\sigma_0^2)^{-\frac{1}{4}} e^{-\frac{z^2}{4\sigma_0^2}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-i\frac{\varphi_0}{2}} \\ \sin\frac{\theta_0}{2} e^{i\frac{\varphi_0}{2}} \end{pmatrix}$$

- **Pure state** : θ_0 and φ_0 fixed
- **Mixed states** : θ_0 and φ_0 randomly drawn

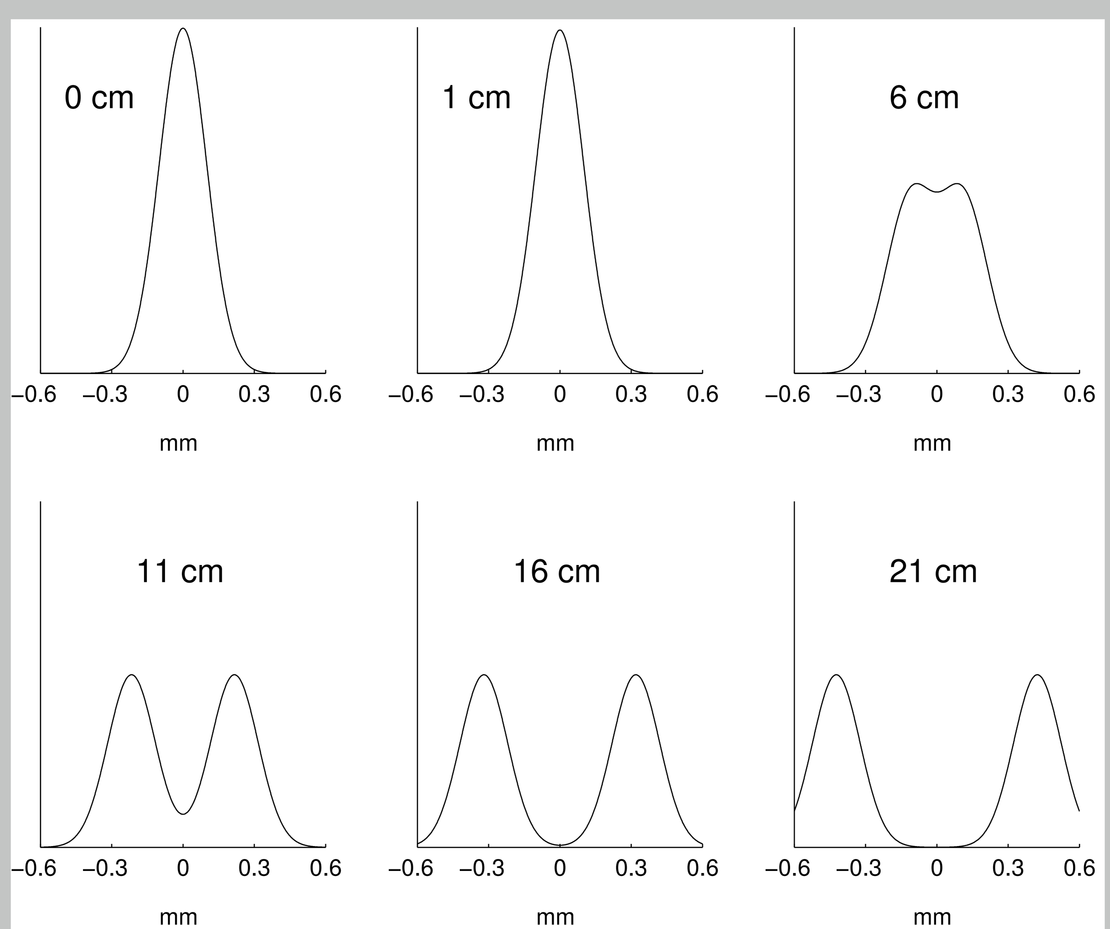
Pauli equation

- After the magnetic field: at $t + \Delta t$

$$\Psi(z, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-\frac{(z-z_\Delta-ut)^2}{4\sigma_0^2}} e^{i\frac{muz+\hbar\varphi_+}{\hbar}} \\ \sin\frac{\theta_0}{2} e^{-\frac{(z+z_\Delta+ut)^2}{4\sigma_0^2}} e^{i\frac{-muz+\hbar\varphi_-}{\hbar}} \end{pmatrix}$$

- Decoherence in Stern-Gerlach experiment

$$\rho(z, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{2}}\frac{1}{2}\left(e^{-\frac{(z-z_\Delta-ut)^2}{2\sigma_0^2}} + e^{-\frac{(z+z_\Delta+ut)^2}{2\sigma_0^2}} \right)$$



- The decoherence time Spots N^+ and N^- appear : $y = vt >$16 cm

$\Rightarrow$ the decoherence time :

$$t_D \simeq \frac{3\sigma_0 - z_\Delta}{u} = \frac{(3\sigma_0 - z_\Delta)mv}{\mu_B B_0' \Delta l} = 3 \times 10^{-4} s.$$

How is the transformation done ?

Mixed states θ_0 and φ_0 randomly drawn	$\Longrightarrow$	Quantized mixture $\theta_0 = \pi$ and $\theta_0 = 0$

Measure quantization's postulates

or

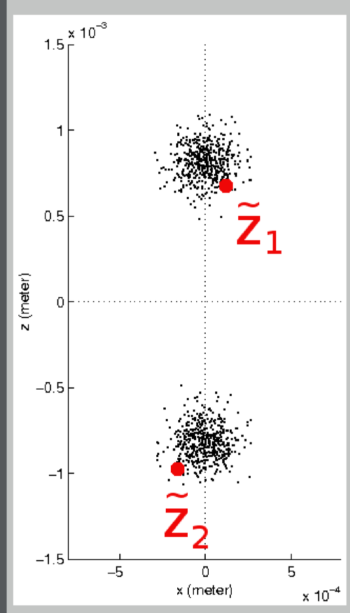
Pauli equation with spinor spatial extension

Decoherence in Stern-Gerlach experiment

Quantization postulates' proof for $S_z = \frac{\hbar}{2}\sigma_z$

$$\Psi(z, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \begin{pmatrix} \cos\frac{\theta_0}{2} e^{-\frac{(z-z_\Delta-ut)^2}{4\sigma_0^2}} e^{i\frac{muz+\hbar\varphi_+}{\hbar}} \\ \sin\frac{\theta_0}{2} e^{-\frac{(z+z_\Delta+ut)^2}{4\sigma_0^2}} e^{i\frac{-muz+\hbar\varphi_-}{\hbar}} \end{pmatrix}$$

Experimentally, one measures the particle position $\tilde{z}$



- $\tilde{z}_1 \in N^+$

$$\Psi(\tilde{z}_1, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \cos\frac{\theta_0}{2} e^{-\frac{(\tilde{z}_1-z_\Delta-ut)^2}{4\sigma_0^2}} e^{i\frac{mu\tilde{z}_1+\hbar\varphi_+}{\hbar}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- $\tilde{z}_2 \in N^-$

$$\Psi(\tilde{z}_2, t+\Delta t) \simeq (2\pi\sigma_0^2)^{-\frac{1}{4}} \sin\frac{\theta_0}{2} e^{-\frac{(\tilde{z}_2+z_\Delta+ut)^2}{4\sigma_0^2}} e^{i\frac{-mu\tilde{z}_2+\hbar\varphi_-}{\hbar}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

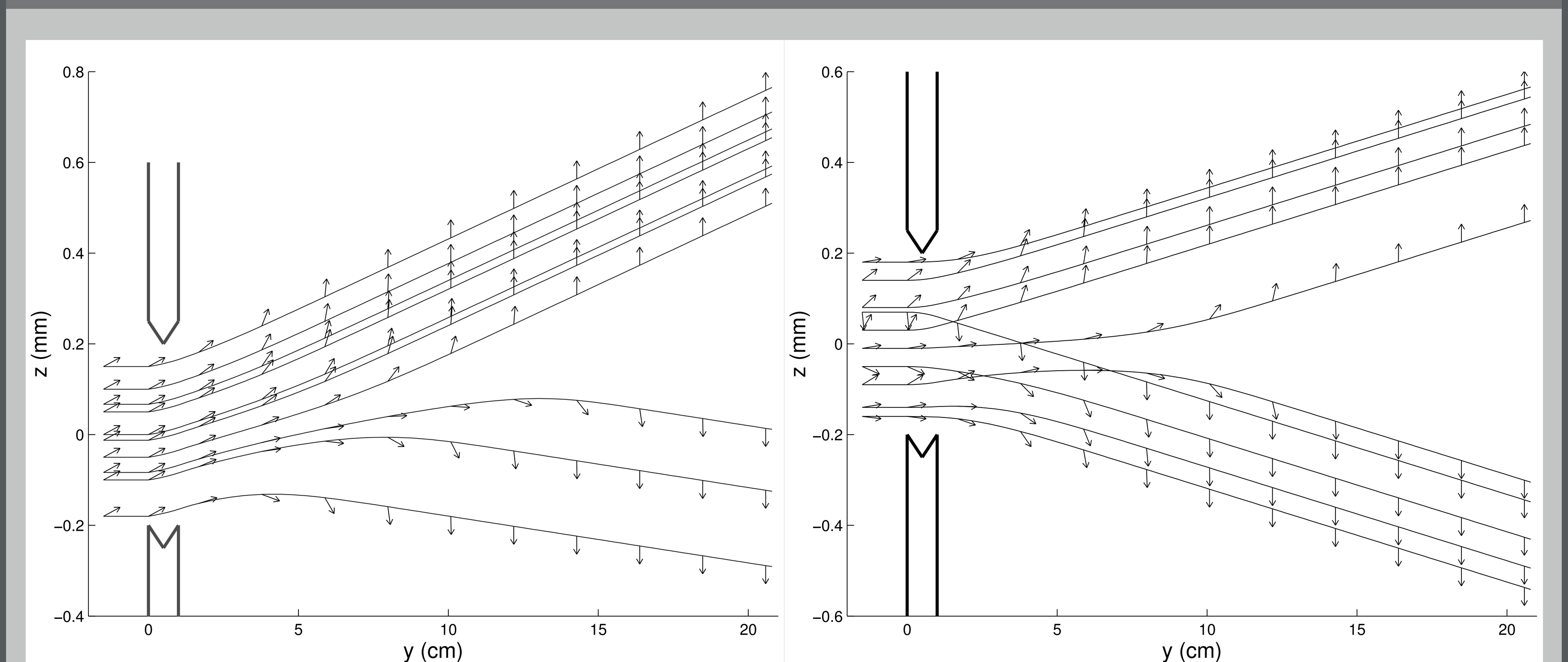
Marginal density matrix of spin variables of a pure state

$$\rho^S(z,t) = \begin{pmatrix} |\psi_+(z,t)|^2 & \psi_+(z,t)\psi_-^*(z,t) \\ \psi_-(z,t)\psi_+^*(z,t) & |\psi_-(z,t)|^2 \end{pmatrix}$$

When $t > t_D$:

$$\rho^S(z,t) \simeq \begin{pmatrix} |\psi_+(z,t)|^2 & 0 \\ 0 & |\psi_-(z,t)|^2 \end{pmatrix}$$

Experimental results : z_0 randomly drawn



Pure state (left) :$\theta_0 = \pi/3$ and $\varphi_0 = \pi/4$

Mixed states (right) : $\theta_0 \in [0;\pi]$ and $\varphi_0 \in [0; 2\pi]$ randomly drawn

Conclusion on quantum computer

- **Spin-based qubit's existence?**
  Space and spin variables are not factorizable during treatment
- **Chuang NMR results' explanation**
  Each wave function must be physically splitted because it takes at least two particles to represent the quantum system
  $\Longrightarrow$ Signal decay with a factor 2 for each additional qubit
- **Statistical qubit (10^8 spins) exists but not individual qubit**
  $\Longrightarrow$ Quantum mechanics is not complete

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