



HAL
open science

Computing the Worst-Case Peak Gain of Digital Filter in Interval Arithmetic

Anastasia Volkova, Christoph Lauter, Thibault Hilaire

► **To cite this version:**

Anastasia Volkova, Christoph Lauter, Thibault Hilaire. Computing the Worst-Case Peak Gain of Digital Filter in Interval Arithmetic. 17th International Symposium on Scientific Computing, Computer Arithmetics and Verified Numerics. , Sep 2016, Uppsala, Sweden. hal-01347634

HAL Id: hal-01347634

<https://hal.sorbonne-universite.fr/hal-01347634>

Submitted on 21 Jul 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Computing the Worst-Case Peak Gain of Digital Filter in Interval Arithmetic

Anastasia Volkova, Christoph Lauter and Thibault Hilaire

Sorbonne Universités, UPMC Univ Paris 06, UMR 7606, LIP6

4, place Jussieu 75005 Paris, France

first_name.last_name@lip6.fr

Keywords: digital filters, interval arithmetic, worst-case peak gain

The Worst-Case Peak Gain (WCPG) of a Linear Time Invariant (LTI) filter is used to determine the output interval of a filter and in error propagation analysis [5].

Consider a stable LTI filter \mathcal{H} in state-space representation:

$$\mathcal{H} \begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k) \end{cases} \quad (1)$$

where $\mathbf{u}(k)$ is the input vector, $\mathbf{y}(k)$ is the output vector, $\mathbf{x}(k)$ is the state vector and matrices \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} contain the filter coefficients.

The WCPG of a linear filter can be computed [1] as the infinite sum $\mathbf{W} := |\mathbf{D}| + \sum_{k=0}^{\infty} |\mathbf{C}\mathbf{A}^k\mathbf{B}|$. In [6] the authors have proposed an algorithm for the reliable evaluation of the WCPG matrix in multiple precision.

However, usually the filter coefficients are rounded prior to implementation, changing \mathbf{A} , \mathbf{B} , \mathbf{C} and \mathbf{D} by rounding. To provide a reliable filter implementation, these rounding errors must be taken into account in the WCPG computation. We represent the rounded coefficients as interval [2] matrices with small radii. Let $\mathbf{M}^I := \langle \mathbf{M}_c, \mathbf{M}_r \rangle$ to be an interval matrix centered in \mathbf{M}_c with radius \mathbf{M}_r . Then, the WCPG matrix of a filter $\mathcal{H} = (\mathbf{A}^I, \mathbf{B}^I, \mathbf{C}^I, \mathbf{D}^I)$ is an interval $\mathbf{W}^I := |\mathbf{D}^I| + \sum_{k=0}^{\infty} |\mathbf{C}^I \mathbf{A}^{I^k} \mathbf{B}^I|$.

In this work we adapt the algorithm presented in [6] to obtain a reliable evaluation of the WCPG interval. The WCPG is computed in two stages: the reliable truncation of the infinite sum and the summation.

We determine the truncation order only for the center matrices but add a correction term after the final step. This step requires to perform an eigenvalue decomposition. To obtain trusted error bounds on the computed eigenvalues we use the Theory of Verified Inclusions developed by S. Rump [4].

The summation is done using Interval Arithmetic in midpoint-radius form. However, powering a dense interval matrix can lead to an interval explosion. Instead of powering \mathbf{A}^I we power an almost diagonal matrix \mathbf{T}^I , for which $\|\mathbf{T}^I\|_2 < 1$ is true. We use an analogue of Gershgorin circle theorem [3] to verify a spectral norm condition that needs to be satisfied for the WCPG sum to converge.

It is obvious that we cannot guarantee an a priori given bound on the WCPG matrix radius \mathbf{W}_r because the radii of the input matrices are the limiting factors. However, when given point coefficient matrices (intervals with zero radii) and an absolute error bound ε we guarantee that the output WCPG interval is not larger than ε in width.

References

- [1] V. Balakrishnan and S. Boyd. On computing the worst-case peak gain of linear systems. *Systems & Control Letters*, 19:265–269, 1992.
- [2] H. Dawood. *Theories of Interval Arithmetic: Mathematical Foundations and Applications*. LAP Lambert Academic Publishing, 2011.
- [3] S. Gershgorin. Über die Abgrenzung der Eigenwerte einer Matrix. *Bull. Acad. Sci. URSS*, 1931(6):749–754, 1931.
- [4] S. M. Rump. New results on verified inclusions. In *Accurate Scientific Computations, Symposium, Proceedings*, 1985.
- [5] A. Volkova, T. Hilaire, and C. Lauter. Determining fixed-point formats for a digital filter implementation using the worst-case peak gain measure. In *2015 49th Asilomar Conference on Signals, Systems and Computers*, Nov 2015.
- [6] A. Volkova, T. Hilaire, and C. Lauter. Reliable evaluation of the worst-case peak gain matrix in multiple precision. In *Computer Arithmetic (ARITH), 2015 IEEE 22nd Symposium on*, pages 96–103, June 2015.