To gain or not to lose? Tuition fees for loss averse students
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Abstract

We model the educational choice of students whose objectives in terms of salary are conditioned by their social origins. We assume that students from a poor background have a lower reference point than students with wealthier origins. We then study the efficiency of a policy of tuition fees as a mechanism to select students on the basis of their academic abilities. We show that, even in the absence of borrowing constraints, the optimal policy consists in lowering tuition fees for poorer students: since prospective students from a disadvantaged background perceives the possibility of joining the university as a gain, they have a tendency to act too cautiously compared to students with higher aspirations, who are ready to take risky choices in order to avoid what they perceive as a failure, i.e. not joining the university.
1 Introduction

Higher education is characterised by numerous issues of information asymmetries, affecting both students and universities (Friedman 1962); the agents are for instance unable to perfectly observe ex ante the “quality” of the other agent (e.g. the academic abilities of the student, the standard of the courses provided by the university... see Jongbloed 2003, Teixeira 2006). This issue of adverse selection justifies (from the point of view of the universities) the implementation of a selection mechanism at the entry of higher education, such that only sufficiently talented students are recruited (Betts 1998, Fernandez and Gali 1999). Two kinds of mechanism are then possible (and sometimes used in combination) to elicit the quality of prospective students: exams and tuition fees (Del Rey and Romero 2004, Fernandez 1998). Tuition fees select the students on their willingness to pay their studies, and therefore on their “real” motivation in joining the university. They should therefore reveal the preferences of the individual, whereas exams — if they are efficient — directly reveal the quality of prospective students.

The implementation of tuition fees has been defended by Gary-Bobo and Tran-noy (2005, 2008), who show that this policy is efficient even in relatively complex environments, for instance when the students have a noisy perception of their own quality (and therefore do not precisely know their probability of graduating). Furthermore, in presence of borrowing constraints, they show that tuition fees are efficient when implemented in combination with exams. Fernandez (1998) also show that, when universities are in competition, the tuition fees they implement reveal their quality: good students therefore join good universities characterised by a high level of tuition fees. However, in presence of borrowing constraints and competition between universities, exams may be preferable if they are sufficiently efficient.

Those approaches are however grounded on a a-psychological and a-sociological conception of students’ behaviours (Flacher and Harari-Kermadec 2013, Flacher et al. 2013, Moulin 2014), assuming that prospective students are rational decision makers. In this paper, we suggest that the crucial hypothesis underlying the efficiency of tuition fees is the rationality of prospective students. The fundamental mechanism of tuition fees indeed consists in giving a monetary incentive to which rational individuals will perfectly respond. Boundedly rational individuals may however misperceive the incentives, and therefore take inadequate decisions. The aim of this paper is to highlight that a proper analysis of tuition fees shall consider the decision biases affecting the educational choices of the individuals. We therefore provide a very simple model without issues of information or borrowing constraints so as to isolate the impact of bounded rationality on the efficiency of a policy of tuition fees. We argue that prospect theory (Kahneman and Tversky 1979) offers a relevant framework to model the decision of individuals differentiated
by their social origins and show that, even in the absence of borrowing constraints, the optimal policy consists in increasing tuition fees with the income of the parents. We therefore stress that assuming that individuals are rational decision makers is a too strong assumption for modelling educational choices: the risk is indeed that students from a disadvantaged background will be relatively more disincentivised than students from a wealthier background, and therefore that good but poor students will be excluded from higher education, and be replaced by weaker but wealthier students.

The paper is organised as follows. Section 2 presents our model of educational choice. Section 3 then discusses the objective of the planner and states our main proposition. We conclude by discussing some policy implications in section 4.

2 Educational choice and loss aversion

Conventional welfare economics assume that individuals choose as if seeking to satisfy coherent (i.e. stable, consistent and context-independent) preferences. In the case of educational choice, those preferences represent the individuals’ evaluations of the possible educational outcomes, which are generally reduced to the expected salary of the individuals. It is however probably dubious to treat educational choices as standard choices between lotteries, since those kind of choices significantly impact the whole life of the individual. In particular, educational outcomes are considered with respect to the social aspiration of the individual, i.e. the standard of living the individual intends to sustain during the rest of her life. It is then likely that two individuals of the same quality (i.e. who have the same academic abilities, and therefore the same chance of graduating from a given university) may take different decisions if their levels of aspiration differ. Consider for instance the educational choice of Marine and Tony. Marine comes from a relatively rich family and is used to live quite comfortably. If she does not go to university, she will not be able to keep her level of consumption and will probably difficultly accept her more modest life. On the contrary, Tony grew up in a relatively poor family and is used to a modest standard of living. While he would greatly appreciate going to university so as to be able to get a high salary, he would not consider as a failure the option of quitting his studies before university. Suppose that Marine and Tony have the exact same chance of graduating if they go to the university: due to their different social aspirations, it is likely that Marine will be more tempted to pursue her studies than Tony, since her desire to avoid downward social mobility is stronger than Tony’s desire for upward social mobility.

Several sociological works suggest that the differences in social aspirations between social classes can explain at least partially the different educational choices
between individuals (Hyman 1953, Kahl 1957, Keller and Zavalloni 1964, Boudon 1974, Bourdieu 1974). Bourdieu for instance argued that “the adolescent will behave such that he will achieve what he perceives as an established fact: when you belong to a disadvantaged background, you cannot join University” (p.6). It indeed appears that the lower the student social class is, the lower her social aspiration will be (Krauss 1964, Page 2005). This difference of aspiration may then lead to differences in educational choices, since it is likely to affect the attitude towards risk of the individual. This kind of perception biases has for instance been theorised by the concept of relative risk aversion (Breen and Goldthorpe 1997, Holm and Jaeger 2008), according to which the risk aversion of an individual directly depends on her social position and the potential social mobility induced by her choices. Relative risk aversion theory is consistent with the sociological work of Boudon (1974, 1994) who showed that inequality between students pathways can be explained by the differences between the strategies of different social classes: the existence of distinct social positions leads to the existence of different systems of expectations and decisions (Boudon 1974, page 211).

An important bias that is likely to affect the decision of the students is therefore their social origins, through the subjective perception they will have of educational success. While poor individuals will perceive the event “going to university” as an improvement of their situation compared to “not going to university” — which constitute for them the benchmark case — students with wealthier origins (with highly-educated parents for instance) will perceive the event “going to university” as the benchmark case, and “not going to university” as an educational failure. Kahneman and Tversky (1979)’s prospect theory seems here to offer a relevant framework to model the educational choice of students whose choice are conditioned by their social origins: the core of prospect theory is indeed that individual choice may depend on the subjective representation of the different lotteries in terms of gains or losses. We will therefore develop a model of educational choice in which prospective students are loss averse, i.e. they are risk averse when considering prospects involving gains, but risk-seeking when facing losses.

Since our aim is to highlight the central role of the assumption of perfect rationality for the efficiency of a policy of tuition fees, we will provide here a very simple framework, so as to isolate the impact of the introduction of cognitive biases from other factors such as information issues or borrowing constraints. We consider a finite population of prospective students \( N = \{1; \ldots; n\} \) indexed by \( i \), characterized by (1) a payoff function \( u_i \), (2) a reference point \( \bar{x}_i \) and (3) an exogenous quality \( p_i \in [0; 1] \). The quality of \( i \) is measured such that an individual \( i \) with quality \( p \) has a probability \( p \) of graduating (\( p \) for instance gives a measure of \( i \)'s academic abilities). We do not endogenise the level of effort of the student once she
joined the university to keep the model as simple as possible\textsuperscript{1}. For sake of clarity, we assume that the payoff function of each student $i$ is linear and corresponds to her salary, i.e. $\forall i \in N, u_i(x) = x$. The educational choice of an individual $i$ can be formulated as follows:

- either stop one’s study, and get a low-qualified work, with a low salary $G_L$;
- or continue one’s study, knowing that (i) $i$ must pay a tuition fee $F_i \in [0; G_H - G_L]$; (ii) she has a probability $p_i \in [0; 1]$ of graduating, such that:
  - if $i$ graduates, she will get a high-qualified work, whose discounted salary $G_H$ is strictly higher than $G_L$;
  - if $i$ fails, she will get a low-qualified work and a low salary $G_L$; since she only gets her first salary after several years of unpaid studies, she perceives this gain as weighted by a discount parameter $\delta < 1$.

This educational choice can therefore be represented as a choice between two prospects $H$ (study) and $L$ (work)\textsuperscript{2}:

\[
\begin{align*}
H &= (G_H - F_i, p_i; \delta G_L - F_i, 1 - p_i) \\
L &= (G_L, 1)
\end{align*}
\]

We consider that each player has a reference point, i.e. a level of outcome $\bar{x}_i$ such that:

- if $i$ gets an outcome $x > \bar{x}_i$, then she will perceive $x$ as a gain;
- if $i$ gets an outcome $x < \bar{x}_i$, then she will perceive $x$ as a loss.

According to the experimental results of Kahneman and Tversky (1979), the individuals have a tendency to be more risk averse when they are facing prospects with gains, and more risk-seeking when facing losses. This reference point can be interpreted as the social aspiration of the individual in terms of outcome: we can therefore assume that this reference point directly depends on the social origins of the prospective students, and that an individual from a disadvantaged background will have a relatively lower reference point than a more favoured individual.

\textsuperscript{1}Endogenising $p$ would not actually change our result. The students would be characterised by exogenous academic abilities and/or a cost of effort, and $p_i$ would be interpreted as the probability of graduating of $i$, knowing her optimal effort in the second stage.

\textsuperscript{2}We reproduce here the notation of Kahneman and Tversky (1979). The prospect $P = (x_1, p_1; \ldots; x_K, p_K)$ means that the player get the outcome $x_k$ with a probability $p_k$, $\forall k \in [1; K]$. $P$ denote the set of prospects.
This implies that $i$ is not an expected utility maximiser, but maximises instead the following function $V_i : P \mapsto \mathbb{R}$:

$$V_i(P) = \sum_{k \in K} p_k v(x_k - \bar{x}_i),$$

with $P$ a prospect, and $v$ a value function that integrates the perception of the outcome as a gain or a loss. Kahneman and Tversky (1992) suggested that this value function is (i) concave for gains, (ii) convex for losses, and (iii) steeper for losses than for gains. We have therefore:

$$\begin{cases} 
  v(0) = 0 \\
  \frac{\partial v}{\partial x}(x) > 0 & \forall x \neq 0 \\
  \frac{\partial^2 v}{\partial x^2}(x) \leq 0 & \text{if } x > 0 \\
  \frac{\partial^2 v}{\partial x^2}(x) \geq 0 & \text{if } x < 0 \\
  \frac{\partial v}{\partial x}(-x) \geq \frac{\partial v}{\partial x}(x) & \text{if } x > 0 
\end{cases}$$

In this framework, the prospective students have the choice between a risky prospect (continue their studies) and a riskless one (get a low qualified work). An individual $i$ will choose to continue her studies if and only if she prefers the prospect $H$ to the prospect $L$, i.e. if and only if:

$$V_i(H) \geq V_i(L) \iff \bar{p}_i \geq \bar{p}_i$$

with $\bar{p}_i = \frac{v[G_H - F_i - \bar{x}_i] - v[\delta G_L - F_i - \bar{x}_i]}{v[G_H - F_i - \bar{x}_i] - v[\delta G_L - F_i - \bar{x}_i]}$.

$\bar{p}_i$ can be interpreted as a threshold of self-selection: prospective students whose quality is less than $\bar{p}_i$ will choose the riskless prospect, whereas the sufficiently talented students will prefer to pursue their studies.

### 3 Optimal policy

We now take the standpoint of the university so as to determine the optimal level of tuition fees. Suppose for convenience that the university is public, and therefore that its tuition fees policy is determined by the social planner. Suppose that the university supports a fixed cost $c > 0$ per student, such that $G_H - c \geq G_L$. There is therefore a net gain for the society if an individual gets a high-qualified work.
In the absence of tuition fees, it is possible that students of a relatively low quality go to university, although their expected gain is lower than the cost supported by the university. The impossibility to observe the quality of the students is therefore likely to lead to a suboptimal situation in terms of social welfare: implementing tuition fees may therefore give a monetary incentive to those individuals to stop their study. A difficulty may however arise in our situation, since there is a difference between \( u_i \), the function that determines the payoff of the individual, and \( V_i \), the function that represents the actual preferences of the individual, whose satisfaction determines the choice of the individual. The definition of the social welfare function is therefore not unambiguous since \( u_i \) and \( V_i \) seem to be both reasonable candidates for measuring the welfare of the individuals. It should however be noticed that the reference point integrated into the function \( V_i \) does not correspond to any measure of welfare (unlike the function \( u_i \)): it is merely a mathematical construction allowing us to model a cognitive bias. It is therefore probably more relevant to consider only \( u_i \) when defining the social welfare function\(^3\). We therefore define the social welfare as the sum of payoff of the individuals minus the net cost of studies supported by the university.

The social planner defines the level of tuition fees \( F_i \) such that only the sufficiently talented students choose the risky prospect, knowing that the decision of the students is determined by the relation \( 4 \). The social welfare function is therefore:

\[
SW(F_i, \forall i \in N) = \sum_{i \in N, p_i \geq \bar{p}_i} [p_i G_H + (1 - p_i)\delta G_L - F_i] + \sum_{i \in N, p_i < \bar{p}_i} G_L + \sum_{i \in N, p_i \geq \bar{p}} (F_i - c)
\]

(5)

Since the self-selection threshold \( \bar{p}_i \) of the individual \( i \) does not depend on the cost of studies \( c \) supported by the university, the planner can try to implement a threshold \( \bar{p}^* \) such that only the sufficiently talented students (whose probability of success \( p_i \) is higher than \( \bar{p}^* \)) go to university. Since the objective of the planner is to

\(^3\)This assumption is actually in line with most of the literature on behavioural welfare economics, such as libertarian paternalism and nudges (Thaler and Sunstein 2008); what matters from the social planner's perspective is the “true preferences” of the individual, i.e. the preferences on which she would have acted were she not misled by cognitive biases. The argument underlying this approach is that the individuals would change their decisions were they aware that their actual decision was influenced by some cognitive biases. If Marine becomes aware that the main reason motivating her choice is the fear of downward social mobility, and that she is likely to end up with an even worse situation (getting a low qualified work after unsuccessful studies), then she may reconsider her choice and accept to stop her studies. Since Marine’s actual preferences could be “corrected” by a thorough introspection on the reasons of her choice, it seem more legitimate to consider Marine’s true preferences in a welfarist perspective rather than her biased preferences.
send to the university only the best students, it implies that a necessary condition of this policy is that two individuals with the same probability \( p_i \) must take \textit{in fine} the same decision. The objective of the planner can therefore be rewritten as follows:

\[
\max_{\bar{p} \in [0;1]} \ SW(\bar{p}) = \sum_{i \in N, p_i \geq \bar{p}} [p_i G_H + (1 - p_i)\delta G_L - c] + \sum_{i \in N, p_i < \bar{p}} G_L. \tag{6}
\]

The planner can then compute the optimal level of fees \( F_i^* \) such that \( \bar{p}_i(F_i^*) = \bar{p}^* \), \( \forall i \in N \), i.e. such that the self-selection threshold of each individual \( i \) corresponds to the optimal threshold. We can determine the optimal threshold \( \bar{p}^* \) that maximises the social welfare:

\[
\bar{p}^* = \frac{G_L(1 - \delta) + c}{G_H - \delta G_L}. \tag{7}
\]

We can firstly notice that, if the students are not loss averse, then the function \( v \) is linear. The optimal level of tuition fees is \( F_i^* = c \) \( \forall i \in N \), i.e. the students must directly support the cost of their studies. In the presence of loss averse students, we can however show the following result (the proof is provided in appendix):

**Proposition 1.** Let \( F_i^* \) denote the level of tuition fees such that \( \bar{p}_i(F_i^*) = \bar{p}^* \), \( \forall i \in N \). \( F_i^* \) increases with the reference point \( \bar{x}_i \) if:

- \( \bar{x}_i \in [x_i^-; x_i^+] \);
- or \( \bar{x}_i \leq x_i^- \) and \( v \) is sufficiently concave for gains;
- or \( \bar{x}_i \geq x_i^+ \) and \( v \) is sufficiently convex for losses;

with \( x_i^- < \frac{G_L + G_H - F_i^*}{2} \) and \( x_i^+ > \frac{G_L + G_H - F_i^*}{2} \) two reference points such that:

\[
\frac{\partial v}{\partial x}(G_L - x_i^+) = \frac{\partial v}{\partial x}(G_H - F_i^* - x_i^-),
\]

\[
\frac{\partial v}{\partial x}(G_L - x_i^-) = \frac{\partial v}{\partial x}(\delta G_L - F_i^* - x_i^+). \tag{8}
\]

Proposition 1 means that, for intermediate values of \( \bar{x}_i \) (we have indeed \( x_i^- < G_L < x_i^+ \)) the optimal level of tuition fees increases with the reference point: the higher the social aspiration of an individual is, the higher her tuition fees should be. For more extreme values of \( \bar{x}_i \) the relation still holds when \( v \) is sufficiently concave for gains and convex for losses. This means that, as soon as the students are sufficiently loss averse, the social planner should systematically decrease the level of tuition fees for poorer students, even in the absence of borrowing constraints. We can indeed show that the level of tuition fees \( F_i \) has always a positive effect on the self-selection threshold \( \bar{p}_i \), and that, under the conditions specified above, \( \bar{p}_i \)
decreases with the reference point $\bar{x}_i$: the individuals from a disadvantaged background — presenting therefore a low social aspiration and a relatively low reference point $\bar{x}_i$ — will have a higher threshold than an individual from a more favoured background. This means that two individuals with the same quality and different reference points can take different decisions, which is not optimal in terms of social welfare. In particular, it means that the planner should correct this deformation by the implementation of different levels of tuition fees, according to the level of the reference point of the individuals. We can highlight here an interesting mechanism, consistent with the findings of Breen and Goldthorpe (1997), Holm and Jaeger (2008): individuals with a relatively high social aspiration are considering both prospects as losses, and are therefore risk-seeking, unlike the students with a more modest objective, who evaluate the outcomes of the prospects as gains, and are therefore risk averse. We can therefore expect that those individuals will behave more cautiously and take less risk than the students from a more favoured background (this is typically what could happen with Marine and Tony: while she wants to avoid at all costs losing a current social status, he is perfectly fine with it and will not express the same desire to improve his status). We have here an interesting justification of the introduction of decreasing tuition fees: individuals from a disadvantaged background must pay less tuition fees, not because they are facing funding issues, but because their modest social aspirations imply a too cautious behaviour. In particular, even if universities manage to develop loan schemes such that poorer students would not face borrowing issues, a policy of differentiated tuition fees would still be needed.

4 Policy implications

Our aim is in this paper was to stress the necessity of introducing psychological biases in the analysis of tuition fees as a selection mechanism, since it affects the efficiency of monetary incentives. We have suggested that a major bias in educational choice was the subjective perception of educational outcome in terms of social achievement or failure. We have shown that the implementation of a unique level of tuition fees can be suboptimal in terms of social welfare: since the individuals from a disadvantaged background have lower social aspirations, they have a tendency to act too cautiously, and relatively good students will not be sufficiently encouraged to join university. The introduction of a simple cognitive bias seriously questions the efficiency of a policy of tuition fees, since its mechanism relies on the assumption that the willingness to pay of the students reveal their true preferences (the function $u_i$) and indirectly their quality. This willingness to pay however depends on the perception of the outcome as gains or losses, which is influenced by the social origins of the individual. The willingness to pay therefore
does not reveal the quality of the student, but a combination of her quality and her social origins (the same willingness to pay can indeed either mean that the individual is poor with a high quality, or rich with a low quality).

Our main result therefore suggests that an optimal policy would consist in the implementation of decreasing tuition fees with the social origins of the students, independently of any borrowing issues. Since individual reference points are defined as one’s social aspiration, and that social aspiration directly depends on one’s social origins, we can use the income of the parents of the student as a proxy for her reference point. We can for instance consider that an individual will perceive her salary as a gain if and only if she gains more than her parents, since it will allow her to sustain at least the standard of living of her parents. Concretely, such kind of indexation of the level of tuition fees on the parents’ income already exist in many universities, since it may also provide a proxy for the immediate financing capacity of the student: tuition fees in those situations however only take into account issues of funding, and probably neglects the possible difference in social aspirations. It is therefore possible that the level of tuition fees designed for poor students, although lower than for rich students, is still too high.

Although we developed a very simple model of educational choice, we would like to stress that our result would remain valid in more complex environments. We suggest therefore now discussing the main hypotheses of our model: (i) the absence of borrowing constraint, (ii) the probability of graduating is exogenous and known by the student, (iii) there is a single public university. Firstly, in presence of borrowing constraints, then the effect of social origins will probably be strengthened, since — in addition to the bias in perception — students from a poor background are also less likely to be able to fund their studies. Secondly, modelling the actual effort of the individuals in the second stage (i.e. once they have joined the university) could also lead to greater inequalities between social classes, since students with higher financial capacities can afford private lessons in addition to their standard courses. Lastly, if we consider private universities in competition rather than a public one, then the fundamental issue remains identical: since universities intend to select the best students, they must take into account the possibility that individuals from a disadvantaged background are relatively more sensitive to tuition fees than wealthier individuals. Whatever the structure of the competition between the universities is, they still intend to select the students on the basis of their quality, and must offer lower tuition fees for poorer students so as to compensate the cognitive bias of the individuals.

Such a policy of decreasing tuition fees may however present an undesirable side effect (highlighted by Epple et al. (2004)). The actual population of students is quite heterogeneous across universities, i.e. the proportion of students from a disadvantaged background is much higher in some universities than others. This
implies that the implementation of decreasing tuition fees will enable universities with a high proportion of students from a favoured background to raise important funds through the payment of tuition fees, whereas the universities for which the proportion of disadvantaged students is high will not collect many funds. Such a policy of decreasing tuition fees, even if it appears to be optimal on an individual level, can be questioned in the presence of heterogeneous universities at the aggregate level, since it will generate serious inequalities between the universities according to the population of their students. Empirical evidence of this phenomenon have been documented by Hsieh and Urquiola (2006) in the context of the Chilean education system. This suggests that the implementation of tuition fees is likely to lead to segregation phenomena in higher education: in presence of differentiated fees, we may observe a segregation between “rich” and “poor” universities, while a single level of fees is likely to lead to a segregation between rich and poor individuals, since the disincentive will be too strong for poor individuals.
References


Appendix

So as to study the impact of the reference point on the optimal level of tuition fees, we will show the two following results:

\[ \frac{\partial \bar{p}_i}{\partial F_i}(\bar{x}_i; F_i) > 0, \quad (9) \]

\[ \frac{\partial \bar{p}_i}{\partial \bar{x}_i}(\bar{x}_i; F_i) < 0. \quad (10) \]

Indeed, the maximisation of the social welfare implies that two students with the same abilities \( p_i \) must take the same decision. If (10) holds, then a student with a low reference point has a higher self-selection threshold, i.e. does not necessarily choose the risky prospect when an individual with the same abilities but a higher reference point does. In this situation, if (9) holds, the optimal level of tuition fees should be lower when the individual has a low reference point, since this will compensate the disincentive effect of her social origin.

The relation (9) holds if and only if:

\[ \frac{\partial \bar{p}_i}{\partial F_i} = \frac{\partial v}{\partial x}(G_H - F_i - \bar{x}_i)(v(G_L - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)) 
\quad + \frac{\partial v}{\partial x}(\delta G_L - F_i - \bar{x}_i)(v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i)) 
\quad > 0 \quad (11) \]

which is true by construction, since \( \frac{\partial v}{\partial x} \) is positive \( \forall x \), and \( G_H - F_i - \bar{x}_i > G_L - \bar{x}_i > \delta G_L - F_i - \bar{x}_i \).

The relation (10) holds if and only if:

\[ \frac{\partial \bar{p}_i}{\partial \bar{x}_i} = \frac{\partial v}{\partial x}(G_H - F_i - \bar{x}_i)(v(G_L - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)) 
\quad + \frac{\partial v}{\partial x}(\delta G_L - F_i - \bar{x}_i)(v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i)) 
\quad + \frac{\partial v}{\partial x}(G_L - \bar{x}_i)(v(\delta G_L - F_i - \bar{x}_i) - v(G_H - F_i - \bar{x}_i)) 
\quad < 0 \quad (12) \]

which is equivalent to:
\[ \frac{\partial v}{\partial x} (G_L - \bar{x}) \geq \frac{\partial v}{\partial x} (G_H - F_i - \bar{x}_i) \frac{v(G_L - \bar{x}) - v(\delta G_L - F_i - \bar{x}_i)}{v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)} + \frac{\partial v}{\partial x} (\delta G_L - F_i - \bar{x}_i) \frac{v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)}{v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)} \]  

(13)

This last condition means that the self-selection threshold diminishes with the reference point if and only if \( \frac{\partial v}{\partial x} \) in \( x = G_L - \bar{x}_i \) is higher than a convex combination of \( \frac{\partial v}{\partial x} \) in \( x = \delta G_L - F_i - \bar{x}_i \) and \( x = G_H - F_i - \bar{x}_i \). We now suggest identifying the conditions under which:

\[
\begin{cases} 
\frac{\partial v}{\partial x} (G_L - \bar{x}_i) > \frac{\partial v}{\partial x} (\delta G_L - F_i - \bar{x}_i), \\
\frac{\partial v}{\partial x} (G_L - \bar{x}) > \frac{\partial v}{\partial x} (G_H - F_i - \bar{x}_i).
\end{cases}
\]  

(14)

We can indeed notice that, when those two conditions are verified, (10) necessarily holds. Knowing that \( v \) is concave for gains, convex for losses and steeper for losses than for gains, we can deduce from the conditions (3):

\[ \frac{\partial v}{\partial x} (y) \geq \frac{\partial v}{\partial x} (x) \quad \forall y \in [-x; x], \quad x \geq 0 \]  

(15)

We have therefore:

\[
\bar{x}_i \geq \frac{G_L + \delta G_L - F_i}{2} \Rightarrow \frac{\partial v}{\partial x} (G_L - \bar{x}_i) \geq \frac{\partial v}{\partial x} (\delta G_L - F_i - \bar{x}_i) \\
\bar{x}_i \leq \frac{G_L + G_H - F_i}{2} \Rightarrow \frac{\partial v}{\partial x} (G_L - \bar{x}_i) \geq \frac{\partial v}{\partial x} (G_H - F_i - \bar{x}_i)
\]  

(16)

These last conditions imply that, \( \forall \bar{x}_i \in \left[ \frac{G_L + \delta G_L - F_i}{2}; \frac{G_L + G_H - F_i}{2} \right], \) the conditions (14) are verified: \( \bar{p}_i \) is therefore decreasing for those \( \bar{x}_i \).

We will now determine the interval of \( \bar{x} \) such that (14) is true. Consider \( \bar{x} < \frac{G_L + \delta G_L - F_i}{2} \); we have therefore \( \bar{x} < G_L \), the individual perceives the outcomes \( G_L \) and \( G_H - F_i \) as “gains”. The concavity of \( v \) for gains implies:

\[ \frac{\partial v}{\partial x} (G_L) > \frac{\partial v}{\partial x} (G_H - F_i). \]  

(17)

Define now \( x_i^- \) as the level of the reference point such that:

\[ \frac{\partial v}{\partial x} (G_L - x_i^-) = \frac{\partial v}{\partial x} (\delta G_L - F_i - x_i^-). \]  

(18)

Due to the concavity of \( v \) for gain, we have:
\[
\frac{\partial v}{\partial x}(G_L - \bar{x}_i) > \frac{\partial v}{\partial x}(\delta G_L - F_i - \bar{x}_i) \quad \forall G_L > \bar{x}_i > x_i^-.
\] (19)

This implies that, \(\forall \bar{x}_i \in [x_i^- ; G_L]\), the conditions (14) are verified.

Consider now the case \(\bar{x}_i > \frac{G_L + G_H - F_i}{2}\): we have now \(\bar{x}_i > G_L\), the individual perceives the outcomes \(G_L\) and \(\delta G_L - F_i\) as “losses”. The convexity of \(v\) for losses implies:

\[
\frac{\partial v}{\partial x}(G_L) > \frac{\partial v}{\partial x}(\delta G_L - F_i).
\] (20)

Define now \(x_i^+\) as the level of the reference point such that:

\[
\frac{\partial v}{\partial x}(G_L - x_i^+) = \frac{\partial v}{\partial x}(G_H - F_i - x_i^+).
\] (21)

Due to the convexity of \(v\) for losses, we have:

\[
\frac{\partial v}{\partial x}(G_L - \bar{x}_i) > \frac{\partial v}{\partial x}(G_H - F_i - \bar{x}_i) \quad \forall G_L < \bar{x}_i < x_i^+.
\] (22)

The conditions (14) are therefore also verified \(\forall \bar{x}_i \in [G_L ; x_i^+]\). We have therefore proven the first part of proposition 1, i.e. the set of reference points such that the optimal level of tuition fees necessarily increases with \(\bar{x}_i\), if \(v\) verifies the conditions (3).

Consider now the case \(\bar{x}_i < x_i^-\). We have \(\frac{\partial v}{\partial x}(G_L - \bar{x}_i) < \frac{\partial v}{\partial x}(\delta G_L - F_i - \bar{x}_i)\): the condition (13) is therefore not necessarily verified for any function \(v\) that respects the conditions (3) (for instance in the limit case in which \(v\) is linear and steeper for losses). We can however verify that, when the concavity of \(v\) increases, the weight \(\frac{v(G_H - F_i - \bar{x}_i) - v(G_L - \bar{x}_i)}{v(G_H - F_i - \bar{x}_i) - v(\delta G_L - F_i - \bar{x}_i)}\) in the relation (13) decreases: highly risk averse individuals for gains will not perceive a significant difference between the outcomes \(G_L\) and \(G_H - F_i\), and will therefore be more tempted to obtain a sure gain \(G_L\). This means therefore that, if \(v\) is sufficiently concave for gains when \(\bar{x}_i < x_i^-\), then (13) is verified.

With a similar argument for \(\bar{x}_i > x_i^+\), we can argue that for a sufficiently convex function \(v\) for losses, (13) will be verified: highly risk-seeking individuals will indeed try to avoid the sure “gain” \(G_L\) (that they perceive as a loss), and tends to perceive the outcomes \(G_L\) and \(\delta G_L - F_i\) as relatively close.

We have therefore:

- \(\forall \bar{x}_i \in [x_i^- ; x_i^+]\), (10) always holds;
- \(\forall \bar{x}_i \leq x_i^-\), (10) holds if \(v\) is sufficiently concave for gains;
- \(\forall \bar{x}_i \geq x_i^+\), (10) holds if \(v\) is sufficiently convex for losses.