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Abstract: The regenerative chatter limits productivity of many cutting processes due to the presence of violent high amplitude vibrations. This self-excited vibration is due to a complex phenomenon defined by the combination of machine and process parameters. Consequently, there are many alternative strategies to avoid these self-excited vibrations. The focus of this study is on the exploitation of the possibilities offered by variable stiffness tuneable mass dampers (VSTMD). These devices are able to dampen a flexible mode to which the dampers are tuned using a mechanism that varies the stiffness. In this work, three different modes of the use of the VSTMD are investigated. First of all, these devices are used like an ordinary passive damper tuned with constant stiffness according to Den Hartog's theory. Secondly, they are tuned on the basis of the so-called Sims' parameters. In the third application, self-tuning algorithm is used to suppress chatter, which is a semi active solution. The idealistic optimal response behavior, what the self-tuning algorithm can achieve, is also derived and verified.
Keywords: stability, chatter, milling, vibration absorber

1. INTRODUCTION

New self-acting and self-characterizing functions of a future machine tool require integrated equipments to improve precision, productivity and remote maintenance capabilities [Monostori, 2014]. This trend in the machine tool industry intends to decrease human interaction in large manufacturing lines and helps further optimizing machining processes taking into account the overall dynamic behavior of the machine tool/tool/workpiece system.

Embedded solutions are becoming cheaper and increasingly affordable for machine tool constructors, which results in the design of independent active [Lu et al., 2014, Munoa et al., 2013] and semi-active [Aguirre et al., 2013] mechatronic devices to attenuate machine tool vibrations. Variable stiffness tuneable mass dampers (VSTMD) dampers offers promising advantages since they can be tuned during the machining process even remotely, and therefore they are excellent candidates to be included in embedded semi-active [Aguirre et al., 2013] and cyber-physical systems.

The original tuned mass dampers (TMD) provide a classical cheap and effective classical solution to increase damping in a mechanical structure. Exact analytical solution exists to optimize the parameters of TMD when the original structure is considered with a single undamped mode by [Den Hartog, 1934]. The above mentioned classic theory provides a sufficiently good approximation if the cutting performance of the machine tool is limited basically by a single dominant mode with low damping in the structure. If the machine has high damping, the exact tuning of the TMD requires semi-analytical or numerical solutions even for a single mode [Asami, et al. 2002]. Several variations of the primary idea have been proposed in the literature. In recent years, a single TMD with multiple degrees of freedom (MDOF) [Zuo, et al. 2006] and multiple TMDs (MTMD) [Li, et al. 2007] and even nonlinear ones [Habib, et al. 2015] have been developed to damp a single mode.

However, the tuning requirements of chatter suppression in machining processes are quite different. The repeating surface pattern on the workpiece causes regeneration that often leads to self-excited vibrations in a given cutting process. This effect can drive the cutting process to loss of its stability leading to large amplitude chatter vibration, which clearly must be prevented. The effect of this regeneration was recognized by pioneers in the field like Tlusty and Tobias [Tlusty et al., 1954, Tobias et al., 1958]. They demonstrated that the minimum stability of a certain process limited by a single mode is directly related to the real part of the frequency response function (FRF) of the system in the cutting point. Mathematically, regenerative chatter is described by delay differential equations DDE [Hale et al., 1977]. Chatter vibrations can grow when the dynamic stiffness of the machine/tool/part system is lower than the cutting stiffness of the cutting process [Merrit, 1965]. Therefore, the higher is the damping of the system is, the stronger stability against chatter vibrations is [Munoa, et al. 2015].

The application of TMD dampers to increase the damping of the machine tool was one of the first proposed solutions for chatter avoidance [Koegnisberger and Tlusty, 1970]. Some experimental studies were performed to find the ideal tuning of TMDs. For instance, [Koegnisberger and Tlusty, 1970] were proposing to build an electrical circuit equivalent to the mechanical system in order to optimize the real part of the FRF. [Rivin, et al. 1992] dealt with the improvement of stability in boring bars considering TMD. Also as an experimental work [Tarng, et al. 2000] and [Rashid, et al. 2008] tuned the natural frequency of their TMD to match with the natural frequency of the structure target mode of the structure. Finally, [Sims, 2007] found an analytical expression for the optimal tuning of a single TMD for chatter avoidance. The damper was tuned to maximize the negative real part of the main frequency response function (FRF) creating two equal peaks. The parameters of a viscoelastic cantilever beam was tuned in [Saffury, et al. 2009] by maximizing the most negative real part of the corresponding FRF. [Yang, et al. 2010] optimizes the MTMD for one dominant mode considering the real part of the FRF.

Recently, new self-tuneable TMD have been proposed based on variable stiffness concepts [Aguirre et al., 2013]. These VSTMD dampers offer the possibility to change the tuning of the damper depending on the cutting conditions.

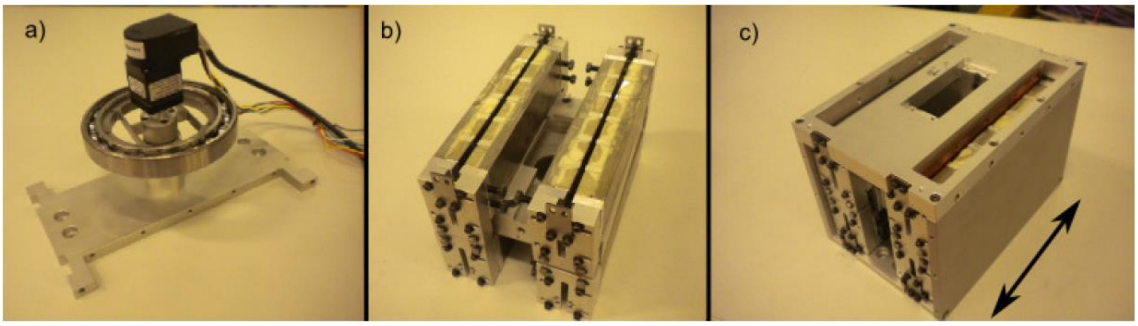

Figure 1;Rotary spring a), the magnets producing Eddy current and their guiding system *b*). The assembled passive damper block *c*).

The aim of this study is to find the ideally best tuning to suppress chatter vibrations by taking the advantage of the variable values of the spring of a VSTMD concept (see Figure 1*a*). The optimal performance of the VSTMD is to be comparable with the results obtained with the best tuning of a single TMD.

In practice, the design of a VSTMD is not an obvious task. In fact, several concepts have been proposed in the literature [Aguirre et al., 2013]. For instance, [Aguirre et al., 2013] created this damper using a movable mass guided by means of a flexures and a rotary spring controlled by a stepped motor. Changing the angular position of the rotary spring, the natural frequency and therefore, the tuning of the damper, could be modified. In parallel, pure viscous damping was introduced in the system by means of the eddy current effect (see Figure 1).

The paper is divided into three parts, which include model development, the determination of the optimal tuning for a rotary spring self-tuning damper using the method of zeroth order approximation (ZOA) [Altintas et al., 1995, Zatarain, et al. 2004] and the verification part. Finally, the concept is verified by using time domain simulations and by using semi-discretization method (SDM) [Insperger et al., 2011]. The model presented in the paper represents an idealistic situation, which needs further more development to fit a in real case scenario.

2. STABILITY MODEL FOR MILLING OPERATIONS WITH A VSTMD

A simple milling model is introduced here in order to describe the dynamic behavior of a VSTMD concept. The model is adequate for applying semi SDM and ZOA with and without taking time averaging in the tooth passing period of the milling operation. The ZOA permits to obtain a fast analytical parametric solution with a good accuracy for continuous milling processes. In the case of interrupted cutting, the presence of double period chatter and mode couplings reduces the precision of the ZOA [Munoa, et al. 2009].

The VSTMD is especially interesting for structural chatter cases in heavy duty operations. In these cases, tools with a high number of flutes and large engagements are used [Iglesias, et al. 2014].

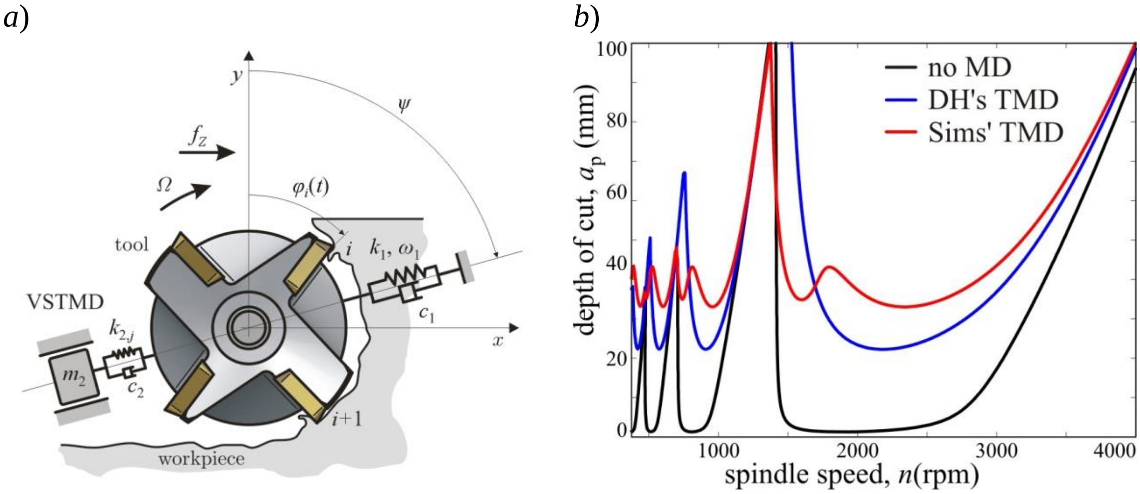

Figure 2; a) sketch of the simple one dimensional milling model. Panel b) shows the effect of mass dampers (MD) *by [Den Hartog, 1934]* (blue) *and [Sims, 2007]* (red).

Therefore, the periodic excitation is close to “flat” (almost constant) and the system can be roughly considered roughly as a time independent autonomous system. This way, the ZOA can also serve acceptable results, and simple assumptions can be made regarding the stability limits (see Figure 2*a*).

The first step to optimize the VSTMD is to develop a milling model that includes the effect of a passive damper. For simplicity, it is assumed that the damper is located at the point where the cutting force is acting. Two different points can also be considered with modal parameters [Munoa et al., 2013] but this increases the complexity of the main expressions and does not help to explain the main innovation in the present work. A single dominant mode parallel to the cutting plane has been considered, too. Finally, a lead angle of 90º has also been assumed. With this set of assumptions, a 2 DOF model has been formed to describe the stability of the milling process.

By assuming linear cutting force characteristics, the resultant cutting force can be derived in the following way for the simple straight fluted milling operation

$$\mathbf{F}(t) = -a_\mathrm{p} \sum_{i=1}^{Z} g_i(\varphi_i(t))\,\mathbf{T}_i(\varphi_i(t))(\mathbf{K}_\mathrm{e} + \mathbf{K}_\mathrm{c} h_i(t)), \tag{1}$$

where a_p and Z are the depth of cut and the number of teeth, respectively. The cutting force is taken into account by using $\mathbf{K}_\mathrm{e} = \mathrm{col}(K_{\mathrm{e},t}, K_{\mathrm{e},r})$ edge coefficients and $\mathbf{K}_\mathrm{c} = K_{\mathrm{c},t}\,\boldsymbol{\kappa}_\mathrm{c}$ ($\boldsymbol{\kappa}_\mathrm{c} = \mathrm{col}(1, \kappa_r)$) cutting coefficients [Altintas, 2000]. A ratio κ_r has been defined between the radial and the tangential force components.

The instantaneous chip thickness is a function of the chip thickness direction $\mathbf{s}_i(t) = \mathrm{col}(\sin\varphi_i(t), \cos\varphi_i(t))$ of the i^{th} tooth:

$$h_i(t) = (f_Z + x(t) - x(t-\tau))\sin\varphi_i(t) + (y(t) - y(t-\tau))\cos\varphi_i(t)\,, \tag{2}$$

where $x(t)$ and $y(t)$ are the relative spatial vibration of the center of the milling tool compared to its secondary motion described by the feed per tooth f_Z. The instantaneous chip thickness is considered at the angle

$$\varphi_i(t) = \Omega t + (i-1)\,{}^{2\pi}\!/_{Z} \tag{3}$$

for each ith teeth. The milling tool rotates with $\Omega = 2\,\pi\,n\,/\,60$ angular spindle frequency.

The cutting force is transformed back from local (tr) to tool (xy) system by $\mathbf{T}_i\,(\varphi_i\,(t))$ in (1) considering $g_i\,(\varphi_i\,(t))$ screen function [Dombovari, et al. 2010b]. This takes into account the radial immersion by the entrance (φ_{en}) and exit angles (φ_{ex}).

The dominant mode direction is considered under the angle ψ relative to the y axis (see Figure 2a), which results in $x(t) = q_1(t)\sin\psi$ and $y(t) = q_1(t)\cos\psi$. By means of the modal mass m_1, damping c_1 and stiffness k_1 of the relevant mode, the one degree of freedom (DOF) milling system has governing equation in the form

$$m_1\,\ddot{q}_1(t) + c_1\,\dot{q}_1(t) + k_1 q_1(t) = \mathbf{p}^{\mathrm{T}}\mathbf{F}(t;q_1(t),q_1(t-\tau)) \text{ with } \mathbf{p} = \begin{bmatrix}\sin\psi\\ \cos\psi\end{bmatrix}. \tag{4}$$

Assuming that the passive damper has the same vibration mode defined by angle ψ, the equation of motion (4) is modified to

$$\mathbf{M}\,\ddot{\mathbf{q}}(t) + \mathbf{C}\,\dot{\mathbf{q}}(t) + \mathbf{K}\,\mathbf{q}(t) = \mathbf{Q}(t;\mathbf{q}(t),\mathbf{q}(t-\tau))\,, \tag{5}$$

where

$$\mathbf{M} = \begin{bmatrix} m_1 & 0\\ 0 & m_2\end{bmatrix},\ \mathbf{C} = \begin{bmatrix} c_1 + c_2 & -c_2\\ -c_2 & c_2\end{bmatrix},\ \mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2\\ -k_2 & k_2\end{bmatrix},\ \mathbf{Q}(t) = \begin{bmatrix}\mathbf{p}^{\mathrm{T}}\mathbf{F}(t)\\ 0\end{bmatrix}.$$

Linearizing around the periodic stationary solution (forced vibration) $\mathbf{q}_{\mathrm{p}}(t) = \mathbf{q}_{\mathrm{p}}(t+T)$ $(T = 2\pi/\Omega/Z)$, the following linearized system can be derived for the perturbation $\mathbf{u}(t) = \mathbf{q}(t) - \mathbf{q}_{\mathrm{p}}(t)$:

$$\mathbf{M}\,\ddot{\mathbf{u}}(t) + \mathbf{C}\,\dot{\mathbf{u}}(t) + \mathbf{K}\,\mathbf{u}(t) = \Delta\mathbf{Q}(t;\mathbf{u}(t),\mathbf{u}(t-\tau))\,, \tag{6}$$

where

$$\Delta\mathbf{Q}(t) = \begin{bmatrix}\Delta Q_1(t)\\ \Delta Q_2(t)\end{bmatrix} = \begin{bmatrix} K_{\mathrm{c},t}a_{\mathrm{p}}\,B(t)\,(u_1(t) - u_1(t-\tau))\\ 0\end{bmatrix} \tag{7}$$

and

$$B(t) = -\mathbf{p}^{\mathrm{T}} \left(\sum_{i=1}^{Z} g_i(\varphi_i(t)) \mathbf{T}_i(\varphi_i(t)) \boldsymbol{\kappa}_{\mathrm{c}} \mathbf{s}_i^{\mathrm{T}}(t) \right) \mathbf{p} \ . \tag{8}$$

A time dependent factor $B(t)$ has been obtained in the expression of the perturbed force. This coefficient is named time domain directional factor and it is a periodic function that collects the projections of the cutting force generated by the different flutes onto the direction of the single vibration mode and the projection of the generated vibration onto the chip thickness direction ($\mathbf{s}_i$) [Zatarain, et al. 2010].

3. CALCULATION METHODS

This time-periodic directional factor can be averaged to obtain a zero order approximation (ZOA) that permits an analytical solution. This approximation is going to be used to investigate the optimal tuning of the different TMDs.

Since ZOA gives back the borders of stability, the SDM is used to calculate exact stability properties on the exact model of milling, which, although it is hardly usable for analytical calculations.

Non-smooth time domain simulation was used on the fly-over [Stepan, et al. 2011] DDE model of milling to determine the real chatter frequency which is defined as the dominant frequency of the large amplitude threshold vibration.

These three methods have been combined to explore the potential of the VSTMD.

3.1 Time averaged model for ZOA

In order to find an optimal tuning for the passive damper, analytical ZOA is used, which is based on the time averaged model of the milling operation. Due to the periodicity of the directional factor, it can be developed in Fourier series. ZOA is obtained when this periodic function is approximated by the average value. Milling operation can be approximated really well by ZOA if the operation is near to full immersion (not interrupted) milling. By the introduction of an equivalent cutting coefficient [Opitz et al., 1970]

$$B_0 := \frac{Z}{2\pi} \beta_0(\varphi_{\mathrm{en}}, \varphi_{\mathrm{ex}}, \psi, \kappa_r) \,, \tag{9}$$

the periodicity is averaged out from the describing milling model. The averaged directional factor is

$$\begin{aligned} \beta_0(\varphi_{\mathrm{en}}, \varphi_{\mathrm{ex}}, \psi, \kappa_r) = -\frac{1}{4}(&\cos(2(\varphi_{\mathrm{ex}} - \psi)) - \cos(2(\varphi_{\mathrm{en}} - \psi)) + \\ &\kappa_r(2(\varphi_{\mathrm{ex}} - \varphi_{\mathrm{en}}) + \sin(2(\varphi_{\mathrm{ex}} - \psi)) - \sin(2(\varphi_{\mathrm{en}} - \psi)))), \end{aligned} \tag{10}$$

The force variation (6) is in the direction of q_1 and can be written as

$$\Delta Q_1(t) := \frac{Z}{2\pi} K_{c,t} a_p \, \beta_0 (u_1(t) - u_1(t-\tau)). \tag{11}$$

From [Tlusty et al., 1954] to [Altintas, 2000], this leads to the following conditions for the border of asymptotic exponential stability

$$a_{\lim}(\omega) = -\frac{\pi}{Z\, K_{c,t}\, \beta_0 \, \mathrm{Re}(H_{11}(\omega))} \text{ and } \tau_k(\omega) = \frac{1}{\omega}\big((2k+1)\pi + 2\psi(\omega)\big), \tag{12}$$

where $k = 1, 2, \ldots$ and $H_{11}(\omega)$ is originated from the equation defined at (6) as

$$\mathbf{U}(\omega) = \mathbf{H}(\omega)\,\Delta\mathbf{Q}(\omega), \tag{13}$$

and

$$\mathbf{H}(\omega) = (-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{C} + \mathbf{K})^{-1} = [H_{ij}(\omega)], \text{ if } i, j = 1, 2,$$

while $\psi(\omega) \in [-\pi, 0]$ is the phase of $H_{11}(\omega)$.

This classical approach relates the stability of the cutting process to the value of real part of the FRF [Mancisidor, et al. 2014]. Therefore, equations at (12) open the way for using simple optimization based on the analytical representation of the FRF $H_{11}(\omega)$ supposing $c_1 \approx 0$ [Sims, 2007]. The stability of regenerative cutting process requires a special tuning where the real part should be shrunk. The average directional factor β_0 can be positive or negative value depending on the engagement and the cutting characteristics. Therefore, if the directional factor is positive, the negative side of the real part should be maximized creating two equal peaks. On the other hand, if the directional factor is negative the positive side should be minimized.

TMD / VSTMD				Structure	
mass ratio	damping ratio	frequency ratio	natural frequency	dimensionless frequency	natural frequency
$\mu := \frac{m_2}{m_1}$	$\chi := \frac{c_2}{2\, m_2\, \omega_1}$	$f := \frac{\omega_2}{\omega_1}$	$\omega_2 = \sqrt{k_2/m_2}$	$g := \frac{\omega}{\omega_1}$	$\omega_1 = \sqrt{k_1/m_1}$

Table 1; definition of dimensionless dynamic parameters.

The real part of a FRF dominated by a single mode can be given by means of several dimensionless parameters (see Table 1):

$$\mathrm{Re}(H_{11}(g)) = \frac{1}{k_1} \frac{(f^2 - g^2)((1-g^2)(f^2-g^2) - \mu f^2 g^2) + 4\chi^2 f^2 g^2 (1 - g^2 - \mu g^2)}{((1-g^2)(f^2-g^2) - \mu f^2 g^2) + 4\chi^2 f^2 (1 - g^2 - \mu g^2)^2}, \tag{14}$$

where f is the tuning of the absorber, μ is the mass ratio, g is the dimensionless frequency and $\chi = c_2 / c_{cr}$ is the damping ratio relative to $c_{cr} = 2\, m_2\, \omega_1$ "critical" damping defined in [Den Hartog, 1934].

3.2 Linear stability of stationary cutting determined by SDM

The SDM is used in this work to obtain the linear stability borders. The complete description of this method can be found in the literature [Insperger et al., 2011]. It compiles the discrete representation of the solution operator (step matrix) of autonomous (time independent) DDEs. The step matrix $\mathbf{B}_l$ is derived for one single arbitrary chosen time step $\Delta t = \Delta\theta = \lceil \tau_{max}/m \rceil$. Time periodicity is taken into account by the use of Floquet theory [Farkas, 1994] over the tooth passing period T by simply multiplying the corresponding step matrices, that is for

$$\mathbf{z}_{l+r} = \mathbf{\Phi} = \mathbf{B}_{l+r-1}\mathbf{B}_{l+r-2}...\mathbf{B}_l\, \mathbf{z}_l \,, \text{ where } \mathbf{z}_l = \operatorname*{col}_{k=1}^{m} \mathbf{x}(t_l - (k-1)\Delta\theta)\,. \tag{15}$$

In equation (15) $r = T/\Delta t$ represents the resolution of the approximation, while $\theta \in [-\tau_{max}, 0]$ is the relative 'delayed time' to describe a state of the delayed system considering the following solution definition $\mathbf{x}(t+\theta) := \mathrm{col}(\mathbf{u}(t+\theta), \dot{\mathbf{u}}\,(t+\theta))$. The eigenvalues of the so-called Floquet multipliers determine if the resulting periodic stationary solution is stable: $|\mu_k| < 1$. The multiplications at (15) can be accelerated by using spare matrices or using special techniques like in [Henninger, et al. 2008] or determining efficiently the eigenvalue problem itself [Zatarain et al. 2014].

3.3 Time domain solution

The final verification of the optimal tuning has been performed using pure time domain simulation. The simulation of the milling process was performed by using the original milling model as a base, (5) is supplemented by the non-smooth behavior of the fly-over effect. Fly-over appears if the local instantaneous chip thickness

$$\begin{aligned} h_i(t; r_i) = (r_i\, f_Z + \sin\psi(q_1(t) - q_1(t - r_i\,\tau)))\sin\varphi_i(t) + \\ \cos\psi(q_1(t) - q_1(t - r_i\,\tau))\cos\varphi_i(t) \end{aligned} \tag{16}$$

is less than zero considering all possible previous cuts.

Considering all possible fly-overs and cuts, the effective chip thickness can be determined as a minimum $h_{ie}(t) := \min_l h_i\,(t; l\,)$. If $h_{ie}(t) \le 0$ the corresponding ith tooth flies-over at the time instant t and the cutting force switches off. This results in the following force definition after (1)

$$\mathbf{F}(t) = -a_{\mathrm{p}} \sum_{i=1}^{Z} g_{\mathrm{fo}i}(\varphi_i(t))\, \mathbf{T}_i(\varphi_i(t))\, (\mathbf{K}_{\mathrm{e}} + \mathbf{K}_{\mathrm{c}} h_{ie}(t))\,, \tag{17}$$

where the switching function $g_{\text{fo}i} = 1$, if the corresponding ith edge is in radial immersion and if that edge is not in fly-over state, that is, $h_{ie}(t) > 0$. Formulating the first order form using the dynamics from (5) with the excitation at (17), the simulation was performed by standard **`dde23`** introducing special initial function (IF). The IF is the corresponding stationary solution $\mathbf{y}_0(\theta) = \text{col}(\mathbf{q}_\text{p}(\theta), \dot{\mathbf{q}}_\text{p}(\theta))$ in order to avoid the effect of weak attraction zones appearing in the non-smooth fly-over system [Dombovari, et al. 2010].

4. VARIABLE STIFFNESS TUNEABLE MASS DAMPER (VSTMD)

In the literature, there are two main analytical results for the ideal tuning of passive dampers. On the one hand, the work of [Den Hartog, 1934] uses two invariant analytical points to achieve the decrease of the receptance magnitude in all frequencies. On the other hand, the work of [Sims, 2007] shows three invariant points to achieve the best possible real parts of the corresponding FRF considering a_lim by ZOA (12) as an objective.

A case study described by [Aguirre et al., 2013] has been used to compare the performance of the different strategies. In this case, the moving mass was $m_2 = 7$ kg and the mass ratio is $\mu = 4.7$ %.

D (mm)	φ_en (deg)	φ_ex (deg)	ψ (deg)
32	20.36	180	0
Material	$K_{\text{c},t}$ (MPa)	$K_{\text{c},r}$ (MPa)	f_Z (mm/tooth)
C45	1459	257	0.1
ω_1 (Hz)	ξ_1 (%)	m_1 (kg)	$\mathbf{p}_1$
94	0.35	150	$[0\ 1\ 0]^\text{T}$

Table 2; Milling process parameters.

The optimal tuning f (14), the optimal stiffness k_2 and damping c_2 can be determined for both methods by considering initial process parameters presented in Table 2 and the optimal parameters described in Table 3. The results are presented in Figure 2*b* about a near full immersion milling process (see Table 2) without any passive damper, a passive damper tuned according to [Den Hartog, 1934], and a passive damper tuned according to [Sims, 2007].

However, all of these theories work with fixed and invariant stiffness of the damper (see k_2 at (5)) for the different spindle speeds, which is not the case for VSTMD. This tuning procedure and the design of the rotary spring (see Figure 1*a*) allow setting the stiffness value iteratively or even continuously between a minimum and a maximum stiffness values, which means different optimal tuning can be realized along the stability limits.

	Frequency ratio	Damping ratio
[Den Hartog, 1934]	$f_{\mathrm{DH}} = \dfrac{1}{1+\mu}$	$\chi \approx \sqrt{\dfrac{3\mu}{8(1+\mu)^3}}$
[Sims, 2007] ($\beta_0 > 0$)	$f_{\mathrm{opt,n}} = \sqrt{\dfrac{\mu+2+\sqrt{2\mu+\mu^2}}{2(1+\mu)^2}}$	$\chi := \sqrt{\dfrac{3\mu}{8(1+\mu)}}$
[Sims, 2007] ($\beta_0 < 0$)	$f_{\mathrm{opt,p}} = \sqrt{\dfrac{\mu+2-\sqrt{2\mu+\mu^2}}{2(1+\mu)^2}}$	

Table 3; Optimal tuning analytical expressions.

By the modification of (6), the equation of motion has the form

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}_j\,\mathbf{u}(t) = \Delta\mathbf{Q}(t;\mathbf{u}(t),\mathbf{u}(t-\tau)), \tag{18}$$

where

$$\mathbf{K}_j = \begin{bmatrix} k_1 + k_{2,j} & -k_{2,j} \\ -k_{2,j} & k_{2,j} \end{bmatrix},\; k_{2,j} \in [k_{2,\min}, k_{2,\max}]. \tag{19}$$

Here j represents the jth set stiffness of the iteration described in [Aguirre et al., 2013]. The iteration is only performed if unstable stationary cutting appears, that is, when the chatter frequency ω_{ch} and its modulations raise up-to a previously defined level compared to the harmonics of the forced vibration. In this case, the embedded VSTMD system acts by rotating the specially designed spring. It tunes the system to the dominant chatter frequency if it is possible without the violations of the bounds (19). The idea is behind to set the “resonant” stable pockets (see Figure 2*b*) to the used spindle speed zone by changing the stiffness of the tunable damper to $k_{2,j} = \omega_{\mathrm{ch}}^2 m_2$.

4.1 Ideal tuning of VSTMD

Regardless of the tuning iteration procedure applied on the VSTMD, it can only be successful if the system can be stabilized at all. This means that there is a special stiffness value where the cutting process (stationary cutting) is stable.

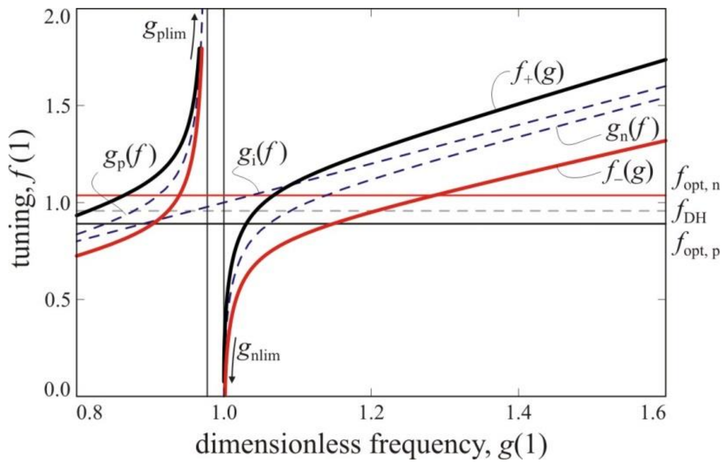

Figure 3; Optimal tuning w.r.t. the dimensionless frequency g.

This fact is convenient because the ideal tuning can be found by means of linear theories presented at (12) and (15). The ZOA and the FRF at (14) can be used for the analytical derivation. In order to ease the notation the magnifying function $M_{11}(g, f) := H_{11}(g)\, k_1$ is introduced.

According to the derivation of ZOA [Altintas, et al. 2008] the arising chatter frequency ω_{ch} can be considered as sampling on the corresponding FRF. Consequently, considering non-interrupted case the FRF can be optimized at each dimensionless frequency $g = \omega_{\mathrm{ch}} / \omega_1$. This means finding an extremum of $\mathrm{Re}(M_{11}(g, f))$ by taking $\mathrm{dRe}(M_{11}(g, f))/\mathrm{d}f$, which leads to five roots for tuning including one at $f_0 = 0$ and four others symmetrically placed. Thus, these latter four can be considered as two extremums as f_+ and f_-

$$f_{\pm}(g) = \sqrt{\frac{g^2 + g^6(1+\mu) \pm 2\sqrt{g^4(g^2-1)(g^2(1+\mu)-1)^3 \chi^2}}{(g^2(1+\mu)-1)^2}}\ . \qquad (20)$$

This new formula shows the best tuning of a damper to have the maximum stability for a certain spindle speed represented indirectly (see (12)) by the dimensionless value (g). This formula can improve the result of Sims' proposal for a single spindle speed.

In Figure 3, Sims' locked frequencies are $g_{\mathrm{p}}(f)$, $g_{\mathrm{n}}(f)$ and $g_{\mathrm{i}}(f) = f$ and their limits are g_{plim} and g_{nlim}. Among the solutions presented at (20), it is always $f_+(g)$ that gives the maximum and $f_-(g)$ that gives the minimum for $\mathrm{Re}(M_{11}(g, f))$. Keeping in mind the results of the ZOA depending on the sign of the directional factor $\beta_0 > 0$ ($\beta_0 < 0$) at (10) $\mathrm{Re}(M_{11}(g, f))$ needs to be maximized (minimized). Namely, the best solution for the tuning is always given by $f_+(g)$ ($f_-(g)$), except between the values of g_{plim} and g_{nlim} where the optimum values determined at (20) are extremes.

a) *b*)

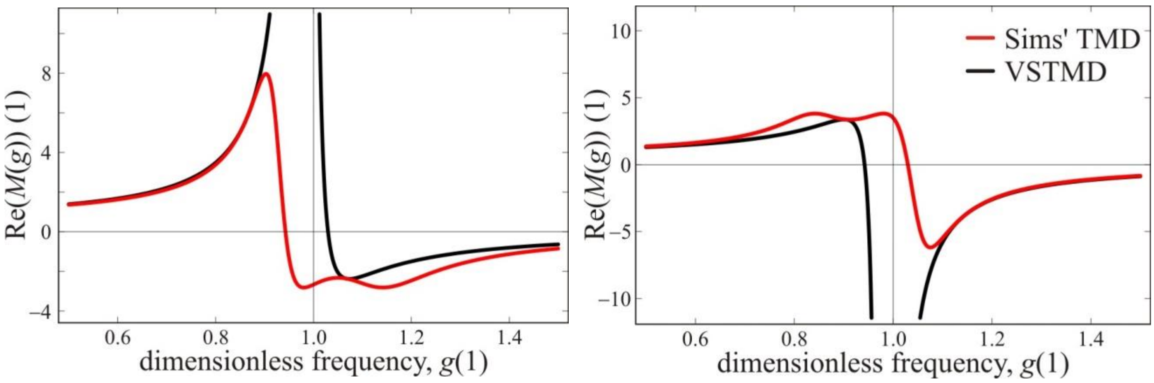

Figure 4; Real parts of the optimally tuned magnifying function for positive directional factor ($\beta_0 > 0$) *a*)*, and the same for negative directional factor* ($\beta_0 < 0$) *compared to Sims' optimal tuning (red) [Sims, 2007].*

In this region, depending on $\beta_0 > 0$ ($\beta_0 < 0$) a minimum f_{min} (maximum f_{max}) tuning is limiting not realizable stiffness values on the rotary spring (see (21) and (22)).

In Figure 3, one can realize that Sims' constant optimal tuning $f_{\text{opt,p}}$ ($f_{\text{opt,n}}$) for $\beta_0 > 0$ ($\beta_0 < 0$) is only optimal at two dimensionless frequencies g, below and above $g = 1$. Although, along the lobes limited frequencies are possible, usually g>~1 (g<~1) if $\beta_0 > 0$ ($\beta_0 < 0$). This means one point along the lobe is optimal for a VSTMD in this simple case (see Figure 4).

4.2 Ideal stability behavior of VSTMD

Based on the calculated optimal tuning function $f_{\pm}(g)$, piecewise smooth definition can be defined, which take into account the physical limitation of the rotary spring. The limitations are only active close to the original natural frequency ω_1 of the structure.

$$\text{When } \beta_0 > 0,\ \ f_{\text{opt,p}}(g) = \max\{f_+(g), f_{\text{min}}\}, \tag{21}$$

$$\text{while } \beta_0 < 0,\ \ f_{\text{opt,n}}(g) = \min\{f_-(g), f_{\text{max}}\}. \tag{22}$$

Using these piecewise smooth definitions, the stability boundaries can be depicted (see Figure 5) by simply applying ZOA shown at (12). For both cases of negative and positive directional factors, the stability is improved by using VSTMD solution. In these ideal examples, one can realize there is a point in the stability limits where the stability is not improved with respect to the Sims' optimal tuning values (see Figure 3). It can be also noticed that the "double lobe" shape is disappears and in this ideal solution the lobes are simple shaped stability boundaries.

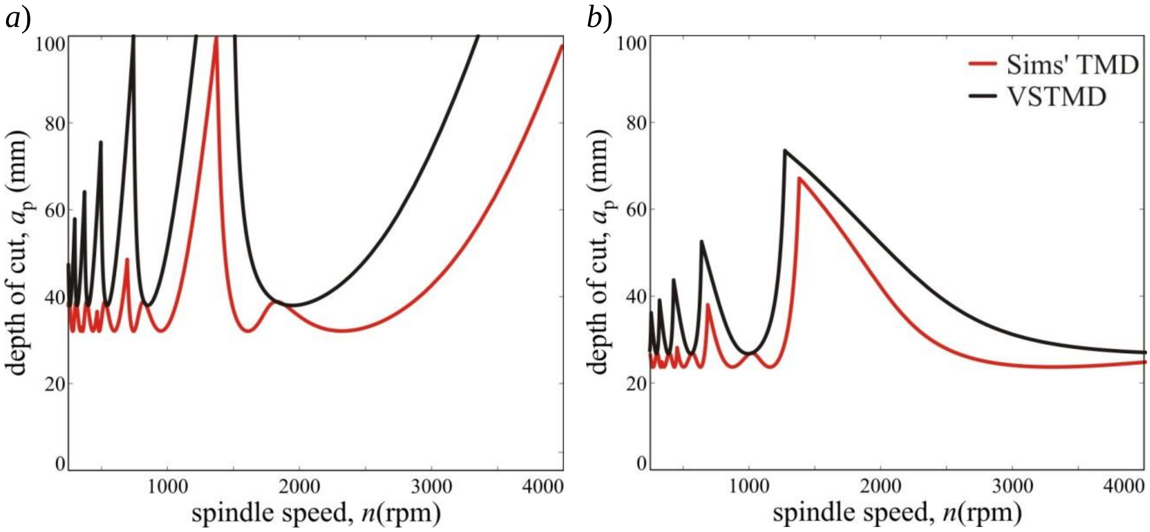

Figure 5; ZOA stability limits based on the optimal FRF for positive directional factor ($\beta_0 > 0$) *a*), *and the same for negative directional factor* ($\beta_0 < 0$) *b*) *compared with Sims' solutions* (*red*).

4.3 Iterative algorithm for optimal tuning VSTMD

It has been demonstrated that if the tuning of the damper is changed for each spindle speed using a VSTMD, the stability of the process can be improved more than an ordinary TMD with the same mass tuned according to Sims' parameter. However, it is a complex procedure to implement it in practice. In this section, an iterative method to approach this optimal solution is going to be proposed.

The first step towards chatter suppression is to detect whether chatter is occurring or not during the machining process. The vibration measured by the accelerometer installed on the structure of the damper is processed in order to find its main frequency components. Here it is important to distinguish between forced vibrations, induced directly by the cutting forces, and chatter, which is due to an unstable regenerative process generated only under certain working conditions.

Forced vibrations appear at harmonics of the tooth passing frequency, but are stable, and thus are usually not a problem for machining, except in finishing operations where surface roughness needs to be improved. Chatter appears at other frequencies than tooth passing frequencies. It is an unstable cutting process, meaning that the cutting forces and vibrations increase with time, leading to unacceptable machining conditions, since they produce very bad surface quality and can lead to damage in the machine.

The chatter detection and suppression algorithm is presented in *Figure 6*

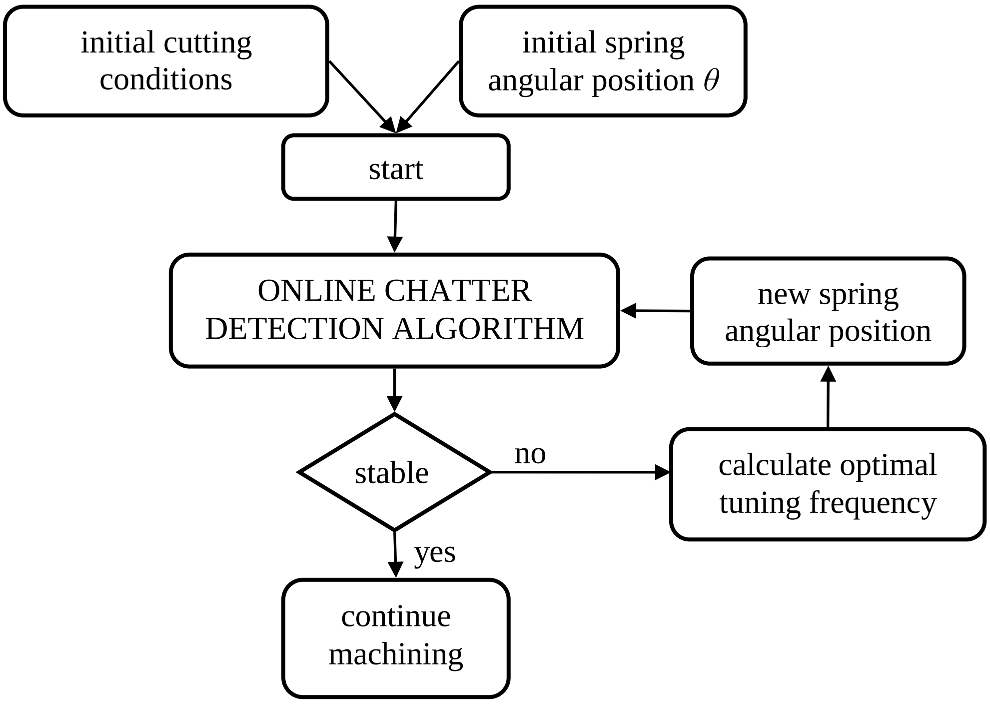

Figure 6; Chatter detection and suppression algorithm.

This algorithm is implemented on a real-time controller. It is running continuously during the machining process, calculating the spectrum of the measured vibration of the machine, as shown in Figure 7*b*. The algorithm detects the frequency of the maximum vibration peak, and compares it with the tooth passing frequency: if it is an integer multiple of the tooth passing frequency, it is considered to be a forced vibration, and no corrective action is taken. If it is not an integer multiple, it is considered to be chatter, and the angular position in the damper is modified in order to tune it to the chatter frequency.

It is very important to clearly distinguish clearly chatter from forced vibrations, so that the damper is only tuned to chatter frequencies. Otherwise, once the damper is tuned to the chatter frequency, the vibration level at this frequency will drop, and the algorithm will detect a forced vibration as main frequency. If the damper is tuned to this new frequency, chatter generation could start again, which needs to be avoided.

The detection algorithm first filters the acquired acceleration signal with a low pass filter. Then the FFT of the filtered signal is calculated, finding the frequency of the highest peak. When the highest peak's frequency does not match any harmonic of the spindle speed, chatter detection is considered positive.

5. SIMULATION RESULTS

In this section, real case simulations are presented by the real tuning procedure of the modelled VSTMD solution. Time domain simulations are only performed if the technological parameters set result in unstable cutting determined by SDM (15). The time domain simulations are started from the perturbed stationary solution $\mathbf{q}_\mathrm{p}$ to reach threshold fly-over [Stepan, et al. 2011] effect fast. Roughly 30-40 periods are simulated and 10-20 periods long of signal from the end of the simulation is taken for FFT analysis. With the removal of the DC component, the chatter frequency can be extracted easily since the stationary cutting solution $\mathbf{q}_\mathrm{p}$ is known and the harmonics of its amplitude can be used as a threshold for chatter. The maximum among the peaks overtaking the limiting amplitude is considered as chatter frequency and the VSTMD is tuned to this main frequency with $k_{2,j} = \omega_\mathrm{ch}^2 m_2$. This tuning is done subsequently until stability determined by SDM or a preset maximum number of iterations are reached.

Sims' and the ideal tuning solutions are presented in Figure 7*a* for milling operation calculated by SDM in this case. Time domain calculations were performed for selected spindle speeds (A, B and C) listed in Table 2 in order to see the tuning iterations of the VSTMD. The time domain simulations combined with the fly-over effect must be performed because there is no sufficiently mature method that can determine the dominant frequency of the threshold periodic, quasi-periodic or completely chaotic motion [Dombovari, et al. 2010].

The 'originally stable' cases in Figure 7 and Table 4 are stable according to Sims', no tunings are necessary. The 'tuned' cases are parameters, when the tuning was effective by using the simulated chatter frequencies. The 'not tuneable' situations are the cases when even the VSTMD was not effective.

A_1: n_A=1600 rpm, $a_{\mathrm{p},1}$=30 mm	B_1: n_B=1830 rpm, $a_{\mathrm{p},1}$=30 mm	C_1: n_C=2500 rpm, $a_{\mathrm{p},1}$=30 mm
Originally stable	Originally stable	Originally stable
A_2: n_A=1600 rpm, $a_{\mathrm{p},2}$=45 mm	B_2: n_B=1830 rpm, $a_{\mathrm{p},2}$=40 mm	C_2: n_C=2500 rpm, $a_{\mathrm{p},2}$=40 mm
$\omega_{\mathrm{ch},j}$ = 96.7, 106.7, 94 Hz Stable in three tuning iterations.	Originally stable	$\omega_{\mathrm{ch},j}$ = 115.6, 111.5 Hz Stable in two tuning iterations
A_3: n_A=1600 rpm, $a_{\mathrm{p},3}$=70 mm	B_3: n_B=1830 rpm, $a_{\mathrm{p},3}$=50 mm	C_3: n_C =2500 rpm, $a_{\mathrm{p},3}$=60 mm
$\omega_{\mathrm{ch},j}$ = 98.7, 97.3, 98.7, … Hz Unstable	$\omega_{\mathrm{ch},j}$ = 106.0, 99.1, 99.9, 99.1, 101.4, … Hz Unstable	$\omega_{\mathrm{ch},j}$ =118.8, 113.5, 121.9, 111.5 Hz Stable in four tuning iterations
		C_4: n=2500 rpm, $a_{\mathrm{p},4}$=70 mm
		$\omega_{\mathrm{ch},j}$ =119.8, 113.5, 124.0, 113.5, 123… Hz Unstable

Table 4;The detailed results of the time domain simulations of the selected points (*see Figure 7a*).

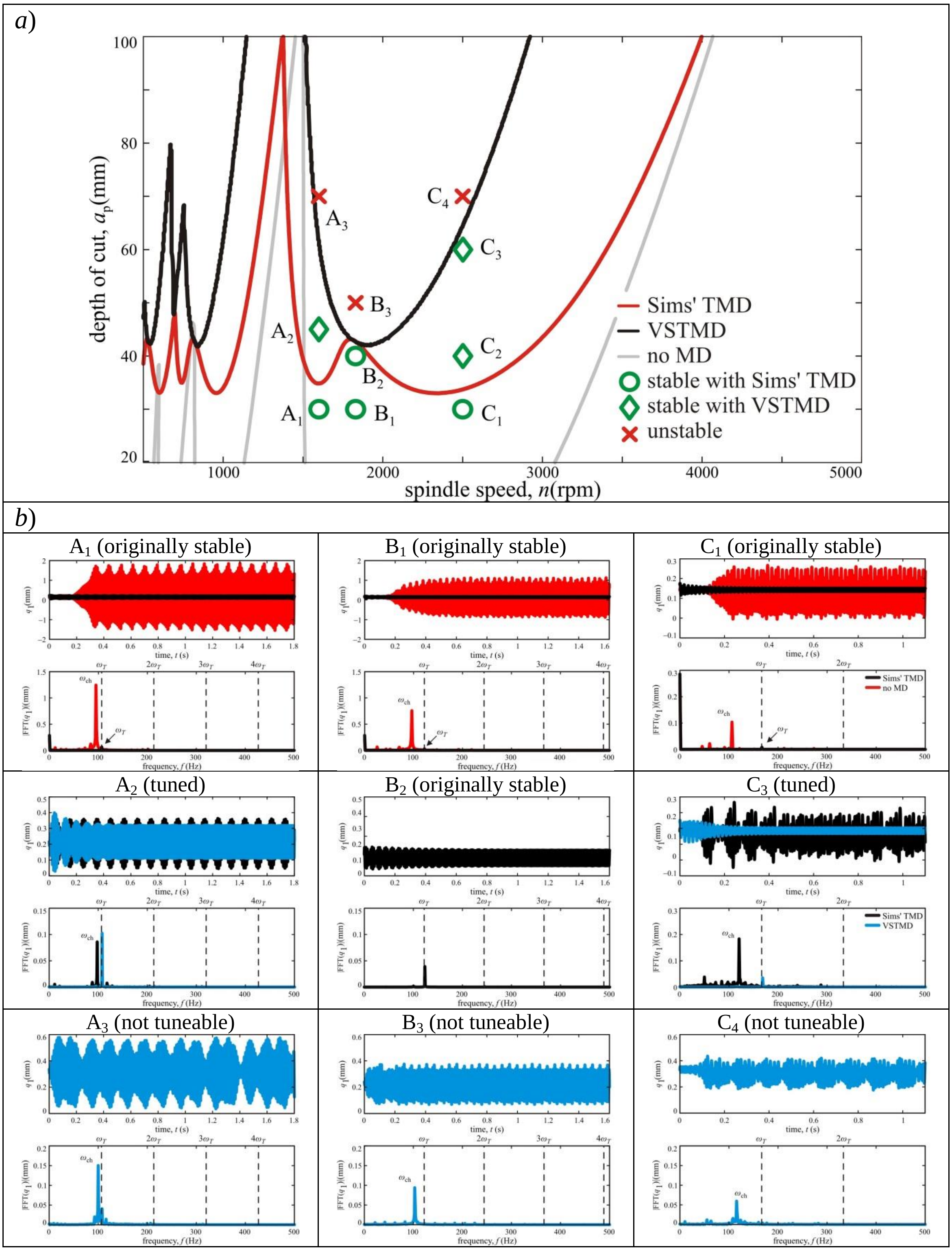

Figure 7; a) shows ideal tuning solutions for VSTMD *(red) performed by* SDM *($\beta_0 > 0$) compared to Sims' case (black). b) shows time domain solution at selected points.*

The specific parameters and tuning data of the simulated cases presented in Figure 7 can be followed in Table 4.

It is important to mention that the authors experienced small attraction zones at some parameter sets like C_3 when the non-smooth fly-over effect introduced weak attraction zone. In this case the accurate selection of IF was of great importance, although this situation hardly appears in realistic cases.

Summarizing, we can say that the time domain simulations confirm that VSTMD can improve the results obtained by a single VSTMD with the optimal design for chatter, and the proposed iterative algorithm can approach the optimal solution defined in the previous sections for a VSTMD.

6. CONCLUSION

In the presented work a variable stiffness tuneable mass damper concept was investigated. The real system is built as an embedded system that is capable to decouple the applied damping and stiffness.

The paper deals with the idealized tuning of this semi-active damper. Sims' idea to use the real part of the corresponding frequency response function was used when but in this case the optimization was performed at each frequency. This way, a new tuning method has been proposed to maximize the stability in a certain spindle speed. Using this concept, the frequency dependent tuning function was derived analytically and the best possible stability for a VSTMD has been obtained. Later the concept was confirmed by using the mathematical model of the tuned mass damper performing semi-discretization and time domain simulations.

An iterative method to approach the optimal solution has been proposed and verified by means of time domain simulations. It can be seen that the method together with the VSTMD solution is able to improve the stability behavior by simply retuning the damper to the best optimal stiffness value in each spindle speed. Time domain simulations sometimes showed weak attraction behavior which effect must be checked more closely in the future.

7. ACKNOWLEDGEMENT

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