Excitation of Oscillations in the Melt by Frequency-Modulated TMF

B Mikhailovich, A Kapusta, A Levy

Excitation of Oscillations in the Melt by Frequency-Modulated TMF

B. Mikhailovich, A.Kapusta, A. Levy

Pearlstone Center for Aeronautical Engineering Studies
Department of Mechanical Engineering, Ben-Gurion University of the Negev
P.O. Box 653, Beer-Sheva, 84105, Israel

Corresponding author: borismic@bgu.ac.il

Abstract
We consider a magnetohydrodynamic flow in a cylindrical vessel caused by frequency-modulated magnetic field traveling along vertical axis in order to estimate the field impact on the flow parameters. To increase the impact efficiency and intensify transfer processes, we propose to generate non-stationary structures of auto-oscillatory type in the melt against the background of a convective flow, whose parameters are connected with the parameters of the modulated electromagnetic field. The forced convective flow generated in liquid metal contributes to its homogenization and can improve the structure obtained at the melt crystallization.

Key words: Traveling magnetic field, liquid metal flow, cylindrical vessel, "external" friction model, frequency modulations, auto oscillations, transfer processes intensification.

Introduction
Magnetohydrodynamic (MHD) impact on liquid metals and alloys is well known and widely used both in laboratory and in various technological processes. At the same time, the need to enhance the efficiency of such an impact remains important and requires some modifications and the development of new approaches. One of such approaches is the use of modulated rotating or traveling magnetic fields, where modulation parameters play the role of additional degrees of freedom in the generation of MHD flows.
In particular, review [1] describes some MHD methods of exciting oscillations of various frequencies in molten metals with the transfer of forced vibrations into the solidifying melt, which contributes to the improvement of their structure. It is obvious that such vibrations introduced into the melt promote a decrease of the boundary layer thickness near the liquid-solid interface. It follows that it is possible to affect the boundary layer thickness, which, in turn, can serve as an effective tool for affecting the parameters of the melting process. Some aspects of such impact have been studied previously in [2], [3] where the oscillations in the melt were excited by modulated electromagnetic forces.
The most important part of the research in the described area is setting a connection between the parameters of the modulated field and the generated MHD flow. In perspective, this will allow generating hydrodynamic structures required for solving technological problems, such as the intensification of melt stirring and its homogenization.
In the present study we examine the impact of frequency-modulated traveling magnetic field (TMF) along the vertical axis of a cylindrical vessel on the liquid metal contained therein. The possibility of MHD flow realization with time-dependent hydrodynamic structures of auto-oscillatory type arising in the melt against the background of a convective flow is analyzed.

Problem statement
We study a turbulent flow arising under the action of frequency-modulated TMF activated by an ideal inductor in a cylindrical vessel of a limited height. The electromagnetic part of the problem is described by dimensionless equations [3]:

$$\Delta a_\varphi = \frac{\varpi}{2\pi}\left\{\frac{\partial a_\varphi}{\partial t} + \frac{1}{\delta_z r}\frac{\partial\left(r\psi_\varphi\right)}{\partial r}\frac{\partial a_\varphi}{\partial z} - \left(\frac{1}{\delta_z r}\frac{\partial \psi_\varphi}{\partial z}\frac{\partial (r a_\varphi)}{\partial r}\right)4\pi\tau\right\} \qquad (1)$$

with boundary conditions: $\frac{1}{r}\frac{\partial (r a_\varphi)}{\partial r}\Big|_{r=1} = -A_t \exp 2\pi i\theta, \qquad a_\varphi\Big|_{r=0} < \infty,$

where a_φ is the φ - component of the vector potential $\vec{a}$ of the magnetic induction ($\vec{b} = rot\,\vec{a}$), ψ_φ is the φ - component of the hydrodynamic stream function $\vec{\psi}$ ($\vec{u} = rot\,\vec{\psi}$), $\vec{b}$ and $\vec{u}$ are the vectors of magnetic induction and flow velocity, respectively, $\theta = \omega t - z/2\tau + \omega_f \sin 2\pi\Omega_f t / 2\pi\Omega_f$, A_t is a dimensionless linear current load, $\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} + \frac{1}{4\pi^2\delta_z^2}\frac{\partial^2}{\partial z^2}$; $\delta_z = Z_0 / R_0$ is a height-radius of the cylinder aspect ratio; ω, Ω_f, ω_f are the carrier frequency, modulation frequency and frequency deviation, τ is a pole pitch of the TMF inductor, $\varpi = \mu_0\sigma\omega_0 R_0^2$.

Using the semi-empirical "external" friction model, we can describe the hydrodynamic flow by the following dimensionless equations [3], [4]:

$$\frac{\mathrm{Re}_\omega}{2\pi}\frac{\partial \Delta\psi_\varphi}{\partial t} - \Delta^2\psi_\varphi + \lambda\Delta\psi_\varphi = -\frac{2Ha^2}{\omega_0} rot_\varphi \vec{f}, \tag{2}$$

$$\psi_\varphi\big|_\Gamma = 0;\ \frac{\partial(r\psi_\varphi)}{\partial r}\bigg|_{r=1} = 0;\ \frac{\partial^2(r\psi_\varphi)}{\partial r^2}\bigg|_{r=0} = 0;\ \frac{\partial\psi_\varphi}{\partial z}\bigg|_{z=0} = 0;\ \frac{\partial^2\psi_\varphi}{\partial z^2}\bigg|_{z=1} = 0,$$

where $\lambda = C_\varepsilon < u_z >^{1-\varepsilon} \mathrm{Re}_\omega^{1-\varepsilon}/\delta_z$; $\mathrm{Re}_\omega = \omega_0\tau R_0/\pi\nu$; $Ha = b_0 R_0\sqrt{\sigma/\eta}$; $u_0 = \omega_0\tau/\pi$; ε, C_ε are empirical constants; index "0" refers to characteristic values.
In the induction-free approximation ($\varpi < 1$) we can rewrite Eq. (1) as:

$$\Delta a_\varphi \cong 0, \tag{3}$$

and seek the solution to Eq. (3) in the form: $$a = \alpha(r)e^{2\pi i\theta}. \tag{4}$$

Substituting (4) into (3), we obtain: $$\alpha'' + \alpha'/r - (1/r^2 + 1/4\delta_z^2\tau^2)\alpha = 0, \tag{5}$$

where $$\frac{1}{r}\frac{\partial(r\alpha)}{\partial r}\bigg|_{r=1} = -A_t.$$

The solution to the problem (5) has the form: $\alpha(r) = -A_t I_1(gr)/gI_0(g)$, where $g = 1/2\delta_z\tau$,
whence it follows that: $$a_\varphi = -A_t I_1(gr)/gI_0(g)e^{2\pi i\theta}. \tag{6}$$

Hence, the components of the magnetic induction are:

$$b_r = \frac{1}{2\pi\delta_z}\frac{\partial a_\varphi}{\partial z} = -\frac{A_t I_1(gr)}{I_0(g)}e^{2\pi i\theta},\quad b_z = \frac{1}{r}\frac{\partial(ra_\varphi)}{\partial r} = -S_{1R}\frac{A_t I_0(gr)}{I_0(g)}e^{2\pi i\theta}, \tag{7}$$

and the electric current density:

$$j_\varphi = -\frac{A_t}{2\pi}\left(\frac{\partial a_\varphi}{\partial t} - u_z b_r + u_r b_z\right) = A_t i\varpi S e^{2\pi i\theta}, \tag{8}$$

where the slip $S = S_R - iS_I$, $S_R = \left(1 + \omega_f\cos 2\pi\Omega_f t - u_z\right)I_1(gr)/I_0(g)$, $S_I = u_r I_0(gr)/I_0(g)$.

Consequently, the components of electromagnetic body force (EMBF):

$$f_r = \mathrm{Re}\, j_\varphi\cdot\mathrm{Re}\, b_z = -A_t^2\varpi I_0(gr)\left[S_I(1+\cos 4\pi\theta) - S_R\sin 4\pi\theta\right]/2I_0(g),$$
$$f_z = -\mathrm{Re}\, j_\varphi\cdot\mathrm{Re}\, b_r = -A_t^2\varpi I_1(gr)\left[S_R(1+\cos 4\pi\theta) - S_I\sin 4\pi\theta\right]/2I_0(g).$$

Azimuthal component of $rot\,\vec{f}$: $$rot_\varphi \vec{f} = \frac{1}{\delta_z}\frac{\partial f_r}{\partial z} - \frac{\partial f_z}{\partial r} = rot_\varphi < \vec{f} > + rot_\varphi \tilde{\vec{f}}, \tag{9}$$

where the stationary part of this component is:

$$rot_\varphi < \vec{f} > = -\frac{\varpi A_t^2}{2I_0^2(g)}\left\{I_1^2(gr) - \frac{\partial}{\partial r}\left[I_1^2(gr) < u_z > - I_1^2(gr)\frac{\partial < u_z >}{\partial r} + \frac{1}{\delta_z}I_0^2(gr)\frac{\partial < u_r >}{\partial z}\right]\right\}. \tag{10}$$

We represent the stream function ψ_φ as a sum of stationary $<\psi>$ and time-dependent $\tilde{\psi}$ components. Substituting (10) into (2), we obtain two equations for determining $<\psi>$ and $\tilde{\psi}$:

$$(\Delta^2 - \lambda\Delta) < \psi > = \frac{2Ha^2 A_t^2}{\varpi} rot_\varphi < \vec{f} >, \tag{11}$$

$$\frac{\mathrm{Re}_\omega}{2\pi}\frac{\partial\Delta\tilde{\psi}}{\partial t} - (\Delta^2 - \lambda\Delta)\tilde{\psi} = -\frac{2Ha^2 A_t^2}{\varpi} rot_\varphi \tilde{\vec{f}} \tag{12}$$

with the boundary conditions:

$$< \psi >\big|_{r=1} = \tilde{\psi}\big|_{r=1} = 0,\quad \tilde{\psi}\big|_{t=0} = 0,\ < \psi >\big|_{z=0,1} = \tilde{\psi}\big|_{z=0,1} = 0, \tag{13}$$

$$D < \psi >\big|_{r=1} = D\tilde{\psi}\big|_{r=1} = 0, \quad \frac{\partial < \psi >}{\partial z}\Big|_{z=0,1} \frac{\partial \tilde{\psi}}{\partial z}\Big|_{z=0,1} = 0.$$

The term $\Delta^2 < \psi >$ in the equation (11) represents the contribution of molecular viscosity to the flow profile formation near the vessel sidewall. In the flow core, it is small in comparison with the terms describing turbulent and MHD effects and can be omitted. Then we get in the flow core:

$$\lambda\Delta < \psi > = -\frac{2Ha^2 A_t^2}{\varpi} rot_\varphi < \vec{f} >, \qquad \frac{\mathrm{Re}_\omega}{2\pi}\frac{\partial \Delta\tilde{\psi}}{\partial t} + \lambda\Delta\tilde{\psi} = -\frac{2Ha^2 A_t^2}{\varpi} rot_\varphi \tilde{\vec{f}} \qquad (14), (15)$$

with the boundary conditions: $< \psi >\big|_{r=1} = \tilde{\psi}\big|_{r=1} = 0, \ \ \tilde{\psi}\big|_{t=0} = 0, \ < \psi >\big|_{z=0,1} = \tilde{\psi}\big|_{z=0,1} = 0$. (16)

Stationary problem solution

We seek the solution of (14), (16) using Galerkin's method as:

$$< \psi > = \sum_{n=1}^{\infty} Z_k(z)\cdot J_1(\gamma_k r), \quad \text{where } \gamma_k \text{ are the roots of the equation } J_1(\gamma_k) = 0. \qquad (17)$$

Substituting (17) into (14) and accomplishing the procedure of Galerkin's method, we obtain:

$$Z_k''(z) - Q_k Z_k(z) = -N_k,$$

where $Q_k = \delta_z^2\gamma_k^2\left(\lambda + C_1 A_t^2 Ha^2\right)/\left(\lambda + C_2 A_t^2 Ha^2\right)$, $N_k = (\delta_z A_t Ha)^2 In_{11} / I_0^2(g) In_{00}\left(\lambda + C_2 A_t^2 Ha^2\right)$,

$$C_1 = \int_0^1 r I_1^2(gr)dr \quad C_2 = \int_0^1 r I_0^2(gr)dr, \ In_{00} = \int_0^1 r J_1^2(\gamma_k r)dr, \ In_{11} = \int_0^1 r\frac{\partial}{\partial r} I_1^2(gr) J_1(\gamma_k r)dr.$$

The solution to $Z_k(z)$ is: $Z_k(z) = N_k[shq_k - shq_k z - shq_k(1-z)] / Q_k shq_k$, $q_k = \sqrt{Q_k}$,

and

$$< \psi > = \sum_{k=1}^{\infty} N_k J_1(\gamma_k r)[shq_k - shq_k z - shq_k(1-z)] / Q_k shq_k. \qquad (18)$$

Transient problem solution

Now we examine transient processes arising in the melt under the action of frequency-modulated TMF. The expression of the transient component $rot_\varphi \tilde{\vec{f}}(t)$ is:

$$rot_\varphi \tilde{\vec{f}} = \varpi(A_1\cos 4\pi\theta - A_2 \sin 4\pi\theta - A_3\cos 2\pi\Omega_f t) / 2\tau\delta_z, \qquad (19)$$

where $A_1 = 0.5\, S_\omega(0.5 - r) + \frac{\tau}{2\pi}\frac{\partial u_r}{\partial z} + \frac{r^2}{8\tau\delta_z}\frac{\partial u_z}{\partial r}$, $\quad A_2 = \frac{u_r}{2} - \frac{\partial u_r}{\partial r} - \frac{1}{4\pi}\frac{\partial u_z}{\partial z}$,

$$A_3 = 0.5\omega_f(0.5 - r), \qquad S_\omega = \omega - u_z / 2\pi\delta_z.$$

The equation for a transient component of the stream function has the form:

$$\frac{\mathrm{Re}_\omega}{2\pi}\frac{\partial \Delta\tilde{\psi}_\varphi}{\partial t} + \lambda\Delta\tilde{\psi}_\varphi = -\frac{Ha^2}{\varpi} rot_\varphi \tilde{\vec{f}} \qquad (20)$$

with the boundary conditions (16).

Rewriting the parameter θ as $\theta = \theta_1(t) - \theta_2$, we obtain from Eq. (20):

$$\frac{\partial \Delta\tilde{\psi}_\varphi}{\partial t} + \frac{2\pi\lambda}{\mathrm{Re}_\omega}\Delta\tilde{\psi}_\varphi = -\frac{\pi Ha^2}{\mathrm{Re}_\omega \delta_z \tau}(f_1\cos 4\pi\theta_1 + f_2 \sin 4\pi\theta_1 - A_3\cos 2\pi\Omega_f t), \qquad (21)$$

where $f_1 = A_1\cos 4\pi\theta_2 + A_2\sin 4\pi\theta_2$, $f_2 = A_1 \sin 4\pi\theta_2 - A_2\cos 4\pi\theta_2$.

We seek the solution to the problem (21), (16) by Galerkin's method in the form:

$$\tilde{\psi}_\varphi = \sum_{k,m=1}^{\infty} T_{km}(t) J_1(\gamma_k r)\sin m\pi z. \qquad (22)$$

Substituting (22) into (21), we obtain: $\dfrac{\partial T_{km}}{\partial t} + GT_{km} = -H_{km}\Phi_{km}(t)$,

where $G = 2\pi\lambda / \mathrm{Re}_\omega$, $H_{km} = 4Ha^2 / \mathrm{Re}_\omega\, \gamma_k^2 m^2 \pi J_2^2(\gamma_k)\delta_z \tau$,

$$\Phi_{km}(t) = \sqrt{I_{1km}^2 + I_{2km}^2}\,\sin(4\pi\theta_1 - arctg I_{1km} / I_{2km}) - I_{3km}\cos 2\pi\Omega_f t \,,$$

$$I_{1km} = \int_0^1\int_0^1 f_1 r J_1(\gamma_k r)\sin m\pi z\, dr\, dz \,,\quad I_{2km} = \int_0^1\int_0^1 f_2 r J_1(\gamma_k r)\sin m\pi z\, dr\, dz \,,$$

$$I_{3km} = \frac{\omega_f}{2}\int_0^1\int_0^1 r(1/2 - r)J_1(\gamma_k r)\sin m\pi z\, dr\, dz \,.$$

At the first step, it is assumed that $\tilde{\psi}_\varphi = \ <\psi>$, and then the change as a function of time is considered.
The solution of the problem is:

$$T_{km} = -e^{-Gt} H_{km}\int_0^t e^{Gx}\Phi_{km}(x)dx \,. \tag{23}$$

Figure 1 gives examples of the flow streamlines in the stationary (a) and transient (b and c) cases. It is noteworthy that such temporal evolution of the streamlines is characterized by a change in their direction, i.e., due to frequency modulations of the TMF, hydrodynamic structures of auto-oscillatory type arise in the melt against the background of a convective flow. These forced oscillations can intensify mass and heat transfer processes in liquid metal, contribute to its homogenization and improve the structure obtained at the melt crystallization.

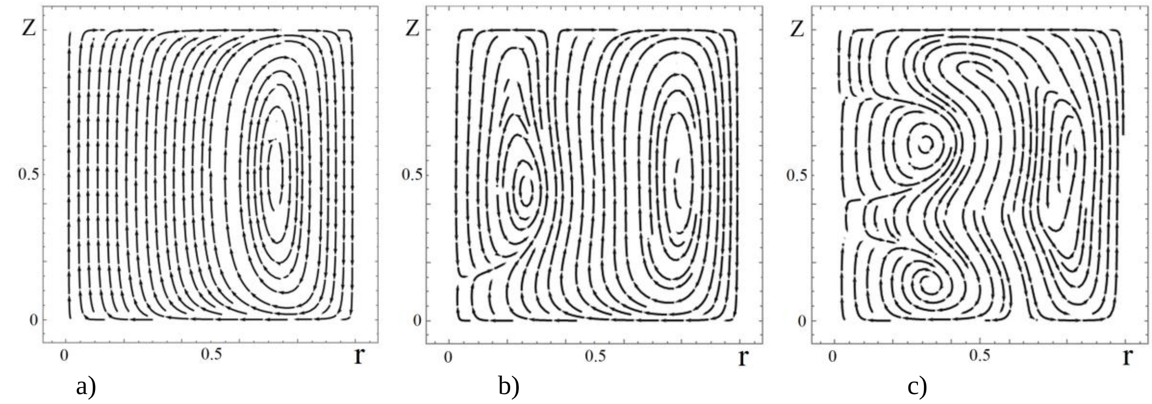

Fig. 1: Streamlines of stationary flow a) and transient flow b), c) in vertical cross-section of the cylindrical vessel at $Ha = 80$ and modulation index $\omega_f / \Omega_f = 2$.

Conclusions
Along with known advantages of MHD methods of impact on the molten metal in various technological processes, the results obtained in the work open new perspectives in the control of melt flow parameters. Changing the index of modulation frequency leads to energy redistribution in the spectrum of electromagnetic forces and to a transformation of the generated hydrodynamic structures. At the same time, there appears an opportunity, for example, to intensify the forced convection, accelerate the melting process and improve the melt homogenization, to suppress the dendrites growth during solidification and refine the final product quality.
The experiments planned for the next stage of the research will clarify the optimal parameters of the described MHD impact.

References

[1] C. Vives (1998), JOM 50 (2), 1-7
[2] X. Wang, R. Moreau, J. Etay, Y. Fautrelle (2009), Metallurgical and Materials Trans. B 40B, 104-113
[3] A. Kapusta, A. Levy, B. Mikhailovich (2012), Journal of Iron and Steel Research Int.19 (1-2), 771-774
[4] E. Golbraikh, A. Kapusta, B. Mikhailovich (2007), Magnetohydrodynamics 43 (1), 35-44