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Magnetic field effects on liquid metal free convection

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Abstract
We provide a numerical analysis of three-dimensional free convection of a liquid in a Rayleigh-Bénard configuration, subject to a steady and uniform magnetic field, using the finite volume code Jadim. The influence of the Hartmann and Rayleigh numbers are studied. We compare our results to several experimental works. As suggested by previous experiments, the magnetic field tends to lower the heat transfer at the walls. This is caused by a significant alteration of the flow structures, due to the Lorentz force. For slightly overcritical Rayleigh numbers, two-dimensional rolls appear but the flow structure rapidly becomes three-dimensional as we increase the Rayleigh number. The magnetic field tends to destroy those structures and the transition to a 3D flow is delayed to higher values of the Rayleigh number, when the Hartmann number is increased. We show that the averaged heat transfer at the walls decreases, although it remains of the same order of magnitude. However the local structure of heat transfer is altered.

Key words: numerical simulations, magnetoconvection, heat transfer, linear stability

Introduction

The Vulcano experiments are carried out at CEA (Commissariat à l'Energie Atomique) in Cadarache, in order to model the corium-concrete interaction, using induction heating of an electrically conducting liquid. In such experiments, the volumetric source of nuclear reactions is simulated by Joule effect, due to induced eddy currents in the liquid. In that case, the alteration of turbulent convection can take two aspects: (i) heating by Joule effect causing local buoyancy forcing, and (ii) momentum forcing due to the Lorentz forces. In this paper, we focus on the second aspect, by considering the free convection of a conducting liquid, subjected to a steady and uniform magnetic field. Our setting consists in a Rayleigh-Bénard convection configuration, which has been widely studied. It consists in a cavity heated from below and cooled from above. Chandrasekhar [1] theorised the onset of convection in the presence of a magnetic field. Considering two infinite horizontal plates, he demonstrated the stabilising effect of a magnetic field on convection. Those results were confirmed by several experiments [2]-[5]. Globe & Dropkin [6] found a correlation between the Nusselt number Nu and the Rayleigh Ra and Prandtl Pr numbers. This work is in agreement with the works of Rossby [2], Aurnou & Olson [3], Burr & Müller [4], [5]. The results from [2]-[5] have shown a reduction of heat fluxes in magnetoconvection, in comparison with ordinary thermoconvection. The reorganisation of the patterns can display a wide range of motifs in classical thermoconvection, depending on geometric properties and experimental conditions [7]. The structure in rolls has been observed in magnetoconvection, and they are more stable than in thermoconvection [8]. We first recall the linear stability analysis from Chandrasekhar [1], in order to assess the damping of the convection by a magnetic field. In the second part of this paper, we use direct numerical

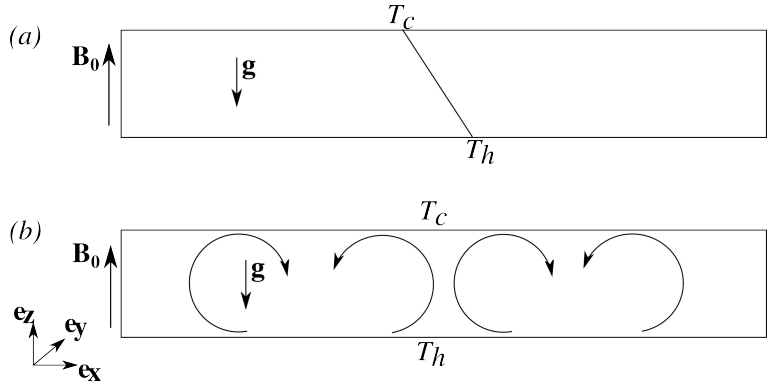


Fig. 1: Stable and unstable cases of a fluid layer heated from below, with an impressed magnetic field.

simulations (DNS) at overcritical Rayleigh numbers to analyse the modification of convective patterns and heat transfer by a vertical magnetic field.

Linear stability analysis

The pioneer work of Chandrasekhar [1] permitted to understand the stabilising effect of a magnetic field on thermal convection. He considered two infinite horizontal plates, maintained at a low temperature T_c for the bottom plate and a high temperature T_h for the upper plate, and separated from a distance e. In addition, a magnetic field $B_0 \boldsymbol{e_z}$ is impressed, $\boldsymbol{e_z}$ being the direction of the gravity, as described in Fig. 1a. If the temperature gradient is low enough, steady conduction will take place. As we increase the temperature gradient over a critical value, convection rolls (or more generally, patterns) will appear, as described in Fig. 1b.

The base state is defined by the absence of motion, a constant temperature gradient and a steady, uniform magnetic field. The linearisation around the base state of the momentum, heat and induction equations, gives the following equations [1]:

$$\begin{aligned} \frac{\partial \Delta w}{\partial t} &= \frac{Ra}{Pr}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right)+\Delta^2 w+\frac{Ha^2}{Pm}\frac{\partial \Delta b_z}{\partial z}, \\ \frac{\partial \theta}{\partial t} &= \frac{1}{Pr}\Delta\theta + w, \\ \frac{\partial b_z}{\partial t} &= \frac{1}{Pm}\Delta\theta + \frac{\partial w}{\partial z}. \end{aligned} \qquad (1)$$

In the previous equations, we use a viscous velocity scale $u_0 = \nu/e$ as the reference velocity. In those equations, w, θ, b_z are dimensionless and represent the vertical component of velocity, the temperature perturbation and the vertical component of the magnetic field perturbation. The first equation is the vertical component of the curl of the vorticity equation. The symbol Δ represents the Laplace operator. The dimensionless parameters that appear are the Rayleigh number Ra, the Hartmann number Ha and thermal and magnetic Prandtl numbers, Pr and Pm. They are defined as follow:

$$Ra = \frac{g\beta(T_h - T_c)e^3}{\nu\alpha},\quad Ha = B_0 e\sqrt{\frac{\sigma}{\rho\nu}},\quad Pr = \frac{\nu}{\alpha},\quad Pm = \mu_0\sigma\nu, \qquad (2)$$

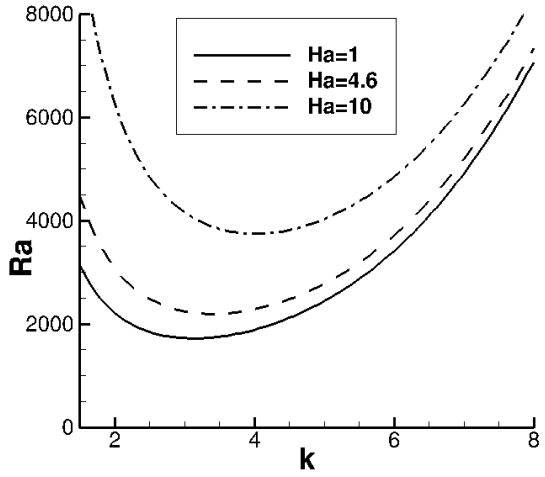


Fig. 2: Marginal stability curves for different Hartmann numbers. The minimum of each curve gives the critical values Ra_{crit} and k_{crit}.

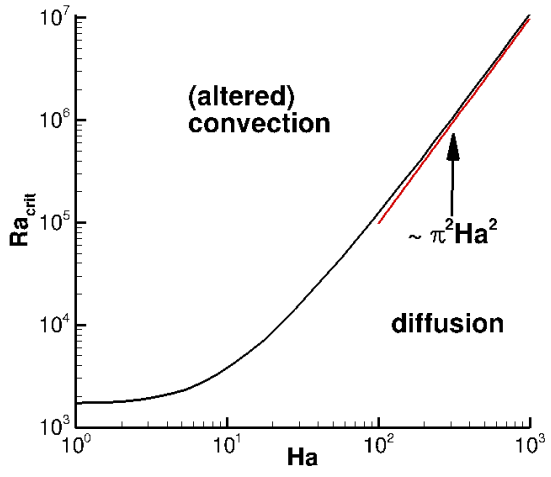


Fig. 3: Critical values of the Rayleigh number Ra_{crit} as a function of the Hartmann number.

with ν and α the fluid kinematic and thermal diffusivity, β is the thermal expansion coefficient, ρ its density, σ its electrical conductivity, and μ_0 is the magnetic permeability of vacuum. We can notice that Pr and Pm only depend on physical properties. Assuming plane wave perturbations $\chi = X(z)\exp(i(k_x x + k_y y) + st)$ (χ standing for w, θ or b_z, and $X(z)$ its amplitude) and focusing on the marginal stability $s = 0$ (i.e. the onset of convection), Chandrasekhar [1] shows that the number of cells (given to a multiplicative constant by the norm of the wavenumber $k = \sqrt{k_x^2 + k_y^2}$) increases with the Hartmann number, as well as the critical value of Ra. The convective motion appears

for $Ra > Ra_{crit}$, and k_{crit} is the wavelength of the unstable mode. Figure 2 shows that both Ra_{crit} and k_{crit} increase with Ha. For Rayleigh numbers below its critical value, the heat transfer will only be conductive, as shown in Fig. 3. But even at overcritical Ra, convection and heat transfer is altered when Ha is high enough. For $Ha \gg 1$, the critical Rayleigh scales as $Ra_{crit} \sim \pi^2 Ha^2$. Therefore our numerical simulations focus on overcritical Rayleigh numbers.

Description of the numerical problem

The Jadim code [9] solves the Navier-Stokes equations, the heat transport equation and the continuity equation, using a Runge-Kutta scheme for temporal integration and a semi-implicit Crank-Nicolson scheme is used for the viscous terms. The pressure is solved using a projection method. For our study, we added the Lorentz force, assuming a steady and uniform magnetic field $\boldsymbol{B} = B_0 \boldsymbol{e_z}$, parallel to the gravity. The Lorentz force yields:

$$\boldsymbol{F_L} = \boldsymbol{j} \times \boldsymbol{B_0} = \sigma(\boldsymbol{u} \times \boldsymbol{B_0}) \times \boldsymbol{B_0}. \quad (3)$$

In the results presented below, the effect of the Joule dissipation is not taken into account. Simulations with the Joule dissipation have shown no significant effect of Joule heating in this situation.

We consider a rectangular cavity, filled with Gallium, whose dimensions are $L \times l \times e = 0.20\,m \times 0.10\,m \times 0.02\,m$. The computational grid is composed of $Nx \times Ny \times Nz = 256 \times 128 \times 64$ regularly spaced points. The temperature is imposed on the lower and upper walls, with a temperature T_c on the upper wall and a temperature T_h on the lower wall, so that $T_h - T_c > 0$. We consider all vertical walls to be insulated. The initial conditions are a uniform temperature, set as T_c. At $t = 0$, we impose the temperature T_h on the lower plate.

Flow structures

At the onset of the instability both with $Ha = 18$ and with $Ha = 0$, and for the same Ra, convection sets in in the form of cells. In a magnetic field, they later reorganise to form unsteady convection rolls. On the other hand without magnetic field, three-dimensional patterns are still observed. If we continue to increase Ra, even with $Ha = 18$, 3D patterns will appear. Figure 4 shows the different patterns that were obtained through simulations.

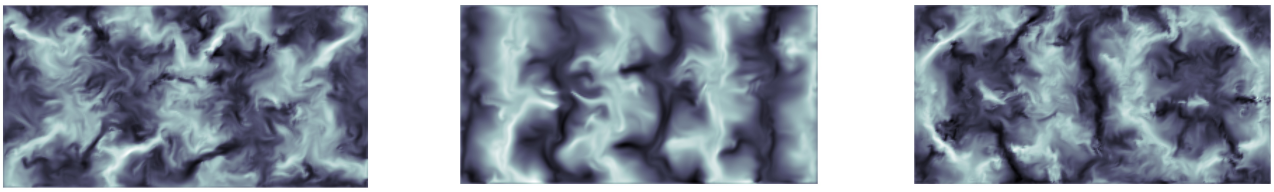


Fig. 4: Instantaneous snapshots of vertical velocity in the mid-plane $z = e/2$, normalised by its maximal value (white is positive and black is negative). The values of Ra and Ha were (from left to right): (i) $Ra = 5 \cdot 10^4$, $Ha = 0$, (ii) $Ra = 5 \cdot 10^4$, $Ha = 18$ (iii) $Ra = 1.5 \cdot 10^5$, $Ha = 18$.

Without or with a purely vertical magnetic field, the onset of convection is in the form of cells, as stated by Chandrasekhar [1]. For Rayleigh numbers near the critical value Ra_c, the cells wills merge into convection rolls. However, for high enough overcritical Ra, the patterns remain three-dimensional. Those results are in agreement with Chandrasekhar's theory and recent experimental [8] and numerical [7] works.

Heat transfer

The heat transfer is of primary importance in our study, since the heat transfer coefficient is proportional to the ablation rate of the concrete in the Vulcano experiments. The local and averaged heat transfer coefficients h and $\bar{h}$ are defined as:

$$h = \frac{k_{th}}{\Delta T} \frac{\partial T}{\partial z}_{walls}, \quad \bar{h} = \frac{k_{th}}{S \Delta T} \iint_S \frac{\partial T}{\partial z}_{walls} dS. \quad (4)$$

We define as well the non-dimensional heat transfer coefficient, called the Nusselt number, as:

$$Nu = \frac{he}{k_{th}}, \quad \overline{Nu} = \frac{\bar{h}e}{k_{th}}. \quad (5)$$

The simulations show that the repartition of the Nusselt number at walls follows the same repartition as the convective patterns, as shown in Fig. 5. On the other hand, the averaged Nusselt number on the walls remains of the same order of magnitude, whether there is a magnetic field or not. This result is of importance for the comprehension of the Vulcano tests. We summed up our results in Fig. 6, that also displays the experimental correlations of Rossby [2], Aurnou &

Olson [3], Globe & Dropkin [6]. The next step of this work is the design of an experimental test-rig.

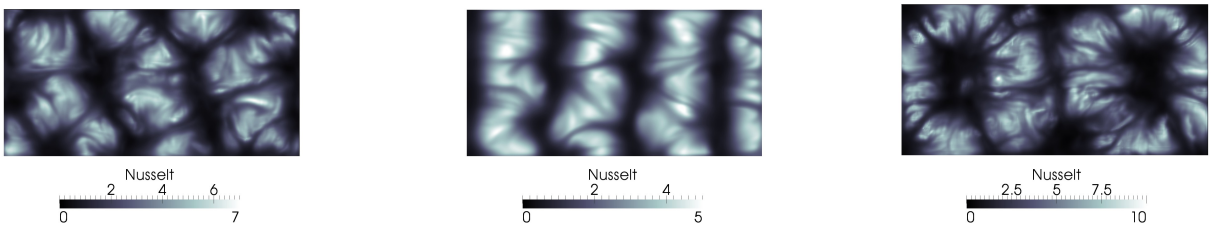


Fig. 5: Instantaneous local Nusselt number at the top walls for (from left to right): Ra and Ha were (from left to right): (i) $Ra = 5 \cdot 10^4$, $Ha = 0$, (ii) $Ra = 5 \cdot 10^4$, $Ha = 18$ (iii) $Ra = 1.5 \cdot 10^5$, $Ha = 18$.

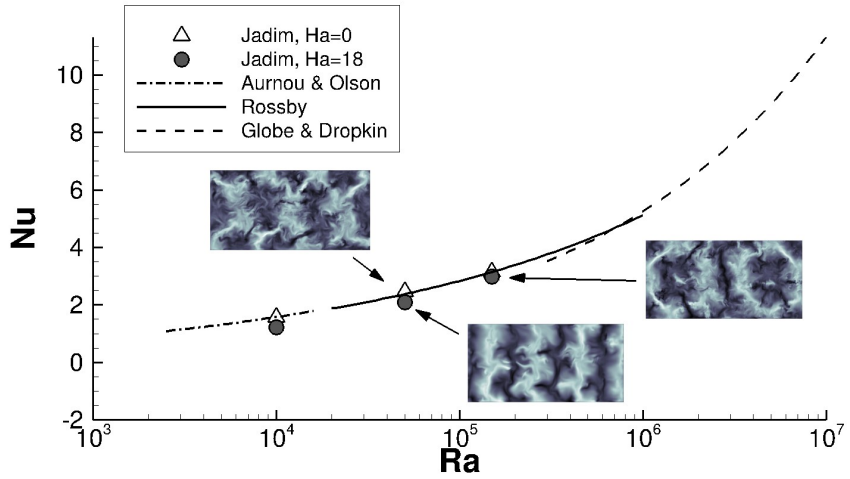


Fig. 6: Nusselt number as a function of the Rayleigh number for $Ha = 0$ and $Ha = 18$.

Conclusion and perspectives

Those results confirm the tendency of a steady magnetic field to stabilise a thermoconvective flow. It significantly modifies the flow structures and reorganise it in rolls. On the other hand, the averaged heat transfer is slightly decreased. In the Vulcano experiments, another interesting question is what happens in an AC magnetic field. Indeed, the Joule dissipation generates local source terms, which is susceptible to modify the onset of convection on one hand, and the flow dynamics and heat transfer on the other hand.

References

[1] Chandrasekhar, S. (1981). *Hydrodynamic and hydromagnetic stability*. Dover Publications Inc., New-York.

[2] Rossby, H. T. (1969). A study of Bénard convection with and without rotation. *Journal of Fluid Mechanics*, *36*(02), 309-335.

[3] Aurnou, J. M., & Olson, P. L. (2001). Experiments on Rayleigh–Bénard convection, magnetoconvection and rotating magnetoconvection in liquid gallium. *Journal of Fluid Mechanics*, *430*, 283-307.

[4] Burr, U., & Müller, U. (2001). Rayleigh–Bénard convection in liquid metal layers under the influence of a vertical magnetic field. *Physics of Fluids (1994-present)*, *13*(11), 3247-3257.

[5] Burr, U., & Müller, U. (2002). Rayleigh–Bénard convection in liquid metal layers under the influence of a horizontal magnetic field. *Journal of Fluid Mechanics*, *453*, 345-369.

[6] Globe, S., & Dropkin, D. (1959). Natural convection heat transfer in liquids confined by two horizontal plates and heated from below. *Journal of Heat Transfer*, *81*(1), 24-28.

[7] Getling, A. V., & Brausch, O. (2003). Cellular flow patterns and their evolutionary scenarios in three-dimensional Rayleigh-Bénard convection. *Physical Review E*, *67*(4), 046313.

[8] Yanagisawa, T., Hamano, Y., Miyagoshi, T., Yamagishi, Y., Tasaka, Y., & Takeda, Y. (2013). Convection patterns in a liquid metal under an imposed horizontal magnetic field. *Physical Review E*, *88*(6), 063020.

[9] Legendre, D. and Magnaudet, J. (1998). The lift force on a spherical bubble in a viscous linear shear flow. *Journal of Fluid Mechanics, 368, 81-126.*