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Effect of a dipolar magnetic field on the stability of spherical Taylor vortex flow

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Abstract
The spherical Couette flow system with imposed magnetic fields offers the possibility of exploring a wide variety of instabilities and has a major importance in the understanding of geophysics and astrophysics phenomena. The flow of an electrically conducting fluid in an annulus between two concentric rotating spheres subjected to a dipolar magnetic field is investigated numerically using a three-dimensional computational fluid dynamics (CFD). The outer sphere is stationary while the inner one rotates freely about a vertical axis through its center. The spherical shell is completely filled with liquid gallium. The numerical studies are performed for the medium gap width $\beta=0.18$, and carried out for a wide range of Hartmann numbers ($Ha$). It is established that the dipolar magnetic field deeply affects the onset of Taylor vortex flow and its structure in which significant topological changes on the flow configuration are observed for different Hartmann number. As $Ha$ increases, it is found that the Taylor vortex flow regime is delayed.

Key words: CFD simulation, Spherical Couette flow, Taylor vortex flow, Dipolar magnetic field

Introduction
The flow patterns in an annulus between two concentric rotating spheres has a major interest in many branches of physics and technology where centrifugal force plays a dominant role. The transitional phenomena encountered in this flow are of fundamental relevance for the understanding of global processes in the planetary atmospheres as well as in astrophysical and geophysical motions. Many theoretical, experimental and numerical studies have been conducted on spherical Couette flow in order to understand the flow behavior and determine a general map of the laminar-turbulent transition, Wimmer [1], Marcus and Tuckerman [2], Hollerbach et al. [3], Nakabayashi et al [4], Bühler [5], Yuan [6].

On the other hand, the presence of a magnetic field imposed on this flow, known as MHD spherical Couette flow, crucially affects the flow structures and the appearance of instabilities. Numerous researchers have extensively studied this problem, both numerically and experimentally under a variety of imposed fields and magnetic boundary conditions. Among these are Hollerbach [7, 8], Hollerbach and Skinner [9], Schmitt et al. [10], Nataf et al. [11], Travnikov et al. [12], Gissinger et al. [13], Kaplan [14].
Numerical studies of Dormy et al. [15] and Hollerbach et al. [16] highlight the nature and structure of the flow that appear when the magnetic field lines are not parallel to the rotation axis, as for a dipolar field. The spherical Couette system has also been proposed for producing a dynamo (Cardin et al. [17], Kelley et al. [18], Sisan et al. [19]).
In the present work, a numerical investigation is conducted in spherical Couette flow with an imposed dipolar magnetic field when the inner sphere rotates freely about a vertical axis through its center, from west to east with a prescribed angular velocity $\Omega_1$, while the outer sphere is kept stationary. The calculations are carried out for a wide range of Hartmann number ($Ha$). This problem has relevant applications in geophysical and astrophysical (stars, planetary interiors), where configurations similar to MHD spherical Couette flow are often encountered.

Numerical modeling
Consider the motion of a viscous incompressible fluid (liquid gallium) of kinematic viscosity $\nu$, density $\rho$, magnetic diffusivity $\eta$ and magnetic permeability $\mu_0$ contained in a spherical annulus between two rotating concentric spheres, as shown in figure 1(b). The geometry is fully determined by the gap width $\beta = (R_2-R_1)/R_1$ which has been kept constant and equal to 0.18 throughout this study, where $R_1$ and $R_2$ are the radii of the inner and outer spheres, respectively. Both outer and inner spheres are considered insulating. There are two dimensionless numbers that completely determine the flow behavior: the Reynolds number and Hartmann number defined as follow:

$$Re = (\Omega_1 R_1^2)/\nu$$  \hspace{1cm} (1)

$$Ha = (B_0 R_1^2)/ (\mu_0 \eta \nu)^{1/2}$$  \hspace{1cm} (2)

Points in the domain are defined by the spherical coordinates $(r, \theta, \varphi)$ with:
The governing equations for this problem are as follows:

\[ \Delta V = 0 \]  
\[ \rho \left( \partial V / \partial t + (V \cdot \nabla) V \right) = -\nabla P + \rho \nabla^2 V + 1/\mu_0 (\nabla \times B) \times B \]  
\[ \partial B / \partial t = \nabla \times (V \times B) + \eta \nabla^2 B \]  
\[ \nabla \cdot B = 0 \]

Where \( P \) and \( V \) are the pressure and velocity components \((u, v, w)\), respectively. Time is denoted by \( t \). \( B \) is the magnetic field, which contains the imposed dipolar magnetic field \( B_d \) defined as:

\[ B_d = B_0 (r/R_2)^3 (2\cos \theta \mathbf{e}_r + \sin \theta \mathbf{e}_\theta) \]  

\( B_0 \) is the intensity of the field at the equator on the outer surface of the fluid \((r = R_2)\).

The boundary conditions used in this study are:

- radial position: \( r \in [R_1, R_2] \) with \( R_1 \leq r \leq R_2 \)
- meridional position: \( \theta \in [0, \pi] \) with \( 0 \leq \theta \leq \pi \)
- circumferential position: \( \phi \in [0, 2\pi] \) with \( 0 \leq \phi \leq 2\pi \)

The converged solution from the coarse mesh was interpolated for use as an initial solution for the medium mesh, and likewise the converged solution of the medium mesh was used as an initial solution for the finer mesh.

**Main Results**

**Flow patterns**

Figure 4 shows the contours of the pressure field on the outer sphere for several Hartmann number. We have firstly examined spherical Couette flow without an imposed magnetic field \((Ha=0)\), in order to validate our numerical calculations as well as verifying the correctness of CFD code. When the Reynolds number reaches a critical value, \( Re_c = 980 \), two steady rolls appear, one on each side of the equator, called one-vortex flow. Our numerical result agrees very well, both qualitatively and quantitatively, with previous work for the same geometry \([2, 4and 5]\). However, when the rotational Reynolds number of the inner sphere is fixed and a magnetic field is applied to this hydrodynamical state, additional magnetohydrodynamic effects are generated. As illustrated in figure below, the inhibiting role of the magnetic field; the rolls at the equator move toward the pole when the Hartmann number is further increased. This flow mode is strongly damped by the applied field.
Figure 3 shows the tangential velocity distribution at a fixed Reynolds number $Re_c=980$, and Hartmann number increasing from 0 up to 4600. The velocity profiles are plotted in dimensionless form in which the tangential velocity is divided by the surface speed, and the radial distance from the wall of the inner sphere is divided by the annular gap (gap width=$\frac{(r-R_1)}{d}$, where $d$ is annular space between the coaxial spheres). The salient features of the numerical profiles are summarized as follow:

- For the case without imposed magnetic field, $Ha=0$ and $Re_c=980$, the profile is monotonically decreasing and it is analogous to the laminar profile in classical Taylor–Couette flow.
- At low Hartmann number, $Ha=6$, the profile is still monotonic but slightly deformed near the outer sphere.
- As the intensity of the magnetic field is further increased, it is observed that the velocity profiles are completely deformed, having negative values in the whole annular gap. Note also that the tangential velocity becomes zero at the outer sphere.

![Flow patterns for different Hartmann number at fixed Reynolds number, Re$_c$=980](image)

**Tangential velocity distribution**

Static pressure distribution

<table>
<thead>
<tr>
<th>$Ha$</th>
<th>Pressure Distribution</th>
</tr>
</thead>
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</tr>
<tr>
<td>60</td>
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</tr>
<tr>
<td>4600</td>
<td><img src="image" alt="Static pressure Ha=4600" /></td>
</tr>
</tbody>
</table>
Conclusion
We have investigated numerically the flow behavior of an electrically conducting fluid in an annular gap between two rotating concentric spheres, when the inner sphere rotates and the outer one is stationary, subject to a dipolar magnetic field. The fluid field was simulated using a three-dimensional CFD, Fluent software. The specific case considered is that with a medium gap width $\beta = 0.18$ and a wide range of Hartmann number, $0 \leq Ha \leq 6000$. In each case, we have increased the Hartmann number for a fixed Reynolds number, and studied the resulting balance and competition between magnetic and inertial effects. We found that increasing the Hartmann number stabilizes the flow and delayed the occurrence of the Taylor vortices at the equator.

References