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DNS of natural convection in liquid metal with strong magnetic fields in rectangular geometry

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Abstract

Direct numerical simulations (DNS) of natural convection in liquid metal within closed boxes heated from below with uniform strong vertical magnetic fields are conducted. The main aim is to explore the possibilities and mechanisms of convection instabilities in such flows, which are referred as MHD convection. It is shown that vertical magnetic field can suppress convection at the center and reside the convection rolls attach to side walls. Stronger magnetic field can even generate new convection rolls which are aligned with the direction of magnetic field.

Key words: convection, magnetohydrodynamics

Introduction

We discuss the interaction between a buoyancy-induced flow and a strong magnetic field. It is known that turbulence can be fully suppressed by strong magnetic fields, leading to a laminar regime. However, this does not mean that laminar steady-state regime is always correct. The results in experiment [1] where the flows through a pipe both with magnetic field and temperature gradient are considered show that temperature fluctuations disappear at moderate magnetic fields, but reappear at stronger magnetic fields in form of high-amplitude low-frequency oscillations indicating that convection-related instabilities may occur. This phenomenon is expected to appear in all systems with strong magnetic fields and buoyancy effects, particularly, in the case of liquid metal blankets of a fusion reactor, where Hartmann and Rayleigh numbers are very high. A detailed investigation of thermal convection in liquid metal flows with strong magnetic fields appears necessary. In this work, we consider the influence of strong magnetic field on natural convection in liquid metal within a rectangular box, which is relevant to the situation with weak or zero imposed flow, for example, in the case of the HCLL blanket, where the liquid metal is used only for breeding. We focus on fundamental analysis rather than on specific applications.

Equations

We consider flows in a closed box with imposed vertical magnetic field and vertical temperature gradient created by maintaining upper and bottom walls at constant and different temperatures, as shown in figure 1. The other four walls are thermally insulated and all walls are perfectly electrically insulating. The fluid is considered as an electrically conducting, incompressible, Newtonian fluid, e.g. a liquid metal. The Boussinesq approximation is applied for the temperature-related buoyancy force. And we assume that the magnetic Reynolds number and the magnetic Prandtl number are both much smaller than one.

![Fig.1: Sketch of geometry and reference system.](image_url)

The non-dimensional governing equations are:
\[ \nabla \cdot \mathbf{u} = 0 \]  
\[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{\sqrt{Gr}} \nabla^2 \mathbf{u} + F_b + F_i \]  
\[ \frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla) \theta = \frac{1}{\sqrt{GrPr}} \nabla^2 \theta \]

Where \( F_b = -\theta \mathbf{e}_b \) presents the buoyancy and \( F_i = \frac{Ha^2}{\sqrt{Gr}} j \times \mathbf{e}_b \) presents Lorentz forces. The electric current is governed by the Ohm’s law \( j = -\nabla \phi + \mathbf{u} \times \mathbf{e}_b \), and the electric potential is a solution of Poisson equation \( \nabla^2 \phi = \nabla \cdot (\mathbf{u} \times \mathbf{e}_b) \). The boundary condition for velocity is no slip boundary condition, that is, the velocity is set zero on all the six walls. For temperature, one pair of the opposite walls (e.g. zz-walls) is set as isothermal walls with constant temperature (e.g. \( \theta = 0.5 \) at \( z = -1 \); \( \theta = -0.5 \) at \( z = 1 \)). All other walls are adiabatic with \( \partial \theta / \partial n = 0 \) (e.g. \( \partial \theta / \partial y = 0 \) at \( y = \pm 1 \); \( \partial \theta / \partial x = 0 \) at \( x = \pm 1 \)). The dimensionless control parameters are Prandtl number, Grashof number or Rayleigh number, Hartmann number, and aspect ratio, which are defined by

\[ Pr = \frac{\nu}{\kappa} ; \quad Gr = \frac{Ra}{Pr} = \frac{g \beta (T_s - T_i) H^3}{\nu^3} ; \quad Ha = B_b H \sqrt{\frac{\sigma}{\rho \nu}} ; \quad \Gamma = L_x : L_y : H . \]  

In our case there is no imposed flow, the velocity is scaled by freefall velocity \( U_f = \sqrt{g \beta (T_s - T_i) H} \) and length is scaled by the height of the geometry \( H \). Time is consequently scaled by the freefall time \( T_f = H / U_f \). Furthermore, the global momentum transfer in the system can be described by Reynolds number \( Re = u_{rms} R e_f \), where \( R e_f = U_f H / \nu = \sqrt{Ra / Pr} \) is freefall Reynolds number and \( u_{rms} = \sqrt{\langle u_i^2 \rangle} \) is the root mean square of the velocity. The global heat transfer in the system can be described by Nusselt number \( Nu_{(c)} = \sqrt{Ra Pr} \langle u_i T \rangle_{A_{(c)}} - \partial \langle T \rangle_{A_{(c)}} / \partial z \).

The problem is solved by direct numerical simulation with FORTRAN code solver based on the projection-type and nearly fully conservative finite difference scheme described in [2]. In the present study, a modified version for non-periodic boundary conditions is used and the detailed discussion of the method can be found in [3]. In addition, we use semi-implicit scheme for solving temperature transport equation, where convective terms are computed explicitly while diffusive terms are computed implicitly. The code is also adapted for massive-parallel computers, including hybrid parallelization with MPI and OpenMP interfaces.

**Results**

Fully 3D DNS of convection in closed boxes are conducted. The effects of parameter range on the flow structures, i.e. variations in aspect ratio and Hartmann number, are analyzed. Values of \( Pr = 0.025 \), typical for liquid metals, \( Ra = 10^6 \), \( \Gamma = 1:1:1, 1:4:1, 4:4:1 \) and \( Ha \) from 100 to 1000 are used in the analysis. And the resolutions are \( 128 \times 128 \times 128 \) for \( \Gamma = 1:1:1, 128 \times 512 \times 128 \) for \( \Gamma = 1:4:1 \) and \( 512 \times 512 \times 128 \) for \( \Gamma = 4:4:1 \).

DNS results of natural convection in closed box without magnetic field are shown in figure 2. Different aspect ratios are considered. We can see that large-scale circulations exit in the boxes. They are typical of Rayleigh-Bénard convection. And with \( Ra = 10^6 \), the convection structures are turbulent. Comparing the results of the different aspect ratios, we find that bounded side walls can suppress the development of convection rolls and have obvious influence on the flow structures. In a long rectangular box, the preferred mode is a finite number of large circulation rolls with axes parallel to the short side, e.g. figure 2(b), where \( \Gamma = 1:4:1 \). While in a square box, the convection structures can develop in both horizontal directions with large-scale circulations oscillating around corners, e.g. figure 2(c), where \( \Gamma = 4:4:1 \).

Uniform magnetic fields in the vertical direction are imposed to investigate the influence of magnetic fields on the system and the magnetic fields are applied at the very beginning. The fully developed convection structures under the influence of magnetic fields with \( Ha = 300 \) are shown in figure 3. We can see that the magnetic fields change the structures of convection and orientation of convection rolls completely. With vertical magnetic field, the original convection rolls tend to reside near the side walls, while the convective motion at the center is suppressed.
Fig. 2: Convection patterns without magnetic field for different aspect ratios. Instantaneous temperature field (upper row) and streamtraces (lower row) are showed. Arrows indicate the direction of gravity acceleration and temperature gradient.

Fig. 3: Convection patterns with magnetic field for different aspect ratios. Instantaneous temperature field (upper row) and streamtraces (lower row) are showed. Direction of magnetic field is additionally indicated.
Influence of different Hartmann numbers is analyzed as well. Here we consider the case with aspect ratio 1:1:1, i.e. the cubic box. The Hartmann number changes in the range between 100 and 1000. DNS results under vertical magnetic fields with different Hartmann number are shown in figure 4. We can see that with stronger vertical magnetic field, the convection rolls are more regular and more likely to attach to the side walls. Furthermore, new convection rolls appear at the corners which are aligned with the direction of the magnetic field. Once the magnetic field is strong enough, convection in the system can be almost fully suppressed, only leaving very weak flow at the corner.

![Convection patterns in cubic boxes under vertical magnetic fields](image)

Fig.4: convection patterns in cubic boxes under vertical magnetic fields with different Ha. Instantaneous temperature field (upper row), 3D streamtraces (middle row) and top view of streamtraces (lower row) are showed.

**Conclusions**

We have conducted direct numerical simulations of natural convection in liquid metal within closed boxes heated uniformly from below with uniform strong vertical magnetic field. The influences of magnetic field and aspect ratio on the convection structures have been analyzed. The magnetic fields can completely change the structures and orientation of convection rolls. The original convection rolls tend to reside near the side walls under vertical magnetic field, while the convective motions at the center tend to be suppressed and new structures aligned magnetic fields appear. Bounded walls can influence the development of convection rolls and flow structures. More detailed and quantitative analysis about the influence of the aspect ratio will be conducted.

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**References**

