Contraction, propagation and bisection on a validated simulation of ODE

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Keywords: contractor programming, ordinary differential equations, guaranteed numerical integration.

Introduction

Many tools used for the validated simulation of initial value problem of ordinary differential equations (ODE) provide abstraction of the solution under the form of a list of boxes obtained by time discretization and local polynomial interpolation. Majority of them are based on Taylor methods (Vnode-LP or CAPD) or on Runge-Kutta methods (DynIbex). If some information on the system state at a given time are given (*e.g.*, from measurements, or guard intersection [1]), it is complex to take this information into account with a low computation cost. We propose a contraction/propagation algorithm to use this information in an elegant manner. This approach will allow one to avoid some costly steps which would appear in running a new simulation (time-step computation and Picard operator).

Main idea

We consider an interval initial value problem of ODE of the form:

$$\dot{y} = f(y,p) \text{ with } y(0) \in [y_0] \text{ and } p \in [p] \ . \tag{1}$$

A validated simulation of (1) is then given in form of two lists of boxes: i) the *a priori* enclosures: $\{[\tilde{y}_0], \ldots, [\tilde{y}_N]\}$, with $y(t) \in [\tilde{y}_i]$ $\forall t \in [t_i, t_{i+1}]$

[1]This research benefited from the support of the "Chair Complex Systems Engineering - Ecole Polytechnique, THALES, DGA, FX, DASSAULT AVIATION, DCNS Research, ENSTA ParisTech, Télécom ParisTech, Fondation ParisTech and FDO ENSTA"

and ii) the tight enclosures $\{[y_0], \ldots, [y_N]\}$, with $y(t_i) \in [y_i]$.
If an information provides $y(t^*) \in [y^*]$, then a contractor is used at $t = t^*$ following the two steps

- add a k^{th} integration step to the time discretization:
  $\{[y_0], \ldots, [y_i], [y_k], [y_{i+1}], \ldots, [y_N]\}$ s.t. $y(t^*) \in [y_k]$ and $t_k = t^*$
- apply the basic contractor $[y_k] := [y_k] \cap [y^*]$

Then a Picard contractor [2] on $[\tilde{y}_i]$ and a validated Runge-Kutta contractor [2] on $[y_i]$ can be apply on each integration step i, in order to propagate (in forward for $t > t^*$ and backward for $t < t^*$) this information on the whole simulation, *i.e.*, on all the boxes in the lists.

Remarks The bisection can be seen as a copy followed by two contractions, then the bisection of a simulation w.r.t. a given time is available with our approach. A propagation of a contraction on one state is a contraction on a simulation [3]. It is easy to generalize to a contraction on parameters. We can also generalize to an interval of time during one we obtain information.

Applications Bisection on initial state to avoid an obstacle; attainability of an objective at a given time; parameter synthesis w.r.t. some constraints; etc.

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