



# Contraction, propagation and bisection on a validated simulation of ODE

Julien Alexandre Dit Sandretto, Alexandre Chapoutot

► **To cite this version:**

Julien Alexandre Dit Sandretto, Alexandre Chapoutot. Contraction, propagation and bisection on a validated simulation of ODE. Summer Workshop on Interval Methods, Jun 2016, Lyon, France. <hal-01325068>

**HAL Id: hal-01325068**

**<https://hal.archives-ouvertes.fr/hal-01325068>**

Submitted on 1 Jun 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Contraction, propagation and bisection on a validated simulation of ODE<sup>1</sup>

Julien Alexandre dit Sandretto<sup>†</sup>, and Alexandre Chapoutot<sup>†</sup>

<sup>†</sup> U2IS, ENSTA ParisTech, Université Paris-Saclay  
828, bd des maréchaux, 91120, Palaiseau, France  
{chapoutot,alexandre}@ensta.fr

**Keywords:** contractor programming, ordinary differential equations, guaranteed numerical integration.

## Introduction

Many tools used for the validated simulation of initial value problem of ordinary differential equations (ODE) provide abstraction of the solution under the form of a list of boxes obtained by time discretization and local polynomial interpolation. Majority of them are based on Taylor methods (Vnode-LP or CAPD) or on Runge-Kutta methods (DynIbex). If some information on the system state at a given time are given (*e.g.*, from measurements, or guard intersection [1]), it is complex to take this information into account with a low computation cost. We propose a contraction/propagation algorithm to use this information in an elegant manner. This approach will allow one to avoid some costly steps which would appear in running a new simulation (time-step computation and Picard operator).

## Main idea

We consider an interval initial value problem of ODE of the form:

$$\dot{y} = f(y, p) \text{ with } y(0) \in [y_0] \text{ and } p \in [p] . \quad (1)$$

A validated simulation of (1) is then given in form of two lists of boxes:  
i) the *a priori* enclosures:  $\{[\tilde{y}_0], \dots, [\tilde{y}_N]\}$ , with  $y(t) \in [\tilde{y}_i] \forall t \in [t_i, t_{i+1}]$

---

<sup>1</sup>This research benefited from the support of the “Chair Complex Systems Engineering - Ecole Polytechnique, THALES, DGA, FX, DASSAULT AVIATION, DCNS Research, ENSTA ParisTech, Télécom ParisTech, Fondation ParisTech and FDO ENSTA”

and ii) the tight enclosures  $\{[y_0], \dots, [y_N]\}$ , with  $y(t_i) \in [y_i]$ .

If an information provides  $y(t^*) \in [y^*]$ , then a contractor is used at  $t = t^*$  following the two steps

- add a  $k^{\text{th}}$  integration step to the time discretization:  
 $\{[y_0], \dots, [y_i], [y_k], [y_{i+1}], \dots, [y_N]\}$  s.t.  $y(t^*) \in [y_k]$  and  $t_k = t^*$
- apply the basic contractor  $[y_k] := [y_k] \cap [y^*]$

Then a Picard contractor [2] on  $[\tilde{y}_i]$  and a validated Runge-Kutta contractor [2] on  $[y_i]$  can be apply on each integration step  $i$ , in order to propagate (in forward for  $t > t^*$  and backward for  $t < t^*$ ) this information on the whole simulation, *i.e.*, on all the boxes in the lists.

**Remarks** The bisection can be seen as a copy followed by two contractions, then the bisection of a simulation w.r.t. a given time is available with our approach. A propagation of a contraction on one state is a contraction on a simulation [3]. It is easy to generalize to a contraction on parameters. We can also generalize to an interval of time during one we obtain information.

**Applications** Bisection on initial state to avoid an obstacle; attainability of an objective at a given time; parameter synthesis w.r.t. some constraints; etc.

## References

- [1] A. EGGERS, N. RAMDANI, N. S. NEDIALKOV, AND M. FRÄNZLE, Improving the SAT modulo ODE approach to hybrid systems analysis by combining different enclosure methods, *Software and System Modeling*, 2015.
- [2] J. ALEXANDRE DIT SANDRETTO, AND A. CHAPOUTOT, Validated Explicit and Implicit Runge-Kutta Methods, *Reliable Computing*, 2016. To appear.
- [3] A. BETHENCOURT, AND L. JAULIN, Solving Non-Linear Constraint Satisfaction Problems Involving Time-Dependant Functions, *Mathematics in Computer Science*, 2014.