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An Advanced Numerical Model in Solving Thin-Wire Integral Equations by Using Semi-Orthogonal Compactly Supported Spline Wavelets

Guido Ala, Maria Luisa Di Silvestre, Elisa Francomano, and Adele Tortorici

Abstract—In this paper, the semi-orthogonal compactly supported spline wavelets are used as basis functions for the efficient solution of the thin-wire electric field integral equation (EFIE) in frequency domain. The method of moments (MoM) is used via the Galerkin procedure. Conventional MoM directly applied to the EFIE, leads to dense matrix which often becomes computationally intractable when large-scale problems are approached. To overcome these difficulties, wavelets can be used as a basis set so obtaining the generation of a sparse matrix; this is due to the local supports and the vanishing moments properties of the wavelets. In the paper, this technique is applied to analyze electromagnetic transients in a lightning protection systems schematized as a thin-wire structure. The study is carried out in frequency domain; a discrete fast Fourier transform algorithm can be used to compute time profiles of the electromagnetic interesting quantities. The unknown longitudinal currents are expressed by using multiscale wavelet expansions. Thus, the thin-wire EFIE is converted into a matrix equation by the Galerkin method. Results for linear spline wavelets along with their comparison with conventional MoM that uses triangular basis functions and the point matching procedure are presented, for two case studies. Good agreement has been reached with a strong reduction of the computational complexity.

I. INTRODUCTION

ANALYSIS of electromagnetic (EM) transients can be carried out by employing a field approach in frequency domain, based on an appropriate integral equation. This approach is a powerful method for the analysis of EM antennas and scatterers. In particular, in the case of a lightning discharge that strikes a lightning protection system (LPS), thin-wire integral equations can be used. In fact, the whole structure can be simulated by means of a set of thin cylindrical conductors in which the lightning current flow. This current distribution can determine significant electromagnetic interference (EMI) with electric and electronic apparatus placed inside the structure to be protected; dangerous conditions for person can also arise. Moreover, parasitic EM coupling phenomena may cause: abnormal signals in telecommunication cables; electric stresses dangerous for the insulation of electric components, disruptive discharges, danger to person, a significant voltage between any two points

of the earth electrode. Dangerous transient step and touch voltages can arise on the ground surface due to transient currents injected into the soil by the earth electrode. The EM characterization of the LPS environment can be carried out through the realistic evaluation of the lightning current flowing in the various parts of the LPS. Then, from this knowledge, it is possible to correctly estimate electric and magnetic fields and their effects at any point inside the protected volume.

In order to study this electromagnetic compatibility (EMC) problem, integral equation method seems to be very appropriate [1]–[10]. For small and medium scale problems, the discretization of the integral equation by the method of moments (MoM), enables to obtain efficient numerical solutions. On the other hand, for large-scale problems the numerical algorithms resolving dense matrix equations have heavy computational cost. In fact, in the conventional MoM the resolving matrix is dense due to the integral operator; thus the solution of such a system of linear equations needs a large memory requirement and a high computational time. To overcome these difficulties, the use of wavelet theory has been recently proposed in technical literature [11]–[14].

Wavelets theory is a relatively new and an emerging area in mathematical research. It has been applied in a wide range of engineering disciplines; particularly, wavelets are very successfully used in signal analysis for waveform representations and segmentations, time-frequency analysis and fast algorithms for easy implementation [15]. Recently, the use of wavelets and wavelets-like basis functions in computational electromagnetics has received considerable attention whether for time-domain differential algorithms [16]–[19], and for the numerical analysis of integral equations [11], [20]. In particular, by employing wavelets, the unknowns in the integral equation are expressed as a twofold summation of shifted and dilated forms of properly chosen basis functions. Moreover, the wavelets have localization properties in both the space and frequency domain and used as basis functions generate a strong de-correlation among the expansion coefficients and weaken the coupling effects in the integral equation. However, it has been underlined that, also with wavelets bases the resultant matrix is dense but, due to local supports and vanishing moment properties of wavelets, many of the matrix entries are very small compared to the largest ones. Consequently, these elements can be set to zero with an opportune threshold technique without significantly affecting the solution.

Fig. 1. Typical scenario of EM transients in an LPS.

Thus, by using the multiresolution analysis (MRA) a sparse matrix equation can be obtained from an integral equation.

Most of the technical papers on wavelets in electromagnetics deal with orthogonal wavelets, which usually do not have closed form expressions. In [13], the thin-wire electric field integral equation (EFIE) is solved by the hybrid wavelet expansion and boundary element method [21], using orthogonal wavelets on the interval $[0,1]$. The same method has been employed in [22] to study the EM scattering from conducting bodies of revolution by means of the magnetic field integral equation.

In this paper, the semi-orthogonal (SO) compactly supported spline wavelets are used as basis functions for the efficient solution of the thin-wire EFIE in frequency domain. Very few references have been found in technical literature dealing with these kind of wavelets [11] and their application to thin-wire integral equation. The MoM is used via Galerkin procedure that is, wavelets are employed as testing and weighting functions. This, in order to investigate EM transients as those observed in grounding systems due to switching operations or a lightning stroke, or in a LPS during its operation. In Fig. 1 a typical scenario of an earthing system integrated with a LPS in an electric power substation, is shown. A discrete fast Fourier transform (DFFT) algorithm can be employed to compute time profiles of the electromagnetic interesting quantities. The unknown longitudinal currents in each conductor of the structure are expressed by using multiscale wavelet expansions. Thus, the thin-wire EFIE is converted into a matrix equation by the Galerkin method.

The use of SO compactly supported spline wavelets is justified by their interesting properties [11]. Among them the following can be explicitly cited: they satisfy all the properties on a bounded interval that are verified by the usual wavelets on the real line, but they do not determine the difficulty related to enforce boundary conditions explicitly, when applying such wavelets to problems in finite bounded domain; unlike most of the continuous orthogonal wavelets, the SO compactly supported spline wavelets have closed form expressions.

Results for linear spline wavelets along with their comparison with conventional MoM that uses piecewise linear basis functions and the point matching procedure are presented, for two case studies. Good agreement has been reached with a strong reduction of the computational complexity, the computer CPU time and the memory requirement.

II. WAVELETS AND MRA

This section is dedicated to briefly present the basic concepts of the wavelet theory [23]–[25]. A natural framework for wavelet theory is the MRA that characterizes wavelets in a general way. The goal of MRA is to express a function $f(x) \in L^2(R)$ at various levels of details, $L^2(R)$ as the space of square integrable function on R .

A multiresolution approximation of $L^2(R)$ is characterized by a set of nested subspaces V_j in $L^2(R)$

$$V_j \subset V_{j+1} \quad \forall j \in Z \quad (1)$$

which satisfy the following properties:

$$\bigcup_{j \in Z} V_j = L^2(R) \quad \bigcap_{j \in Z} V_j = \{0\} \quad (2)$$

$$f(x) \in V_j \leftrightarrow f(2x) \in V_{j+1} \quad \forall j \in Z \quad (3)$$

where Z is the set of integers. For each j , exists a function referred to as scaling function $\varphi(x)$ such that

$$\{\varphi_{jk}(x)\}_{k \in Z} = \{2^{j/2} \varphi(2^j x - k)\}_{k \in Z} \quad (4)$$

is a basis for V_j .

An approximation of a function $f(x) \in L^2(R)$ at a resolution of 2^{-j} , can be defined as the orthogonal projection of $f(x)$ into V_j , i.e.,

$$\begin{aligned} (P_j f)(x) &= \sum_{k \in Z} \langle f(x), \varphi_{jk}(x) \rangle \varphi_{jk}(x) \\ &= \sum_{k \in Z} c_k^{(j)} \varphi_{jk}(x) \in V_j \end{aligned} \quad (5)$$

where $\langle f(x), \varphi_{jk}(x) \rangle$ represents the inner product of $f(x)$ and $\varphi_{jk}(x)$.

Since $\{\varphi_{jk}(x)\}_{k \in Z}$ is a basis of V_j , but not of $L^2(R)$, some information is being lost in the projection reported in (5). This lost information can be interpreted as the details in $f(x)$ that are finer in resolution than 2^{-j} . It is therefore necessary to describe a function in term of its approximation at the resolution 2^{-j} and the remaining orthogonal terms containing local measures of the finer details. This is formally facilitated by the fact that exists an orthogonal complementary subspace W_j of V_j in V_{j+1} :

$$V_{j+1} = V_j \oplus W_j \quad (6)$$

as well as a function $\psi(x)$, referred to as wavelet, such that the set

$$\{\psi_{jk}(x)\}_{k \in Z} = \{2^{j/2} \psi(2^j x - k)\}_{k \in Z} \quad (7)$$

is a basis for W_j , whilst $\{\psi_{jk}(x)\}_{j, k \in Z}$ is a basis for $L^2(R)$.

Therefore, by considering

$$\begin{aligned} (Q_j f)(x) &= \sum_{k \in Z} \langle f(x), \psi_{jk}(x) \rangle \psi_{jk}(x) \\ &= \sum_{k \in Z} d_k^{(j)} \psi_{jk}(x) \in W_j \end{aligned} \quad (8)$$

as the orthogonal projection of a function $f(x) \in L^2(R)$ into W_j , from the equation (6) it follows that each function $f_{j+1}(x) \in V_{j+1}$, can be written as follows:

$$f_{j+1}(x) = \sum_{k \in \mathbb{Z}} c_k^{(j)} \varphi_{jk}(x) + \sum_{k \in \mathbb{Z}} d_k^{(j)} \psi_{jk}(x). \quad (9)$$

By iterating this decomposition on an arbitrary number of levels, a description of $f(x)$ in terms of its approximation at the resolution $2^{-s} < 2^{-j}$ and its finer details can be written as

$$(P_s f)(x) = (P_j f)(x) + \sum_{i=j}^{s-1} (Q_i f)(x). \quad (10)$$

The scaling function $\varphi(x)$ and the wavelet function $\psi(x)$ satisfy the fundamental *two-scale relations* [23]

$$\varphi(x) = \sum_{k \in \mathbb{Z}} p_k \varphi(2x - k); \quad \psi(x) = \sum_{k \in \mathbb{Z}} q_k \varphi(2x - k). \quad (11)$$

The sequences $\{p_k\}$, $\{q_k\}$, are filters for the scaling function $\varphi(x)$ and for the wavelet function $\psi(x)$, respectively. In particular, for a good approximation and a data compression the following condition have to be satisfied:

$$\langle x^j, \psi(x) \rangle = 0, \quad j = 0, \dots, t-1 \quad (12)$$

where “ t ” is the number of *vanishing moments*.

Let $B_m(x)$ be the m th-order cardinal B-spline function [15, p. 52]

$$B_m(x) = \int_0^1 B_{m-1}(x - \tau) d\tau, \quad m \geq 2$$

with

$$B_1(x) = \begin{cases} 1, & x \in [0, 1) \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

and $\text{supp}[B_m(x)] = [0, m]$.

It is well known that these compactly supported functions generate an MRA [15] with two scale relations

$$B_m(x) = \sum_{k=0}^m 2^{-m+1} \binom{m}{k} B_m(2x - k). \quad (14)$$

Then, the corresponding m th-order cardinal SO compactly supported B-wavelet function $\psi_m(x)$ is given by [15, p. 102]

$$\psi_m(x) = \sum_{k=0}^{3m-2} q_k B_m(2x - k),$$

with

$$q_k = \frac{(-1)^k}{2^{m-1}} \sum_{l=0}^m \binom{m}{l} B_{2m}(k - l + 1). \quad (15)$$

III. WAVELET AND SCALING FUNCTIONS ON THE INTERVAL

Wavelet and scaling functions can be used to expand the unknowns of the integral equation, selected for modeling the electromagnetic problem. This integral equation can be expressed in a general way as

$$\int_{\Omega} I(y) K(x, y) dy = b(x) \quad (16)$$

where $I(y)$ is the unknown function, $K(x, y)$ is the known kernel, $b(x)$ is the known function, and Ω is the domain of integration.

Wavelet and scaling functions are defined on the entire real line so that they could be outside of the integration domain. This behavior may require an explicit enforcement of the boundary conditions. In order to avoid this occurrence, SO compactly supported spline wavelets, constructed for the bounded interval $[0, 1]$, have been taken into account in this paper. These wavelets satisfy all the properties verified by the usual wavelets on the real line [15].

Let m and n be positive integers and

$$\begin{aligned} c = x_{-m+1} = \dots = x_0 < x_1 < \dots < x_n \\ = \dots = x_{n+m-1} = d \end{aligned} \quad (17)$$

an equally-spaced knots sequence. The functions

$$\begin{aligned} B_{m, \nu; \mathbf{x}}(x) &= \frac{x - x_{\nu}}{x_{\nu+m-1} - x_{\nu}} B_{m-1, \nu; \mathbf{x}}(x) \\ &+ \frac{x_{\nu+m} - x}{x_{\nu+m} - x_{\nu+1}} B_{m-1, \nu+1; \mathbf{x}}(x), \\ \nu &= -m+1, \dots, n-1 \end{aligned} \quad (18)$$

and

$$\begin{aligned} B_{1, \nu; \mathbf{x}}(x) &= \begin{cases} 1, & x \in [x_{\nu}, x_{\nu+1}) \\ 0, & \text{otherwise} \end{cases} \\ \nu &= 0, \dots, n-1 \end{aligned} \quad (19)$$

are called cardinal B-spline functions of order $m \geq 2$ for the knots sequence $\mathbf{x} = \{x_i\}_{i=-m+1}^{n+m-1}$, and $\text{supp}[B_{m, \nu; \mathbf{x}}(x)] = [x_{\nu}, x_{\nu+m}] \cap [c, d]$. For the sake of simplicity, the interval $[c, d] \equiv [0, n]$ and $x_k = k$, $k = 0, \dots, n$. The $B_{m, \nu; \mathbf{x}}(x) \equiv B_m(x - \nu)$, $\nu = 0, \dots, n - m$, are interior B-spline functions, while the remaining $B_{m, \nu; \mathbf{x}}(x)$, $\nu = -m+1, \dots, -1$ and $\nu = n - m + 1, \dots, n - 1$ are boundary B-spline functions, for the bounded interval $[0, n]$. Since the boundary B-spline functions at 0 are symmetric reflections of those at n , it is sufficient to construct only the first half functions by simply replacing x with $n - x$.

By considering the interval $[c, d] \equiv [0, 1]$, at any level $j \in \mathbb{Z}^+$, the discretization step is 2^{-j} , and this generates $n = 2^j$ number of segments in $[0, 1]$ with knots sequence

$$\mathbf{x}^{(j)} = \begin{cases} x_{-m+1}^{(j)} = \dots = x_0^{(j)} = 0, \\ x_k^{(j)} = k/2^j, & k = 1, \dots, n-1. \\ x_n^{(j)} = \dots = x_{n+m-1}^{(j)} = 1, \end{cases} \quad (20)$$

Let j_0 be the level for which $2^{j_0} \geq 2m - 1$; for each level $j \geq j_0$ the scaling functions of order “ m ” can be defined as follows [11], [15]:

$$\varphi_{m,\nu}^{(j)}(x) = \begin{cases} B_{m,\nu;\mathbf{x}^{(j)}}(2^{j-j_0}x), & \nu = -m+1, \dots, -1 \\ B_{m,2^j-m-\nu;\mathbf{x}^{(j)}}(1-2^{j-j_0}x), & \nu = 2^j-m+1, \dots, 2^j-1 \\ B_{m,0;\mathbf{x}^{(j)}}(2^{j-j_0}x-2^{-j_0}\nu), & \nu = 0, \dots, 2^j-m. \end{cases} \quad (21)$$

Let $S_{\mathbf{x}m}$ be the space of spline functions of order m with knots sequence $\mathbf{x}$ on $[c, d]$. The $S_{\mathbf{x}m}$ is a proper subspace of the space $S_{\mathbf{t}m}$ on the finer knots sequence $\mathbf{t}$ obtained by inserting a knot t_{2i+1} between x_i and x_{i+1} , $i = 0, \dots, n-1$, and $t_{2i} = x_i$, $i = 0, \dots, n$. The $\{B_{m,\nu-m;\mathbf{x}}\}_{\nu=1}^{n+m-1}$ and $\{B_{m,\nu-m;\mathbf{t}}\}_{\nu=1}^{2n+m-1}$ are bases for the $S_{\mathbf{x}m}$ and $S_{\mathbf{t}m}$ spaces, respectively. So, the orthogonal complementary subspace of $S_{\mathbf{t}m}$ relative to $S_{\mathbf{x}m}$ has order n and a basis is formed by the functions $\{\psi_{m,\nu-m;\mathbf{x},\mathbf{t}}\}_{\nu=1}^n$, i.e., the B-wavelets with knots sequence $\mathbf{t}$ relative to knots sequence $\mathbf{x}$. The relation (15) suggests the following formulations for the m th-order SO compactly supported B-wavelet functions [15, p. 149]

$$\psi_{m,\nu-m;\mathbf{x},\mathbf{t}}(x) = \sum_{k=\nu}^{2\nu+2m-2} q_{\nu,k} B_{m,k-m;\mathbf{t}}(x) \quad \nu = 1, \dots, m-1 \quad (22)$$

$$\psi_{m,\nu-m;\mathbf{x},\mathbf{t}}(x) = \sum_{k=2\nu-m}^{2\nu+2m-2} q_{\nu,k} B_{m,k-m;\mathbf{t}}(x) \quad \nu = m, \dots, n-m+1 \quad (23)$$

$$\psi_{m,\nu-m;\mathbf{x},\mathbf{t}}(x) = \sum_{k=2\nu-m}^{n+\nu+m-1} q_{\nu,k} B_{m,k-m;\mathbf{t}}(x) \quad \nu = n-m+2, \dots, n \quad (24)$$

where $q_{\nu,k} = q_{k-2\nu}$.

Hence, there are $2(m-1)$ boundary wavelets and $(n-2m+2)$ inner wavelets in the boundary interval $[c, d]$.

Finally, by considering the level j with $j \geq j_0$, the B-wavelet functions in $[0, 1]$ can be expressed as follows [15, p. 184]:

$$\psi_{m,\nu}^{(j)}(x) = \begin{cases} \psi_{m,\nu;\mathbf{x}^{(j-1)},\mathbf{x}^{(j)}}(2^{j-j_0}x) & \nu = -m+1, \dots, -1 \\ \psi_{m,2^j-2m+1-\nu;\mathbf{x}^{(j-1)},\mathbf{x}^{(j)}}(1-2^{j-j_0}x) & \nu = 2^j-2m+2, \dots, 2^j-m \\ \psi_{m,0;\mathbf{x}^{(j-1)},\mathbf{x}^{(j)}}(2^{j-j_0}x-2^{-j_0}\nu) & \nu = 0, \dots, 2^j-2m+1. \end{cases} \quad (25)$$

The scaling functions $\varphi_{m,\nu}^{(j)}(x)$ occupy m segments and the wavelet functions $\psi_{m,\nu}^{(j)}(x)$ occupy $(2m-1)$ segments. There-

fore the condition $2^j \geq 2m-1$, must be satisfied in order to have at least one inner wavelet.

In the following, the scaling and wavelet functions used in the paper, for $j_0 = j = 2$ and $m = 2$, are reported:

Boundary scaling

$$\varphi_{2,-1}^{(2)}(x) = 1 - 4x, \quad x \in [0, 1/4] \quad (26)$$

$$\varphi_{2,3}^{(2)}(x) = 4x - 3, \quad x \in [3/4, 1] \quad (27)$$

Inner scaling

$$\varphi_{2,0}^{(2)}(x) = \begin{cases} 4x & x \in [0, 1/4] \\ 2 - 4x, & x \in [1/4, 1/2] \end{cases} \quad (28)$$

$$\varphi_{2,1}^{(2)}(x) = \begin{cases} 4x - 1, & x \in [1/4, 1/2] \\ 3 - 4x, & x \in [1/2, 3/4] \end{cases} \quad (29)$$

$$\varphi_{2,2}^{(2)}(x) = \begin{cases} 4x - 2, & x \in [1/2, 3/4] \\ 4 - 4x, & x \in [3/4, 1]. \end{cases} \quad (30)$$

Boundary wavelets

$$\psi_{2,-1}^{(2)}(x) = \begin{cases} -1 + \frac{46}{3}x, & x \in [0, 1/8] \\ \frac{7}{3} - \frac{34}{3}x, & x \in [1/8, 1/4] \\ -\frac{5}{3} + \frac{14}{3}x, & x \in [1/4, 3/8] \\ \frac{1}{3} - \frac{2}{3}x, & x \in [3/8, 1/2] \end{cases} \quad (31)$$

$$\psi_{2,2}^{(2)}(x) = \begin{cases} -\frac{1}{3} + \frac{2}{3}x, & x \in [1/2, 5/8] \\ 3 - \frac{14}{3}x, & x \in [5/8, 3/4] \\ -9 + \frac{34}{3}x, & x \in [3/4, 7/8] \\ \frac{43}{3} - \frac{46}{3}x, & x \in [7/8, 1]. \end{cases} \quad (32)$$

Inner wavelets

$$\psi_{2,0}^{(2)}(x) = \begin{cases} \frac{2}{3}x, & x \in [0, 1/8] \\ \frac{2}{3} - \frac{14}{3}x, & x \in [1/8, 1/4] \\ -\frac{19}{6} + \frac{32}{3}x, & x \in [1/4, 3/8] \\ \frac{29}{6} - \frac{32}{3}x, & x \in [3/8, 1/2] \\ -\frac{17}{6} + \frac{14}{3}x, & x \in [1/2, 5/8] \\ \frac{1}{2} - \frac{2}{3}x, & x \in [5/8, 3/4] \end{cases} \quad (33)$$

$$\psi_{2,1}^{(2)}(x) = \begin{cases} -\frac{1}{12} + \frac{2}{3}x, & x \in [1/8, 1/4] \\ \frac{5}{4} - \frac{14}{3}x, & x \in [1/4, 3/8] \\ -\frac{9}{2} + \frac{32}{3}x, & x \in [3/8, 1/2] \\ \frac{37}{6} - \frac{32}{3}x, & x \in [1/2, 5/8] \\ -\frac{41}{12} + \frac{14}{3}x, & x \in [5/8, 3/4] \\ \frac{7}{12} - \frac{2}{3}x, & x \in [3/4, 7/8]. \end{cases} \quad (34)$$

Thus, the unknown function in (16) can be expanded in terms of the selected scaling and wavelet functions as follows:

$$I(y) = \sum_{\nu=-m+1}^{2^{j_0}-1} c_{\nu}^{(j_0)} \varphi_{m,\nu}^{(j_0)}(y) + \sum_{j=j_0}^J \sum_{\nu=-m+1}^{2^j-m} d_{\nu}^{(j)} \psi_{m,\nu}^{(j)}(y). \quad (35)$$

By substituting this expression in an integral equation of the type (16) but with the domain of integration $\Omega \equiv [0, 1]$

$$\int_0^1 I(y)K(x, y) dy = b(x) \quad (36)$$

and by employing the Galerkin method, the following set of linear system of order $(2^{j_z+1} + m - 1)$ is generated:

$$\begin{bmatrix} LL & LH \\ HL & HH \end{bmatrix} \begin{bmatrix} c_p^{(j_0)} \\ d_\eta^{(j)} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad (37)$$

where

$$LL = \left[\int_0^1 \varphi_{m,\nu'}^{(j_0)}(x) \int_0^1 \varphi_{m,\nu}^{(j_0)}(y) K(x, y) dy dx \right]_{\nu'\nu} \quad (38)$$

$$LH = \left[\int_0^1 \varphi_{m,\nu'}^{(j_0)}(x) \int_0^1 \psi_{m,\eta}^{(j)}(y) K(x, y) dy dx \right]_{\nu'\eta j} \quad (39)$$

$$HL = \left[\int_0^1 \psi_{m,\eta'}^{(j')}(x) \int_0^1 \varphi_{m,\nu}^{(j_0)}(y) K(x, y) dy dx \right]_{\eta'j'\nu} \quad (40)$$

$$HH = \left[\int_0^1 \psi_{m,\eta'}^{(j')}(x) \int_0^1 \psi_{m,\eta}^{(j)}(y) K(x, y) dy dx \right]_{\eta'j'\eta j} \quad (41)$$

where $\nu, \nu' = 1 - m, \dots, 2^{j_0} - 1$; $j, j' = j_0, \dots, j_z$; $\eta = 1 - m, \dots, 2^j - m$; and $\eta' = 1 - m, \dots, 2^{j'} - m$.

In the integrals (38)–(41) much smaller limits than $[0, 1]$ are involved in the computations, because of the finite support of the wavelet and scaling functions. The integrals involving wavelet functions are quite small, due to the SO property of the wavelets; therefore, the entries with relevant magnitude are in the LL sub-matrix. In this way, a significant number of entries in the MoM wavelet matrix in (37) can be set to zero using an opportune threshold technique.

It is interesting to underline that in the conventional MoM all the entries of the matrices are comparable in magnitude. As a consequence, no threshold can be used to sparsify the system matrix.

IV. NUMERICAL SCHEME OF EMC PROBLEM

The “thin-wire” approximation [2] is used in order to schematize a set of metallic straight cylindrical conductors embedded into a homogeneous medium with assigned specific electric parameters. In order to obtain an integral equation in the unknown longitudinal currents distribution, the start point is the well-known relation that expresses the total electric field as the sum of the incident $\mathbf{E}^{\text{inc}}$ and scattered $\mathbf{E}^{\text{sc}}$ components [2], in the frequency domain. The incident field can take into account, for example, of the influence of the lightning channel. The following statement in a point on the surface of the generic excited wire can be used as a boundary condition:

$$(\mathbf{E}^{\text{sc}} + \mathbf{E}^{\text{inc}})_{tg} = \dot{z}_s I_a \quad \text{on } S. \quad (42)$$

The left-hand side of (42) represents the tangential component of the total electric field, parallel to the axis of the excited wire; $\dot{z}_s$ is the per-unit-length surface impedance of a cylindrical

Fig. 2. Geometric references for exciting and excited wires.

straight wire; I_a is the longitudinal unknown current on the axis of the excited wire. The scattered field can be expressed by means of the retarded scalar Φ and vector $\mathbf{A}$ potentials

$$\mathbf{E}^{\text{sc}} = -j\omega\mathbf{A} - \nabla\Phi. \quad (43)$$

The longitudinal currents and the surface charges of each wire of the structure, are the source of $\mathbf{A}$ and Φ , respectively; for a wire, the expression of $\nabla\Phi$ and $\mathbf{A}$, are the following:

$$\begin{aligned} \mathbf{A} &= \bar{u}' \mu \int_{\text{wire length}} I_a(l') g(\mathbf{r}, \mathbf{r}') dl' \\ \nabla\Phi &= -\nabla \left\{ \int_{\text{wire length}} \frac{dI_a(l')}{dl'} \frac{g(\mathbf{r}, \mathbf{r}')}{j\omega\epsilon} dl' \right\} \end{aligned} \quad (44)$$

where, in a general rectangular coordinate system (O, X, Y, Z) (Fig. 2), $\mathbf{r}$ and $\mathbf{r}'$ are the position vectors of the observation and of the source points, respectively; l' is along the axis of the exciting wire; μ is the magnetic permeability, σ is the conductivity, $\epsilon = \epsilon_r \epsilon_0 + \sigma/(j\omega)$ is the complex permittivity of the medium; $\bar{u}'$ is a unity vector along the axis of the exciting conductor; $I_a(l')$ is the longitudinal current on the axis of the exciting wire; $g(\mathbf{r}, \mathbf{r}')$ is the scalar *Green's function* for an unbounded region $k = \sqrt{-\omega^2 \mu \epsilon}$ as the wave number

$$g(\mathbf{r}, \mathbf{r}') = \frac{e^{-k|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}. \quad (45)$$

In the second equation of (44), the continuity equation with thin-wire approximation has been taken into account; this equation expresses the divergence of the current density vector as a function of the first time derivative ($j\omega$, in time-harmonic variation) of the charge volume density. In the local cylindrical coordinate system ($\bar{u}', \bar{a}', \bar{\theta}'$) of the exciting wire (Fig. 2), $\nabla\Phi$ can be expressed as the vector sum of the axial and radial components (in this case the azimuthal component does not exist)

$$\nabla\Phi = \bar{u}' \nabla\Phi_{x_p} + \bar{a}' \nabla\Phi_{a_p} = \bar{u}' \frac{\partial\Phi}{\partial x_p} + \bar{a}' \frac{\partial\Phi}{\partial a_p}. \quad (46)$$

By introducing (43), (44), and (46) in (42), the following EFIE is obtained [11]–[13]:

$$\begin{aligned} E_{tg}^{\text{inc}}(\mathbf{r}) - j\omega\mu\bar{u} \cdot \left[\bar{u}' \int_{\Omega} I_a(l') g(\mathbf{r}, \mathbf{r}') dl' \right] \\ + \frac{1}{j\omega\epsilon} \bar{u} \cdot \left[\bar{u}' \frac{\partial}{\partial x_p} \int_{\Omega} \frac{dI_a(l')}{dl'} g(\mathbf{r}, \mathbf{r}') dl' \right], \quad \text{on } S \end{aligned}$$

Fig. 3. Mapping process scheme among the wires domains and $\zeta \in [0, 1]$ for $N_b = 12$.

$$+ \frac{1}{j\omega\epsilon} \bar{u} \cdot \left[\bar{u}' \frac{\partial}{\partial a_p} \int_{\Omega} \frac{dI_a(l')}{dl'} g(\mathbf{r}, \mathbf{r}') dl' \right] = \dot{z}_s I_a(\mathbf{r}) \quad (47)$$

where Ω schematizes the domain of integration (i.e., the LPS arrangement and the related complex images [10]); $\bar{u}$, $\bar{u}'$, $\bar{a}'$ are unit vectors associated with the oriented wires (Fig. 2) and “ $\cdot$ ” is the inner product; $I_a(\mathbf{r})$ is the longitudinal current associated to the excited wire; $I_a(l')$ is the longitudinal current on the axis of the generic wire.

The two elementary components of $\nabla\Phi$ in a point on the surface of the excited wire due to an elemental “ dx ” of the exciting wire, can easily be obtained by using the partial derivative with respect to the distance $R = |\mathbf{r} - \mathbf{r}'|$ and referring to the local coordinates x_p and a_p (Fig. 2), as follows:

$$\begin{aligned} |d(\nabla\Phi_{x_p})| &= \frac{\partial\Phi}{\partial R} \frac{\partial R}{\partial x_p} \\ &= \frac{(x_p - x)}{j\omega 4\pi\epsilon} \frac{dI_a(x)}{dx} (1 + \dot{k}R) \frac{e^{-\dot{k}R}}{R^3} dx \\ |d(\nabla\Phi_{a_p})| &= \frac{\partial\Phi}{\partial R} \frac{\partial R}{\partial a_p} \\ &= \frac{a_p}{j\omega 4\pi\epsilon} \frac{dI_a(x)}{dx} (1 + \dot{k}R) \frac{e^{-\dot{k}R}}{R^3} dx \quad (48) \end{aligned}$$

with “ x ” as the local axial coordinate of the source point on the axis of the exciting wire (Fig. 2).

By substituting (45), and (48) in (47), the following integral equation on the surface of the generic wire is obtained:

$$\begin{aligned} E_{tg}^{inc}(\mathbf{r}) - j\omega \frac{\mu}{4\pi} \bar{u} \cdot \left[\bar{u}' \int_{\Omega} I_a(x) \frac{e^{-\dot{k}R}}{R} dx \right] \\ + \frac{1}{j\omega 4\pi\epsilon} \bar{u} \cdot \left[\bar{u}' \int_{\Omega} (x_p - x) \frac{dI_a(x)}{dx} (1 + \dot{k}R) \frac{e^{-\dot{k}R}}{R^3} dx \right] \\ + \frac{1}{j\omega 4\pi\epsilon} \bar{u} \cdot \left[\bar{a}' \int_{\Omega} a_p \frac{dI_a(x)}{dx} (1 + \dot{k}R) \frac{e^{-\dot{k}R}}{R^3} dx \right] \\ = \dot{z}_s I_a(\mathbf{r}). \quad (49) \end{aligned}$$

In solving (49), the unknown longitudinal current $I_a(x)$ on each conductor of the thin-wire arrangement, has to be expressed by means of a convenient set of basis functions. In order to use the SO spline wavelets on the interval $[0, 1]$ as basis functions, the following domain transformations have to be made. Let us introduce a linear mapping between the local axial coordinate x_i of the i th wire ($i = 1, \dots, N_b$) and the ξ_i domain, with $\xi_i \in [0, 1]$ so reporting each conductor length to the unit value. Moreover, let us introduce another mapping relation among the ξ_i domains and the ζ domain, with $\zeta \in [0, 1]$ [13]. The following relations hold:

$$\begin{aligned} \xi_i &= \frac{x_i}{L_i} \\ \zeta &= \zeta_{i-1} + (\zeta_i - \zeta_{i-1})\xi_i \\ x_i(\zeta) &= L_i \left(\frac{\zeta - \zeta_{i-1}}{\zeta_i - \zeta_{i-1}} \right) \quad (50) \end{aligned}$$

with $i = 1, \dots, N_b$, where N_b is the number of wires, L_i is the length of the i th conductor, ζ_i are equally spaced knots (Fig. 3) in the resulting ζ domain. The mapping process scheme is reported in Fig. 3 for $N_b = 12$.

The geometric quantities $\mathbf{r}$, R , x_p , a_p can be also expressed as a function of the variables ζ and $\tilde{\zeta}$; these last are related to the positions of the source point on the exciting wire and to the field point on the excited wire, respectively. In such a way in (49), the transformed domain $\zeta \in [0, 1]$ is completely introduced. The domain of integration Ω is also reported to the transformed domain ζ . In fact, Ω schematizes the contribution of all the thin conductors to the field point $\tilde{\zeta}$ of the selected excited wire. Thus, in conformity with the position of the source point on the axis “ x ” of the exciting wire, the corresponding value in the ζ domain belongs to the related subinterval of $[0, 1]$ (so as in Fig. 3).

Moreover, in conformity with the position of the field point on the surface of the excited conductor, the values of the geometric quantities $\mathbf{r}$, R , x_p , a_p will change in accordance as a function of $\tilde{\zeta}$, also. The information related to the real mutual geometric position of the oriented wires of the structure is taken into account by means of the inner products $\bar{u} \cdot \bar{u}'$ and $\bar{u} \cdot \bar{a}'$, which are computed successively between the exciting and excited conductors, depending on the values of ζ and $\tilde{\zeta}$ in the integrals of

(49). The following further assumptions are made. For sake of simplicity, the incident field $\mathbf{E}^{\text{inc}}$ is supposed to be zero. On the other hand, the term $\mathbf{E}^{\text{inc}}$ in (49) enables us to easily take into account both aerial field and ground current in the case of both direct and indirect stroke on a LPS, as reported in [28], [29]. Thus, in this paper, a lightning current that directly strikes the LPS structure is considered; this current can be schematized by an ideal current source generator. The Sommerfeld half-space problem is approximated by using modified image theory with mirror complex image sources [10], [30]; detailed description of such employed schematization is reported in [10] to which the reader is invited to refer. Based on the previous considerations, (49) is so modified

$$\begin{aligned}
& -\bar{u} \cdot \sum_{i=1}^{N_b} \left[\bar{u}' \frac{j\omega\mu L_i}{(\zeta_i - \zeta_{i-1})} \int_{\zeta_{i-1}}^{\zeta_i} I_a(\zeta) \frac{e^{-kR(\zeta, \tilde{\zeta})}}{4\pi R(\zeta, \tilde{\zeta})} d\zeta \right] \\
& + \bar{u} \cdot \sum_{i=1}^{N_b} \left\{ \bar{u}' \int_{\zeta_{i-1}}^{\zeta_i} [x_p(\zeta, \tilde{\zeta}) - x_i(\zeta)] \frac{dI_a(\zeta)}{d\zeta} \right. \\
& \quad \times \frac{[1 + kR(\zeta, \tilde{\zeta})]e^{-kR(\zeta, \tilde{\zeta})}}{j\omega 4\pi \epsilon R^3(\zeta, \tilde{\zeta})} d\zeta \Big\} \\
& + \bar{u} \cdot \sum_{i=1}^{N_b} \left\{ \bar{u}' \int_{\zeta_{i-1}}^{\zeta_i} \frac{dI_a(\zeta)}{d\zeta} \right. \\
& \quad \times \frac{[1 + kR(\zeta, \tilde{\zeta})]a_p(\zeta, \tilde{\zeta})e^{-kR(\zeta, \tilde{\zeta})}}{j\omega 4\pi \epsilon R^3(\zeta, \tilde{\zeta})} d\zeta \Big\} \\
& = \dot{z}_s I_a(\tilde{\zeta}). \tag{51}
\end{aligned}$$

Each of the three summations of the integrals in the left side of (51) extend the domain of integration over the entire bounded interval $[0, 1]$. Equation (51) is of the form reported in (36) with different integral kernels. As reported in Section III, the unknown current $I_a(\zeta)$, is expressed in terms of scaling and wavelet functions as follows:

$$I_a(\zeta) = \sum_{\nu=-m+1}^{2^{j_0}-1} c_{\nu}^{(j_0)} \varphi_{m,\nu}^{(j_0)}(\zeta) + \sum_{j=j_0}^{j_z} \sum_{\nu=-m+1}^{2^j-m} d_{\nu}^{(j)} \psi_{m,\nu}^{(j)}(\zeta). \tag{52}$$

$j_0 = j_z = 2$ are chosen in this paper for the applications (see Section V). As reported in [11], the highest level j_z verify the condition that $(1/2^{j_z+1})$ does not exceed $0.1/\lambda$, where λ is the minimum wavelength associated with the maximum work frequency. Because of their bandpass filter characteristics the wavelets $\psi_{m,\nu}^{(j)}(\zeta)$ extract successively lower and lower components of the unknown function with increasing values of the level parameter “ j ”, while $\varphi_{m,\nu}^{(j_0)}(\zeta)$ retains the coarsest approximation of the original function.

The Galerkin method is used to solve the integral equation (51). Thus, the functions of the expansion (52) are used as weighting functions also. Equation (51) is first rearranged by

handling the integrals in the second and third summations of the left-hand side; these integrals are of the following type:

$$\int_{\zeta_{i-1}}^{\zeta_i} \eta(\zeta, \tilde{\zeta}) d[\tau(\zeta)] \tag{53}$$

where

$$\begin{aligned}
& \eta(\zeta, \tilde{\zeta}) \\
& = \begin{cases} \rightarrow \eta_x(\zeta, \tilde{\zeta}) = [x_p(\zeta, \tilde{\zeta}) - x_i(\zeta)] \\ \quad \times \frac{[1 + kR(\zeta, \tilde{\zeta})]e^{-kR(\zeta, \tilde{\zeta})}}{R^3(\zeta, \tilde{\zeta})}, & \text{second term in (51)} \\ \rightarrow \eta_a(\zeta, \tilde{\zeta}) = a_p(\zeta, \tilde{\zeta}) \\ \quad \times \frac{[1 + kR(\zeta, \tilde{\zeta})]e^{-kR(\zeta, \tilde{\zeta})}}{R^3(\zeta, \tilde{\zeta})}, & \text{third term in (51)} \end{cases} \tag{54}
\end{aligned}$$

$$d[\tau(\zeta)] = \frac{dI_a(\zeta)}{d\zeta} d\zeta. \tag{55}$$

Integral (53) can be solved as follows:

$$\begin{aligned}
\int_{\zeta_{i-1}}^{\zeta_i} \eta(\zeta, \tilde{\zeta}) d[\tau(\zeta)] & = \eta(\zeta, \tilde{\zeta}) \tau(\zeta) \Big|_{\zeta_{i-1}}^{\zeta_i} \\
& \quad - \int_{\zeta_{i-1}}^{\zeta_i} \tau(\zeta) d[\eta(\zeta, \tilde{\zeta})] \tag{56}
\end{aligned}$$

where $\tau(\zeta) = I_a(\zeta)$. In the first term of the right side of (56) the values of the current functions at the ends of the subinterval $[\zeta_{i-1}, \zeta_i]$ of $[0, 1]$, appear. These current values are those at the ends of the i th conductor of the thin-wire structure. In this way, for each node of the real structure, it is possible to force the incident currents to satisfy the appropriate Kirchhoff's current law [4], [5], [9], [10].

By applying systematically (56) to all the integrals in the second and the third summation of (51), and by using the following identity:

$$\beta(\zeta, \tilde{\zeta}) = \frac{L_i}{(\zeta_i - \zeta_{i-1})} g(\zeta, \tilde{\zeta}) \tag{57}$$

(51) is so modified

$$\begin{aligned}
& -\frac{j\omega\mu}{4\pi} \bar{u} \cdot \sum_{i=1}^{N_b} \left\{ \bar{u}' \int_{\zeta_{i-1}}^{\zeta_i} \tau(\zeta) \beta(\zeta, \tilde{\zeta}) d\zeta \right\} + \frac{1}{j\omega 4\pi \epsilon} \bar{u} \cdot \sum_{i=1}^{N_b} \\
& \quad \times \left\{ \bar{u}' \left\{ \eta_x(\zeta, \tilde{\zeta}) \tau(\zeta) \Big|_{\zeta_{i-1}}^{\zeta_i} - \int_{\zeta_{i-1}}^{\zeta_i} \tau(\zeta) d[\eta_x(\zeta, \tilde{\zeta})] \right\} \right\} \\
& \quad + \frac{1}{j\omega 4\pi \epsilon} \bar{u} \cdot \sum_{i=1}^{N_b} \left\{ \bar{u}' \left\{ \eta_a(\zeta, \tilde{\zeta}) \tau(\zeta) \Big|_{\zeta_{i-1}}^{\zeta_i} - \int_{\zeta_{i-1}}^{\zeta_i} \tau(\zeta) \right. \right. \\
& \quad \quad \left. \left. \times d[\eta_a(\zeta, \tilde{\zeta})] \right\} \right\} \\
& = \dot{z}_s \tau(\tilde{\zeta}) \quad \text{on } S. \tag{58}
\end{aligned}$$

Fig. 4. MoM wavelet matrix for $m = 2$ and different values of the decomposition level j_z ; N as the unknowns number.

Now, it is possible to solve the integral equation by the Galerkin method: by multiplying each side of (58) for the selected wavelet and scaling functions and integrating over the bounded interval $[0,1]$, with respect to the external variable $\tilde{\zeta}$, with $\tilde{\zeta} \in [0, 1]$, (58) is so modified

$$\begin{aligned}
& -j\omega \frac{\mu}{4\pi} \int_0^1 \kappa(\tilde{\zeta}) \bar{u} \cdot \sum_{i=1}^{N_b} \left\{ \bar{u}' \int_{\zeta_{i-1}}^{\zeta_i} \tau(\zeta) \beta(\zeta, \tilde{\zeta}) d\zeta \right\} d\tilde{\zeta} \\
& + \frac{1}{j\omega 4\pi \epsilon} \int_0^1 \kappa(\tilde{\zeta}) \bar{u} \cdot \sum_{i=1}^{N_b} \\
& \times \left\{ \bar{u}' \left\{ \eta_x(\zeta, \tilde{\zeta}) \tau(\zeta) \Big|_{\zeta_{i-1}}^{\zeta_i} - \int_{\zeta_{i-1}}^{\zeta_i} \tau(\zeta) d[\eta_x(\zeta, \tilde{\zeta})] \right\} \right\} d\tilde{\zeta} \\
& + \frac{1}{j\omega 4\pi \epsilon} \int_0^1 \kappa(\tilde{\zeta}) \bar{u} \cdot \sum_{i=1}^{N_b} \\
& \times \left\{ \bar{a}' \left\{ \eta_a(\zeta, \tilde{\zeta}) \tau(\zeta) \Big|_{\zeta_{i-1}}^{\zeta_i} - \int_{\zeta_{i-1}}^{\zeta_i} \tau(\zeta) d[\eta_a(\zeta, \tilde{\zeta})] \right\} \right\} d\tilde{\zeta} \\
& = \dot{z}_s \int_0^1 \kappa(\tilde{\zeta}) \tau(\tilde{\zeta}) d\tilde{\zeta} \quad \text{on } S. \tag{59}
\end{aligned}$$

In (59) $\kappa(\tilde{\zeta})$ is the selected weighting function (i.e., scaling φ or wavelet ψ). Based on the properties described in Section III, each of the integrals over the interval $[0,1]$ is split into several integrals over subintervals depending on the small support of the functions. Moreover, a lot of the integrals become zero due to the SO and vanishing moments properties of the wavelet functions. In fact, the entries with significant magnitude are in the (LL) submatrix (38) which is of order $r = 2^{j_0} + m - 1$ not depending on the decomposition level j_z (m as the order of the used cardinal B-spline functions, and j_0 the initial level) (Fig. 4).

Moreover, let δ be a small number so that $0 \leq \delta < 1$, and $N = 2^{j_z+1} + m - 1$ the unknowns number, a thresholding operation on the matrix elements can be performed by choosing the following ε :

$$\varepsilon = \delta \cdot \max_{1 \leq i, j \leq N} |a_{i,j}| \tag{60}$$

as a threshold parameter, where $a_{i,j}$ are the MoM wavelet matrix entries [see (37)]. So operating, a large number of entries in the $(N \times N)$ MoM wavelet matrix can be set to zero, and a reasonable correct solution is already performed by working with $(r^2 + N - r)$ entries of the LL and diag (HH) submatrices as it

will be shown in the next section. In this way, a strong increase of the efficiency, for both speed and memory, is reached.

As already underlined, in the conventional MoM, all the entries of the matrices are comparable in magnitude. As a consequence, no threshold can be used to sparsify the system matrix. Moreover, the dimension of the MoM wavelet LL submatrix results independent either from the specific length of each conductor of the structure under consideration and from the wavelength associated to the maximum work frequency. On the contrary, in the conventional MoM a proper segmentation of each thin wire of the structure has to be chosen, depending either from the wire length and from the minimum wavelength in frequency domain.

V. APPLICATIONS AND MODEL VALIDATION

Equation (59) is solved for each frequency of the discretized Fourier spectrum of the selected lightning current source; a DFFT algorithm can be then applied in order to obtain time profile of interesting electromagnetic quantities (i.e., currents distribution in the wire structure, electric and magnetic fields in the surrounding medium and on the ground surface [10]). A double exponential wave form ($1.2/50 \mu s$) has been considered for the lightning current time profile $i(t)$. The discretized Fourier spectrum of this function contains significant frequency up to few megahertz. In particular, in order to obtain a correct reconstruction by means of a DFFT algorithm, the maximum frequency value has to be set around some tens of megahertz: the value of 10 MHz has been selected so obtaining good results in the time reconstruction [10]. In this way, a rectangular window function of amplitude in time domain $T_0 = 1638.4 \mu s$ is used with a sampling time $T = 0.05 \mu s$, and $N_s = 32768$ samples. In the frequency domain, $(N_s/2 + 1)$ samples have to be used in order to satisfy the Nyquist theorem [31]. Thus, (59) is solved for all the $(N_s/2 + 1)$ sampled frequency values [10].

It is clear that, based on the consideration reported in Sections III and IV, the method proposed in this paper determines a strong reduction of the computer CPU time and memory requirement for a fixed problem, with respect to the conventional MoM solution.

In order to validate the proposed model, the following examples application in frequency domain are considered; computation results obtained with the cardinal B-spline functions of order $m = 2$, $j_0 = j_z = 2$, and by working with the LL and diag (HH) submatrices of the MoM wavelet matrix, are compared with the conventional MoM solution that uses a direct approach with piecewise linear basis functions and point matching procedure [4], [5], [9], [10]. The details related to conventional MoM solution for LPS structures are reported in [10].

At first, a single buried conductor in homogeneous medium, ten meters length, has been considered for two significant frequencies of the Fourier spectrum of the current source directly injected into one end (Fig. 5).

Moreover two separated wires, of 10-m length, buried in a homogeneous soil, have been considered. One of the wire is directly injected by a current source generator simulating a direct lightning stroke; the second wire is in the EM field of the exciting one (Fig. 6). Only one significant frequency of the Fourier

Fig. 5. Case study 1: single buried conductor in homogeneous medium with a current source generator.

Fig. 6. Case study 2: two separated wires buried in homogeneous soil: a current source generator excites one conductor; the second wire is in the EM field of the first one.

Fig. 7. Magnitude current profile for the case study 1 with frequency value 50 Hz. Results obtained with this paper (dotted line) conventional MoM (plain line).

spectrum of the source has been considered. In the case studies, each wire is partitioned in 50 segments in conventional MoM solution, and all the wires are supposed to be of copper with their radius equal to 5 mm.

As is shown in Figs. 7–9, a good agreement has been reached. In order to better explain the benefits of the performed computational model, the following quantities have been also considered:

$$c = \frac{\|f_M - f_W\|_2}{\|f_M\|_2} \quad s = \frac{t_W}{t_M} \quad \text{mem} = \frac{N_W}{N_M} \quad (61)$$

where f_M , t_M , and N_M are the solution, computational time, and number of matrix entries with conventional MoM, respectively, whilst f_W , t_W , and $N_W = (r^2 + N - r)$ are the solution, computational time, and number of matrix entries involved in the wavelet MoM process, i.e., LL and $diag(HH)$ entries. N_W entries hold back in the MoM wavelet matrix after having introduced an opportune threshold value δ : the range $\delta < 1\%$; has been selected for the case studies. In Table I, for the case studies previous reported, the parameters c , s , and mem are shown in percentage.

Further improvements are expected with the use of cardinal B-spline functions of order $m > 2$ and by increasing the scale level [11].

Fig. 8. Magnitude current profile for case study 1 with frequency value 1 MHz. Results obtained with this paper (dotted line) conventional MoM (plain line).

Fig. 9. Magnitude current profile along the axis of the two parallel wires of case study 2, with frequency value 50 Hz. Results obtained with this paper (dotted line); conventional MoM (plain line).

TABLE I
PRECISION OF SOLUTION, SPEED OF SOLVER, AND MEMORY REQUIREMENT FOR REPORTED CASE STUDIES

	$c\%$	$s\%$	$\text{mem}\%$
Case study 1: 50 Hz	1.603	47.263	1.16
Case study 1: 1 MHz	4.404	49.521	1.16
Case study 2: 50 Hz	4.693	68.289	0.29

VI. CONCLUSION

An advanced numerical model in solving thin-wire EFIE, in the frequency domain, by means of SO compactly supported spline wavelets, has been presented. The MoM is used via the Galerkin procedure. The model is applied to study EM transients in grounding systems or, in a more general way, in a LPS. A DFFT algorithm can be used in order to obtain the time profiles of the EM interesting quantities. By virtue of different mapping processes the thin-wire structure is transformed into the bounded interval $[0,1]$. In this way the SO spline wavelets defined on the interval $[0,1]$ have been used. A sparse matrix equation is obtained from the thin-wire EFIE by using this technique. Thus operating, the traditional drawback of the integral equation methods, due to the dense matrices generated by the conventional approach, is overcome. Two case studies have been presented to illustrate the merits of the used approach.

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