A Stochastic Continuous Time Model for Microgrid Energy Management

Benjamin Heymann, J Frédéric Bonnans, Francisco Silva, Guillermo Jimenez

A Stochastic Continuous Time Model for Microgrid Energy Management

Benjamin Heymann[1], J. Frédéric Bonnans[2], Francisco Silva[3],
and Guillermo Jimenez[4] *Senior Member, IEEE*

Abstract— **We propose a novel stochastic control formulation for the microgrid energy management problem and extend previous works on continuous time rolling horizon strategy to uncertain demand. We modelize the demand dynamics with a stochastic differential equation. We decompose this dynamics into three terms: an average drift, a time-dependent mean-reversion term and a Brownian noise. We use BOCOPHJB for the numerical simulations. This optimal control toolbox implements a semi-Lagrangian scheme and handle the optimization of switching times required for the discrete on/off modes of the diesel generator. The scheme allows for an accurate modelling and is computationally cheap as long as the state dimension is small. As described in previous works, we use a trick to reduce the search of the optimal control values to six points. This increases the computation speed by several orders. We compare this new formulation with the deterministic control approach introduced in [1] using data from an isolated microgrid located in northern Chile.**

I. INTRODUCTION

A microgrid is a small network of loads and energy ressources controlled by an Energy Management System (EMS). It can be either connected to the main network or isolated. The coordination of the microgrid units requires the resolution of an optimization problem. This problem is described in the literature as the microgrid management problem. Palma-Behnke et al. introduce in [2] a microgrid EMS based on a rolling horizon strategy for which the microgrid management problem is formulated as a Mixed Integer Programming (MIP) problem. Heymann et al. show in [1] that this MIP formulation could be replaced by a continuous Optimal Control (OC) formulation. We extend this last approach to the stochastic case by introducing a stochastic dynamics for the load.

In [3] the microgrid energy management problem is formulated as a two-stage stochastic programming model based on optimization principle. Then, the optimization model is decomposed into a mixed integer quadratic programming problem by using discrete stochastic scenarios to approximate the continuous random variables. A scenarios generation approach based on a time-homogeneous Markov chain model is proposed to simulate time-series of renewable energy generation and load demand. Similar approaches are considered in [4] and [5], especially in [4] uncertainty is characterized by a scenarios generation approach based on autoregressive moving average (ARMA) model according to the probability density function of each random variable. In [6], uncertainty is addressed using a two-stage decision process combined with a receding horizon approach. The first stage decision variables (unit commitment) are determined using a stochastic mixed-integer linear programming formulation, whereas the second stage variables (optimal power flow) are refined using a nonlinear programming formulation. Other approaches appear, such as the one described in [7] where uncertainties related to renewable distributed generation are modeled by proper probability distribution functions and are managed by reserve provided by both DGs and loads.

The reader may refer to [8] to get an overview of stochastic control theory and the applications of the dynamic programming principle, and to [9] and [10] for more details about the discretization scheme we use. As in [1] we solve the microgrid management problem using Bellman's Dynamic Programming Principle (DPP). We compute an approximation of the value function using a time and space discretization, and then we use this value function to reconstruct an optimal control. The DPP approach presents numerous advantages. First, no starting point is required to initiate the optimization algorithm. Second, the algorithm computes a global optimum, as opposed to other methods relying for instance on first order optimality conditions that only compute local optima. Third, we can deal with integer variables such as the on/off status of a device (in our case the diesel generator). Fourth, we use directly the original non-linear model. This simplifies the implementation, and may also give more accurate solutions. Fifth, since we derive the optimal controls from the value function, those controls are in feedback form. Last but not least, if the state dimension is low (in our case, two), it is competitive from a computational perspective. In particular, the computational burden is linear in the number of time steps. We perform numerical simulations with BOCOPHJB, a C++ open source numerical solver for stochastic optimal control problems (see [11]). We point out that this solver does not solve stochastic problems with scenario trees but instead solves an associated deterministic second order partial differential equation. The data for our problem come from the Huatacondo microgrid. Huatacondo is an isolated village in the Atacama desert (northern Chile). The village relies completely on the microgrid for its energy supply.

This paper is organized as follows: Section II describes the microgrid and the demand model, and then formulates

*Accepted for ECC 2016

[1]Benjamin Heymann is a PhD student at the Ecole polytechnique, France.

[2]J. Frédéric Bonnans is a researcher at Inria and Ecole polytechnique, France.

[3]Francisco Silva is a professor at the University of Limoges.

[4]Guillermo Jimenez is a researcher at the Universidad de Chile.

the stochastic optimal control problem. Section III presents the numerical method as well as the simulation methodology. We explain the parameters estimation in Section IV, and in Section V we display and comment the simulations results. Finally, the conclusion sums up the main results and presents ongoing research in the continuation of this work.

II. Model Presentation

A. System Description

On the supply side, the microgrid includes a photovoltaic power plant, a wind turbine, a diesel generator and a Battery Energy Storage System (BESS). The photovoltaic power plant and the wind turbine are non dispatchable units. Since one can accurately predict the climate in the desert region of the microgrid, we assume that we know the future production of renewable energy. This is why it is deterministic in this work. The diesel generator and the battery play the role of the dispatchable units. The marginal cost of the energy the generator produces is decreasing, i.e. the fuel consumption cost is a strictly concave function of the energy produced by the diesel generator. We derived this production function in [1] from the constructor data sheet. The diesel generator is either on or off. When on, the diesel generator cannot work below a given threshold (due to its physical properties). When switched on, the generator needs an additional amount of energy to warm up, which is modeled as a fixed switching cost. Since any ON switching is followed by an OFF switching and conversely, we account for half this switching cost for any switch, ON or OFF. We can store the energy surplus in the BESS when production is greater than demand and supply this energy to the system when demand is greater than production. This storage is not free as some energy is lost in the charge/discharge process. We will not take into account the battery aging in this work. On the demand side, the load comes from the villagers domestic needs. In our model proposal the randomness comes from the demand side. Since the village is small, the load is volatile. We modelize the load dynamics with a Stochastic Differential Equation (SDE). The grid is isolated, so there cannot be any flows from or to the outer world. We neglect the transmission losses because the village is small. Our objective is to find a strategy that minimizes the operating cost (diesel and switching cost) and produce enough electricity for the village.

B. Load Model

The microgrid historical load might suggest a random model with several jumps. Nonetheless such type of models requires a large number of parameters: the sizes and probabilities of jumps need to be fitted. As a first step toward the integration of stochastic modeling within our framework we propose a simpler model based on a Brownian motion. Our proposal is similar to an Ornstein-Uhlenbeck process (which is the continuous time equivalent of the AR(1) model) because it is a Brownian dynamics with a mean reversion. The difference is that the mean and volatility parameters are time dependent. We model the load process $L(t)$ with the Stochastic Differential Equation (SDE)

$$\mathrm{d}L(t) = (\dot{\Lambda}(t) + b(\Lambda(t) - L(t))\mathrm{d}t + \sigma(t,L)\mathrm{d}W(t), \quad (1)$$

where $\Lambda(t)$ is a deterministic load process (in kW), $b \geq 0$ is a unitless mean reversion coefficient, $\sigma(t,L)$ is the volatility (in $kW.t^{-0.5}$), $W(t)$ is a Wiener process and $L(0) = L_0$. The volatility σ has a bounded support in $[0,T] \times [0, L_{max}]$. Since Λ is bounded and $b \geq 0$, the load L remains bounded (and positive):

$$L(t) \leq \max(\sup_t \{\Lambda(t)\}, L_{max}). \quad (2)$$

This allows us to refer to section 3 of [10] for the mathematical properties of the system. Setting $Y(t) = L(t)e^{bt}$ and applying Itô's formula we get that $L(t)$ is equal to

$$\Lambda(t) + e^{-bt}(L_0 - \Lambda(0)) + \int_0^t e^{b(\tau - t)}\sigma(\tau, L(\tau))\mathrm{d}W(\tau).$$

So, $L(t)$ has expectation $\Lambda(t) + (L_0 - \Lambda(0))e^{-bt}$ and, by Itô isometry, a variance of at most $\sup(\sigma^2)(1 - e^{-2bt})/(2b)$. In Section IV we will discuss the computation of σ and b and provide an empirical justification of the model.

C. Notations

We denote by t_0 the initial time and by T the time horizon. The state variables will be represented with capital letters. We denote by C the state of charge of the BESS and by L the load. We point out that only C is controlled, since the dynamics (1) of L does not depend on any decision. The diesel generator mode (on or off) at time t is represented with the variable $m(t) \in \{0,1\}$ (0 for off and 1 for on). The control variables will be represented with lower-case letters. We write d the diesel generator output and s an artificial slack variable (to ensure the feasibility of the problem). The variable s represents the excess ($s < 0$) or missing ($s > 0$) power. We penalize decisions with a non zero slack variable by an integrable cost proportional to the absolute value of s. We impose s to be non positive if the diesel generator is off and bigger than a fixed constant if it is positive. We denote by $n(t)$ the counting variable equals to the number of switches that occurred between time t_0 and t. It is non decreasing over time and for all t, $n(t) \in \mathbb{N}$. We associate to each switch (OFF to ON and ON to OFF) a cost K equals to half the real cost needed to fire the diesel generator on. We write P_S the quantity of renewable energy produced at time t. As explained in II-A, this is a deterministic function of time since we assume we have a reliable deterministic forecast. If we denote by P_I and P_O the quantities that go in and out of the BESS, then the power equilibrium equation writes

$$d + P_O + P_S + s - L - P_I = 0, \quad (3)$$

so that P_I and P_O can be written as non linear functions of s, L, P_S and d:

$$\begin{aligned} P_O &= -\min(0, P_S + d - L + s) \\ P_I &= \max(0, P_S + d - L + s). \end{aligned} \quad (4)$$

We denote by Q_B the maximum capacity of the battery, while ρ_I and ρ_O are the efficiency ratios for the charge and discharge processes. We write ℓ the cost function associated to the diesel consumption. The final cost function g ensures a minimal value of the final state of charge:

$$\begin{cases} g(C) = 0 & \text{if } C \geq C_0 \\ g(C) = M & \text{otherwise} \end{cases}. \tag{5}$$

where M is a large penalty parameter. Setting $C_0 = C(t_0)$ we ensure that the system finishes the day with as much energy in the BESS as what was store at t_0.

D. Stochastic Control Formulation

We now define the value of the microgrid management problem as

$$V^{m_0}(t_0, C_0, L_0) := \inf_{n,d,s} \mathbb{E} \left(Kn(T) + g(C(T)) + \int_{t_0}^{T} \ell(d(t), s(t)) \mathrm{d}t \right) \tag{6}$$

subject to, for all t:

$$\dot{C}(t) = F_C(L, d, s, t) \tag{7}$$

$$\mathrm{d}L(t) = (\dot{\Lambda}(t) + b(\Lambda(t) - L(t))\mathrm{d}t + \sigma(t, L(t))\mathrm{d}W(t) \tag{8}$$

$$m(t) = \frac{1 + (-1)^{n(t)}(2m_{t_0} - 1)}{2} \tag{9}$$

$$(C(t_0), L(t_0), m(t_0)) = (C_0, L_0, m_0) \tag{10}$$

$$\begin{cases} d(t) = 0 \text{ and } s(t) \leq 0 & \text{if } m(t) = 0, \\ d(t) \in I_d & \text{otherwise.} \end{cases} \tag{11}$$

$$C(t) \in I_c. \tag{12}$$

$$P_O \in I_{P_O}, \tag{13}$$

$$\begin{cases} P_I \in I_{P_I} & \text{if } C(t) < 0.9, \\ P_I \leq A(C(t) - 1)^2 & \text{otherwise,} \end{cases} \tag{14}$$

where $F_C(L, d, s, t) = \frac{1}{Q_B}(P_I \rho_I - P_O / \rho_O)$. We point out that by many ways this stochastic optimal control problem is similar to the deterministic model presented in [1]. Here the decision variables are the diesel output at any instant $d(t)$, the slack variable $s(t)$ and the value of the counting function $n(t)$. Note that optimizing over the counting functions is equivalent to optimizing over the switching times. Implicitly we impose those decisions to be non anticipative, i.e. progressively measurable with respect to the filtration generated by $W(t)$. Constraints (7) and (8) are respectively the power balance for the battery and the load dynamics. Relation (9) expresses the current mode as a function of the initial mode and the number of switches that occurred since t_0. If this number is even, $m(t) = m_0$ and if it is odd, $m(t) = 1 - m_0$. Constraint (10) is the initial condition. Constraint (11) corresponds to the modeling of the diesel generator mode ($ON = 1$ or $OFF = 0$). If OFF, the diesel generator cannot produce anything and $d = 0$, else, the physics of the generator impose d to be in $I_d = [d_1, d_2]$, with $d_1 > 0$. Last, constraints (12), (13) and (14) correspond to physical properties and limitations of the battery, with P_I and P_O defined by equation (4). The sets I_{P_O} and I_{P_I} are segments of $\mathbb{R}_+$ and I_C is included in $[0, 1]$. The parameters A and M are positive constants. Table I contains the numerical values we use.

E. Technical Remark

We already noticed that L is bounded over $[0, T]$. Thus the number of switches n is bounded on any scenario and the slack variable s is uniformly bounded over the scenari (this is of course true for d since $d \in \{0\} \cup I_d$). To our knowledge, there are no general well posedness results for stochastic control with state constraints. Nonetheless, since the controls are bounded and the diffusion is orthogonal to the outer normal of the state constraint we can argue that the viscosity approach developed in [12] for second order fully nonlinear elliptic equations with state constraints could be extended to our case with finite horizon and switching times.

III. Numerical Optimization Method

A. Dynamic Programming

The Dynamic Programming Principle (DPP) states that (see [8])

$$V^{m_0}(t_0, C_0, L_0) = \inf_{d \in D_{m_0}, \tau \in \mathbb{T}_t, s} \mathbb{E} \int_{t_0}^{\tau} \ell(d, s)\mathrm{d}t + \min\{V^{m_0}(\tau, C(\tau), L(\tau)), K + V^{1-m_0}(\tau, C(\tau), L(\tau))\} \tag{15}$$

where the optimization is performed over (7)-(14) and $D_0 = \{0\}$, $D_1 = I_d$, and $\mathbb{T}_t$ is the set of stopping times in $[t_0, T]$. The time dependency of d and s is implicite in the integrand. Note that from (15) and applying Itô's formula we get that the value function formally satisfies the Hamilton-Jacobi-Bellman equation (see for instance [8])

$$\max\left\{-V_t^i - 0.5 V_{LL}^i \sigma^2 - V_L^i(\dot{\Lambda} + b(\Lambda - L)) + H_i, \; V^i - (K + V^{1-i})\right\} = 0, \tag{16}$$

where

$$H_0 = \sup_{s \leq 0} -\{\ell(0, s) + V_C^1 F_C(L, 0, s, t)\} \tag{17}$$

$$H_1 = \sup_{d \in I_d, s} -\{\ell(d, s) + V_C^0 F_C(L, d, s, t)\}. \tag{18}$$

We now explain a weaker version of a trick introduced in [1] for the deterministic case. We assume $s = 0$, i.e. there is a good balance between production capacity and load. If the diesel generator is off then by definition $d = 0$. Otherwise, the dynamics of the system is locally (16) for $i = 1$, so that heuristically, the control should maximize the Hamiltonian H_1 defined at (18). Since we maximize a piecewise convexe function, the optimal controls can take a limited number of values that can be explicitly computed.

- if the diesel is off ($m = -1$), we simply take $d = 0$.
- if the diesel is on ($m = 1$), we test the five cases
  - d is set to the minimum power,
  - d is set to the maximum power,
  - d such that $F_C = \dot{C} = 0$ (battery unused),
  - d such that P_i is maximal (maximal charge),

TABLE I: Numerical parameters

Notation	Value
Q_B	117 kWh
A	1320 kW
M	1000000 CLP
$2K$	1000 CLP
$\ell(d,s)$	$500d^{0.9}+25000\lvert s\rvert$ CLP
ρ_O	0.95
ρ_I	0.95
I_d	[5,120] kW
I_{P_I}	[0,13.2] kW
I_{P_O}	[0,40] kW
I_c	[0.2,1] kW

– d such that P_0 is maximal (maximal discharge).

From a computational perspective it is sufficient to test those values instead of discretizing the control space.

B. Algorithm

We solve the Hamilton-Jacobi-Bellman equation (15) with BOCOPHJB [11]. This open-source software solves second order finite horizon Hamilton-Jacobi-Bellman equations with a semi-Lagrangian scheme and allows for the use of switches. The semi-Lagrangian scheme is obtained by discretizing (15) first in time and then in space: it consists in the backward resolution of a discretized dynamic programming principle The reader may refer to [9] and [10] for the discretization theory. We point out that the semi-Lagrangian scheme does not require to generate scenari (as opposed to other mainstream approaches in stochastic programming), since the Brownian motion is discretized for each time step with deterministic variables (see [11]). For this kind of scheme, the computation burden is exponential in the state dimension (curse of dimensionality), but here this dimension is only two. On the other hand, the complexity increases only linearly with the number of time steps.

IV. PARAMETERS ESTIMATION

We display in Table I the numerical values we have for the model. Most of them are those used in [1] and [2]. The data consist in about ten months ($N_{days}=300$) of historic load and renewable production from Huatacondo. The renewable production data look both smooth and very similar days after days due to the climate in Huatacondo, so we use the average for the optimization and the simulation (see Figure 5). We denote by h the time step (15 minutes), $t_k = kh$, σ_k the volatility and $\hat{L}_k$ the historical load at time t_k. The data being discrete, Equation (8) becomes, for each day i

$$\hat{L}^i_{k+1} - \hat{L}^i_k = \Lambda_{k+1} - \Lambda_k + b(\Lambda_k - \hat{L}^i_k)h + \sigma_k \varepsilon^i_k \sqrt{h}, \qquad (19)$$

where ε^i_k is a standard centered Gaussian variable and $k \in \{1,\dots,96\}$. Note that as discussed in §II-B, Λ_k is the historical average of the load at time t_k, i.e. $\Lambda_k = \sum_{i\in 1..N_{days}} \hat{L}^i_k / N_{days}$. Set for all $k \in \{1,\dots,96\}$ and $i \in \{1,\dots,N_{days}\}$ $d_{k,i} = \hat{L}^i_k - \Lambda_k$, $b' = hb$ and $\sigma'_k = \sigma_k\sqrt{h}$. Then (19) is equivalent to $d_{k+1,i} - d_{k,i}(1-b') = \sigma'_k \varepsilon_k$. We then use a mean square estimator. If we consider σ'_k fixed for all k, then b' should minimize $\sum_{k,i}(d^i_{k+1} - d^i_k(1-b'))^2/\sigma'^2_k$, so that $b' = \{\sum_{k,i} d^2_{k,i}/\sigma'^2_k - \sum d_{k,i}d_{k+1,i}/\sigma'^2_k\}/\{\sum d^2_{k,i}/\sigma'^2_k\}$. On the other hand, if we know b', σ'_k is the standard deviation of $d_{k+1,i} - d_{k,i}(1-b)$ computed over the same epoch of the day i on the data. So we start with $\sigma'_k = 1$ and iterate the two formulas until numerical convergence to get our estimators. We get $b' = 0.174$. We display σ in Figure 1. We display on Figure 2 some random samples of the data and some generated scenari. They qualitatively look alike.

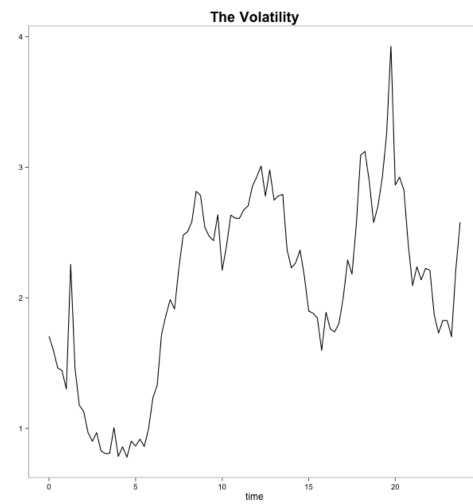

Fig. 1: The estimated volatility

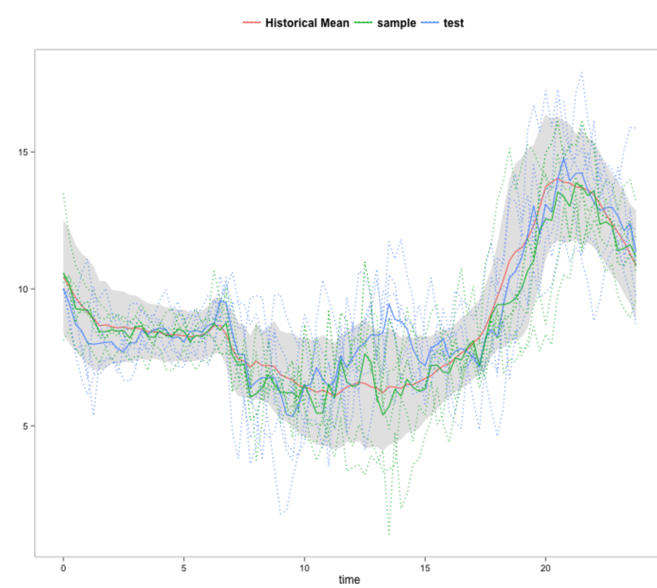

Fig. 2: We simulate some scenari with the load model, and compare them with historical day taken at random. The grey area correspond to one standard deviation.The unit is the kW.

V. SIMULATION

We compare the stochastic extension with the determinist rolling horizon algorithm proposed in [1] on a three day sim-

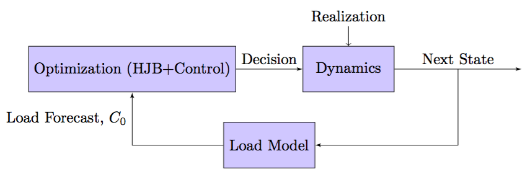

Fig. 3: Deterministic simulation algorithm

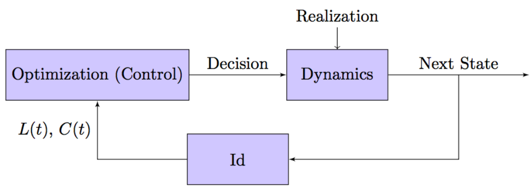

Fig. 4: Stochastic simulation algorithm

ulation. The simulation procedure is summarized in Figure 3 and 4.

The rolling horizon for the deterministic algorithm is set to 24 hours and for each horizon we impose the final state of charge to be at least equal to the initial state of charge at the beginning of the horizon. For every time step, we perform an optimization using an updated load forecast for the next 24 hours. We use as a forecast for the k^{th} step the expectation of the flow of the load process with initial condition L_k, where L_k is the corresponding historical Load.

For the stochastic simulation, we solve only once the Hamilton-Jacobi-Bellman equation, and then use the value function and the load historical realization to produce a trajectory. We impose the state of charge to be at least equal to what is obtained with the deterministic simulation at the end of the three day period.

We display on Figures 6, 8, 7 and 9 the results for the model with real data. On our example the slack variable s is always zero so we do not plot it. The total costs for the deterministic and the stochastic simulation are respectively 66819 CLP and 62342 CLP.

VI. CONCLUSIONS

We have extended the deterministic continuous time model for microgrid management to a stochastic setting and performed a numerical experiment on real data from the Huatacondo microgrid. The total cost of the solution proposed by the stochastic algorithm was lower than the one obtained with a deterministic rolling horizon formulation. Ongoing works on this topic include the study of the long-term aging of the battery as well as a jump model for the load process.

APPENDIX

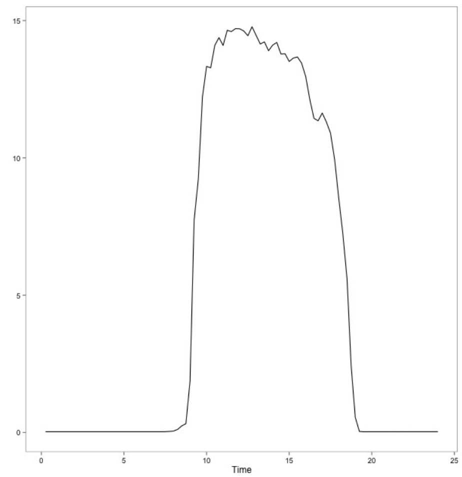

Fig. 5: Real model: Solar Production (in kW)

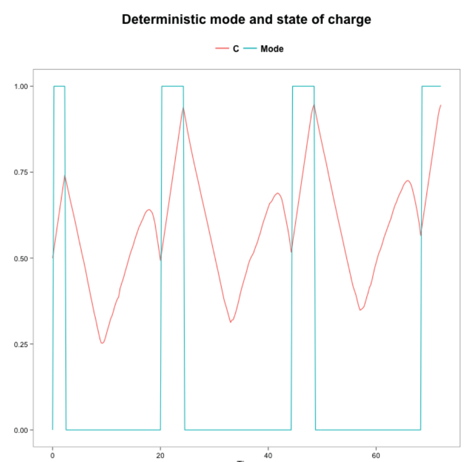

Fig. 6: C (in kW) and mode for the deterministic simulation

Fig. 7: C (in kW) and mode for the stochastic simulation

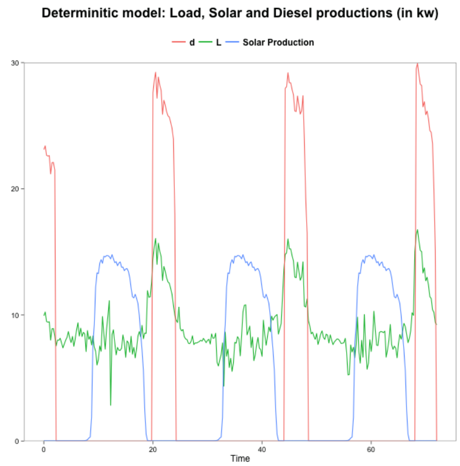

Fig. 8: Load and Production for the deterministic simulation (in kW)

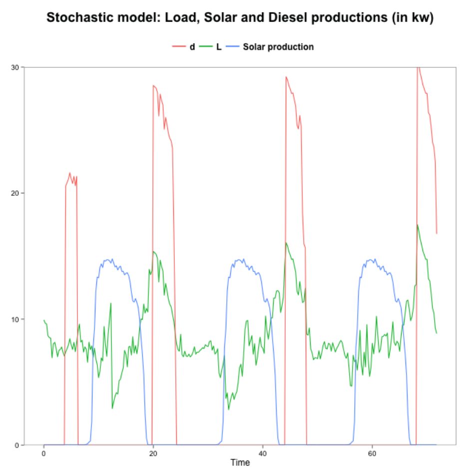

Fig. 9: Load and Production for the stochastic simulation (in kW)

ACKNOWLEDGMENT

This project is the product of a collaboration between the COMMANDS (INRIA, France) and Centro de Energia (Universidad de Chile, Chile) teams. We acknowledge support from PGMO, Icode and the Siebel Scholars Fundation.

REFERENCES

[1] B. Heymann, J. F. Bonnans, P. Martinon, F. Silva, F. Lanas, and G. Jimenez, "Continuous optimal control approaches to microgrid energy management," 2015.

[2] R. Palma-Behnke, C. Benavides, F. Lanas, B. Severino, L. Reyes, J. Llanos, and D. Sáez, "A microgrid energy management system based on the rolling horizon strategy," *Smart Grid, IEEE Transactions on*, vol. 4, no. 2, pp. 996–1006, 2013.

[3] Y. Ji, J. Wang, W. G. S., Yan, and L. H., "Optimal microgrid energy management integrating intermittent renewable energy and stochastic load," *2015 IEEE Advanced Information Technology, Electronic and Automation Control Conference (IAEAC), Chongqing*, pp. pp. 334–338, 2015.

[4] H. P. Li, C. Z. Zang, P. Zeng, H. B. Yu, and Z. W. Li, "Scenario-based stochastic programming strategy for microgrid energy scheduling considering uncertainties," *Applied Mechanics and Materials*, vol. 672-674, pp. 1322–1328, 2014.

[5] A. Eajal, E. F. El-Saadany, Y. Elrayani, and K. Ponnambalam, "Two-stage stochastic power generation scheduling in microgrids," in *Electrical and Computer Engineering (CCECE), 2014 IEEE 27th Canadian Conference on*. IEEE, 2014, pp. 1–6.

[6] D. E. Olivares, J. D. Lara, C. A. Cañizares, and M. Kazerani, "Stochastic-predictive energy management system for isolated microgrids," *Smart Grid, IEEE Transactions on*, vol. 6, no. 6, pp. 2681–2693, 2015.

[7] A. Zakariazadeh, S. Jadid, and P. Siano, "Smart microgrid energy and reserve scheduling with demand response using stochastic optimization," *International Journal of Electrical Power & Energy Systems*, vol. 63, pp. 523–533, 2014.

[8] H. Pham, *Continuous-time stochastic control and optimization with financial applications*. Springer Science & Business Media, 2009, vol. 61.

[9] K. Debrabant and E. Jakobsen, "Semi-lagrangian schemes for linear and fully non-linear diffusion equations," *Mathematics of Computation*, vol. 82, no. 283, pp. 1433–1462, 2013.

[10] F. Camilli and M. Falcone, "An approximation scheme for the optimal control of diffusion processes," *RAIRO-Modélisation mathématique et analyse numérique*, vol. 29, no. 1, pp. 97–122, 1995.

[11] F. Bonnans, D. Giorgi, B. Heymann, P. Martinon, and O. Tissot, "Bocophjb 1.0. 1–user guide," Tech. Rep., 2015.

[12] M. A. Katsoulakis, "Viscosity solutions of second order fully nonlinear elliptic equations with state constraints," *Indiana University Mathematics Journal*, vol. 43, no. 2, pp. 493–520, 1994.