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Privatization and Leverage

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Abstract

This paper studies privatization methods when potential buyers can lever up strategically to maximize their probability of winning. We endogenize the optimal fraction of shares to be auctioned off when privatizing a company. There is a close correlation between the optimal fraction of shares to be sold off and the auction winner’s debt level and hence the risk of bankruptcy.

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1 Introduction

Privatization, the sale by a government of state-owned enterprises (SOEs) or assets to private economic agents, is now widespread. As reported by Megginson and Netter (2001):

*We would like to thank Denis Gromb, seminar participants at Besancon, Bordeaux and Montpellier for helpful comments and suggestions. Correspondence: Avenue de l’Observatoire, 25 030 Besancon, FRANCE. Tel: +33 38166 6747. Email: christian.at@univ-fcomte.fr
...privatization now appears to be accepted as a legitimate tool of statecraft by governments of more than 100 countries.

Various privatization mechanisms are observed around the world. Megginson and Netter (2001) report that most privatizations involve issuing shares directly to the public, issuing vouchers, or selling assets to private parties. The latter approach may take the form of an auction or of private negotiations.¹

In this model, we focus on privatizations via auctions and we address two questions.

• First, should the government sell off 100% of the equity or retain fractional ownership? On average, 74.2% of capital is sold through asset sales (Megginson et al., 2001) and the remaining capital is kept by the government.

• Second, how are the government’s choices affected when potential buyers can raise funds by issuing debt backed by the newly privatized company’s assets and its future revenues, i.e. a leveraged buyout (LBO)? Such transactions have been observed in Albania, Poland, Belarus, Croatia, Macedonia, Romania, Slovenia and Czech Republic (Bennett et al., 2003). An LBO occurs when a person or entity gains control of a majority of a target company’s equity through the use of debt. The assets of the target company are used as collateral for the loans, in addition to the assets of the acquiring company. The purpose of LBOs is to enable companies to

¹Megginson et al. (2001) use a sample of 2477 privatizations in 92 countries from 1977 through 2000; of the 2477 transactions, 1539 were asset sales. La Porta and Lopez-de-Silanes (1999) report that, from a sample of 233 nonfinancial privatizations, the method of privatization employed in Mexico involved first price sealed-bid auctions, with the value of the bid determining the winner in over 98% of all privatized SOEs.
make large acquisitions without having to commit a lot of capital.\footnote{In LBOs, there is usually a ratio of 70\% debt to 30\% equity, although debt may run to 90\% or 95\% of the target company’s total capitalization.}

We consider the SOE is privatized via the commonly ascending auction mechanism. The bidders are assumed to be better managers than the incumbent managers. Where leverage is not used, the government has an incentive to keep the maximum number of shares so as to benefit from the value enhancement to be brought about by the privatized firm. However, we show that the bidders use the debt strategically to maximize their bids. They benefit since a fraction of the debt will be repaid from the future firm’s revenue by the government shareholder. We easily infer that when bidders are compelled to buy more shares they reduce their leverage accordingly. However, the use of debt has a negative effect too: it creates a risk of bankruptcy. The government, confronted with an LBO on an SOE, internalizes this bankruptcy risk and it sells more shares to reduce it. This paper shows that the optimal number of shares sold by the government results from the following trade-off: holding on to the maximum number of shares to benefit from the value enhancement but selling more than the required minimum to reduce the bidders’ leverage and so, the bankruptcy risk. Some examples of large Czech firms privatizations highlight the second part of this trade-off. Some firms as Poldi Kladno and Skoda Plzen have been privatized through LBOs. The new owners could not repaid their debts leading them to financial distress or even to bankruptcy.

The question of foreign investors’ participation is also crucial. Countries in transition diverge on this point. For example, Brada (1996) notes that Hungary’s privatization strategy favored sales of state-owned enterprises to foreigners. At the opposite, while
China offers large investment opportunities for foreign investors because of the reform of SOEs, the government favors domestic investors. We show that the second part of the government’s trade-off is accentuated when it favors domestic bidders over foreign ones, since it internalizes the bankruptcy risk more.

A number of theoretical papers have dealt with important issues of privatization. Cornelli and Li (1997) analyze privatization schemes in the context of optimal auction design; they find that the number of shares sold is a crucial instrument to attract the most efficient investors. In a relatively close paper, Banerji and Errunza (2005) investigate under agency problems, the optimality of different privatization methods via private negotiation; they also derive the optimal fraction of shares the government should sell. Our work complements these models. We also study the important issue that is the optimal number of shares the government should sell. Our contribution is to investigate the government’s decisions when buyers use strategically debt through LBO to maximize their bids. The question of the strategic use of debt has already been addressed by Muller and Panunzi (2004), albeit in the context of takeovers, i.e. when buyers face a continuum of sellers who free-ride. In this case, debt is used to overcome the free-rider problem, not to maximize the buyers’ offers.

The paper is organized as follows. Section 2 outlines the model. Section 3 derives the optimal strategies of bidders and government. Section 4 determines the government’s optimal strategy when: first, the successful bidder can extract some private benefits; second, the government has a preference for local bidders. Section 5 concludes.
2 The model

Consider the privatization of an SOE via an auction. Incumbent management generates a revenue that is normalized to 0. The government decides to sell off a fraction $\alpha \in [0.5, 1]$ of shares and to retain the remaining shares. We consider only two risk-neutral candidates, bidders 1 and 2.

If bidder $i = 1, 2$ gains control, he can generate a revenue $y_i$. It is common knowledge that the revenues $y_i$ are drawn independently from a uniform distribution $F(.)$ on the interval $Y_i = [0, v_i]$ with density $f(.) > 0$, but which is known neither by the government nor by the bidders before the auction. Each bidder privately knows $v_i$ but it is assumed that $v_i$ are i.i.d. over $[0, 1]$ with uniform cumulative $G(.)$ and density $g(.)$.

The sequence of events is as follows:

- $t = 0$, the government decides to privatize an SOE and discloses the fraction $\alpha$ to be sold off.

- $t = 1$, bidder $i = 1, 2$ sets up a new acquisition subsidiary that issues debt with face value $D_i$. We assume that there is no asymmetric information between the creditor and the bidder, and that the government does not observe $D_i$. We assume that the credit market is perfectly competitive.

- $t = 2$, the auction takes place and the shares are allocated to the highest bidder. First, we consider an ascending auction, and later we characterize the optimal auction.

- $t = 3$, the winning bidder generates the revenue $y_w$ for $w = 1, 2$. 

3 The analysis

Numerical example: Let us illustrate our main arguments with a simple numerical example. Consider the financing and bidding strategy of a value-enhancing bidder who can generate the revenue \( y^H = 1 \) with probability 0.5 and \( y^L = 0 \) otherwise. Promising to repay an amount \( D \leq 1 \) permits the bidder to raise funds equal to
\[
d = \text{prob}(y = y^H) \times D + \text{prob}(y = y^L) \times (0 - D^2/2)
\]
where \( D^2/2 \) represents a deadweight bankruptcy cost. In the high state of nature (\( y^H = 1 \)), the creditors receive the nominal debt value \( D \); in contrast, in the low state of nature \( y^L = 0 \), the firm goes bankrupt and the creditors receive the liquidation value of the firm net of the bankruptcy cost.

Consider first that the government wants to sell 75% of shares, therefore the best offer of the bidder is
\[
w = \text{prob}(y = y^H) \times 0.75 \times (1 - D) + \text{prob}(y = y^L) \times 0.75 \times 0 + d,
\]
which represents his expected payoff if he wins the auction. Facing opponents the bidder maximizes his offer with respect to \( D \), which gives \( D^* = 0.25 \). Note that because of bankruptcy cost, the bidder does not issue the maximum of debt he can.

Hence, the bidder is better off issuing \( D^* > 0 \) to pay for the tendered shares since he obtains \( d \) from the debt issue but repay only a fraction (75%) of it, the rest being repaid by the minority shareholder, i.e. the government. We see immediately that the fraction of shares kept by the government affects the optimal level of debt issue: leverage is higher when the government retains more shares, e.g. if the government sells 60% of shares, then \( D^* = 0.4 \). Therefore, the government faces the following trade-off: selling the minimum number of shares to benefit from the value-enhancement but this implies more debt issue, that reduces the value of the shares kept.
In the following, we generalize our numerical example and we investigate further questions.

### 3.1 Financing and bidding strategies

Obviously, the optimal bidding strategy is to remain active during the auction process as long as the price is below what the bidder is willing to pay and the opponent remains active. Hence, the winner is the bidder with the higher willingness to pay.

Each bidder can raise funds up to a maximum $v_i$. The debt may be risky so bidder $i$ raises funds $d_i \leq D_i$. His creditor receives $D_i$ if $y_i > D_i$ and $(1-k)y_i$ otherwise, where $k \in (0,1)$ represents bankruptcy costs. Therefore, a bidder $i = 1, 2$ raising funds, obtains:

$$d_i = (1-k) \int_0^{D_i} y_i dF(y_i) + \int_{D_i}^{v_i} D_i dF(y_i) \quad \forall D_i \in [0, v_i]$$ (1)

The first term represents what the creditor obtains if the privatized firm goes bankrupt. The creditor is paid in full if the privatized firm generates enough revenues, which is captured by the second term.

Bidder $i$ obtains $d_i$ from the debt issue and ends up with the fraction $\alpha$ of the firm’s equity, which is worth $\int_{D_i}^{v_i} (y_i - D_i) dF(y_i)$. Bidder $i$’s willingness to pay for $\alpha$ shares is:

$$w_i(D_i) = \alpha \int_{D_i}^{v_i} (y_i - D_i) dF(y_i) + d_i(D_i)$$ (2)

$$= \alpha \int_0^{v_i} y_i dF(y_i) + (1-\alpha) \left( \int_0^{D_i} y_i dF(y_i) + \int_{D_i}^{v_i} D_i dF(y_i) \right) - k \int_0^{D_i} y_i dF(y_i)$$

By issuing debt, each bidder faces a simple trade-off. On the one hand, an increase in $D_i$ increases his willingness to pay since a portion of the debt will be repaid from the
future firm’s revenue held by the government. On the other hand, it increases expected bankruptcy costs. Therefore, each bidder maximizes his willingness to pay with respect to $D_i$. We obtain the following FOC:

\[(1 - \alpha)(1 - F(D_i^*)) = D_i^* kf(D_i^*)\]  

where the optimal debt level\(^3\) depends on $\alpha$.

**Proposition 1**  The optimal debt level is decreasing in $\alpha$ and $k$.

**Proof.** See appendix A1

As the government retains more shares, each bidder takes on more debt because this increases the benefit derived from the debt, i.e. the fact that a greater portion of the debt will be repaid from the future firm’s revenue earned by the government shareholder. Higher bankruptcy costs reduce the bidder’s benefit from the debt. Hence, we should observe privatized firms with low debt levels when bankruptcy costs are high.

The government may have different objectives in its privatization policy such as bolstering employment and/or the capital market. Hence, the likelihood of bankruptcy in the post-takeover era is an important issue for the government. The highly-leveraged transaction analyzed in this model increases the risk of bankruptcy dramatically. Selling all the shares induces the buyer to take on no debt, so reducing the likelihood of bankruptcy to its minimum value (0 under our assumptions). However, we will see that this is not optimal if the government’s objective is to maximize its revenue. We have:

**Proposition 2**  The probability of bankruptcy is decreasing in $\alpha$ and $k$.

\(^3\)The second order condition $\frac{d^2w_i(D_i)}{dD_i^2} = -(1 + k - \alpha)f(D_i) - D_i kf'(D_i)$ is assumed satisfied.
Proof. See appendix A2. ■

### 3.2 The government strategy

We first consider in this section that the government’s objective is to maximize the revenues it earns through privatization. This objective may be driven by budgetary considerations or by political motives (e.g., a large revenue stream may be necessary to justify domestically the decision to share ownership with foreign firms (Cornelli and Li, 1993). The government obtains direct revenues from the auction and indirect revenues from the fraction \((1 - \alpha)\) of shares kept. This fraction yields revenues generated by the winner of the auction.

Before turning to consider the levered buyer, considering \(D_i = 0 \forall i = 1, 2\) enables us to highlight some basic trade-offs. In this case, the seller maximizes the following total revenue:

\[
E(R) = \alpha E(y_{-w}) + (1 - \alpha)E(y_w)
\]  

(4)

where \(E(y_w)\) are the expected revenues generated by the winner \(w = 1\) or \(2\) of the auction and \(E(y_{-w})\) is the expected price paid, which is the willingness to pay of the second bidder. We have \(\frac{dE(R)}{d\alpha} = E(y_{-w}) - E(y_w) < 0\). Facing unlevered bidders, the government cannot obtain more than the second valuation of the bundle of shares and, since the winning bidder is also the most efficient buyer, the government is better off retaining a maximum number of shares so as to benefit from the value-enhancement of the privatized firm. This result is closely related to the free-rider problem first pointed out by Grossman and Hart (1980). In a widely held company, shareholders, being non pivotal,
are better off retaining their shares in order to benefit from the value enhancement of the raider. In our framework, the government faces countervailing incentives: it has to sell at least a majority of shares but has no interest in selling more.

With levered bidders, this result does not hold since the optimal level of debt depends on the number of shares sold. Debt has a negative effect on share values since it increases bankruptcy costs. Therefore, the government has to consider the negative impact of the debt on its optimal choice. The objective function of the government becomes:

\[
\Pi(\alpha) = \int_0^1 w(v_w) [D_w^*(v_w, \alpha) - w(v_w)] dv_w + (1 - \alpha) \int_0^1 \left( \int_{D_w^*(v_w, \alpha)}^{v_w} (y - D_w^*(v_w, \alpha)) f(y_w) dy_w \right) 2G(v_w) g(v_w) dv_w
\]

The first term is the expected price paid by the winner of the auction, which is the second better valuation. The second term captures the expected revenue net of the debt of the shares the government retains, valued under the winning bidder’s management.

**Proposition 3** Facing levered bidders, the government sells more than the requisite minimum of shares.

**Proof.** See appendix A3.

As before the government is still better off retaining a maximum number of shares to benefit from the future firm’s value enhancement. However, since an indebted bidder reduces the value of the surviving firm’s shares, then this reduces the incentives for the government to retain as many shares as possible. Compared to the no debt case, the government sells more shares to induce the bidders to take on less debt\(^4\).

\(^4\)We could consider that the government imposes a socially optimal leverage for the winner that would
Corollary 1 *The optimal number of shares sold is decreasing in bankruptcy costs* $k$.

When bankruptcy costs are high, bidders take on less debt. The risk of bankruptcy decreases and so, the government can sell fewer shares to benefit more from the value enhancement of those shares it retains. This result implies that we should observe a negative correlation between the number of shares sold by the government and the level of bankruptcy costs.

4 Discussion

4.1 Private benefits and legal protection of shareholders

From Jensen and Meckling (1976) and Grossman and Hart (1980) the idea of private benefits of control plays a central role in the recent theoretical and empirical literature on corporate finance. Private benefits represent some value that is not shared among all the shareholders but that is enjoyed exclusively by the party in control.⁵ Barclay and increase the social welfare. Note that the government would still face the same trade-off: debt increases the price offered but reduces the value of shares kept. From our knowledge, we have never seen such restriction. Imposing leverage for the bidders seems to be very difficult in the reality, because bidders may hide their actual leverage especially when bidders are foreign firms. Moreover, efficient bidders can have to borrow funds because of budget constraints; therefore, imposing leverage could deter too many efficient bidders.

⁵The theoretical literature identifies several sources of private benefits. They can be viewed as the "psychic" value some shareholders attribute simply to being in control (e.g. Harris and Raviv, 1988). Another source of private benefits is the perquisites enjoyed by top executives (Jensen and Meckling, 1976). However, these factors alone cannot justify multimillion dollar bonuses. To generate more sizeable private benefits, the controlling shareholders can use information acquired thanks to their role, to exploit
Holderness (1989) find that on average the value of control (in the US) is worth 20% of the equity value of a firm. Dick and Zingales (2004) estimate the average to be 14% (in 39 countries) with a maximum of 65%.

Let us now assume that the bidder, once he wins the auction, can divert part of the revenues net of the debt, as private benefits. According to the empirical law and finance literature (LaPorta et al., 2000) better legal protection of shareholders increases the difficulty of extracting private benefits. Therefore, we modelize the noncontractible diversion decision as the bidder’s choice of $\phi \in [0, \bar{\phi}]$, where $\bar{\phi}$ is an index of legal protection. The upper-limit $\bar{\phi}$ decreases with the quality of the law. Weak legal protection can be due either to poor quality of the law or to ineffective enforcement.

Security benefits (dividends) are $(1 - \bar{\phi})(y_i - D_i)$ and private benefits are $\bar{\phi}(y_i - D_i)$. Accordingly, the opportunities to extract private benefits increase with the revenues generated, and private benefits extraction does not dissipate value, i.e. it is efficient. Hence, the optimal revenue allocation is straightforward. Unless a successful bidder has acquired all the shares, in which case he is indifferent between any $\phi \in [0, \bar{\phi}]$, he extracts the upper bound $\bar{\phi}$. That is, setting $\phi = \bar{\phi}$ is a successful bidder’s (weakly) dominant strategy.

**Proposition 4.** In regimes with weak legal protection, the bidders take on less debt, the risk of bankruptcy is reduced, and the government sells fewer shares.

**Proof.** See appendix A4. ■

opportunities through another firm they own or are associated with, without sharing the net present value of these opportunities among the remaining shareholders.
The intuition behind this proposition is that private benefits and strategic debt are substitutes. More precisely, strategic debt can be viewed as ex ante expropriation of shareholders while private benefits can be viewed as ex-post expropriation. However, private benefits are more advantageous for the successful buyer than debt because the buyer has to cover part of the debt for the debt-holder, which decreases his net wealth, whereas he enjoys all private benefits in full.

For the government, private benefits have two opposing effects. A negative effect: higher private benefits reduce the value of the shares the government retains. A positive one: higher private benefits reduce the debt level. This first (second) effect induces the government to sell more (fewer) shares. Private benefits are only a monetary transfer without any effect on the total value of the privatized firm, while debt is also a monetary transfer but it reduces the total value of the privatized firm because of bankruptcy costs. We deduce that the positive effect of private benefits is more relevant, therefore the government is better off selling fewer shares when bidders can extract private benefits.

4.2 National preference

Whatever the sales method of SOEs, foreign participation is omnipresent. Boubakri et al. (2004) show that foreign participation as a share of total divestitures in the developing world increased steadily in the 1990s, reaching close to 76% of total privatization proceeds in 1999 and generating an estimated 32.3 billion USD in foreign exchange (World Bank 2001).

The government can derive some political advantage from favoring domestic bidders over foreign ones. In order to address this point, the domestic bidder’s profit is plugged
into the objective function. We analyze how the optimal fraction of shares to be sold is modified when the government wants to favor a domestic bidder over a foreign one. We find that:

**Proposition 5** When the government has national preference, it invariably sells off more shares.

**Proof.** See appendix A5. □

By considering the domestic bidder’s profit in its objective function, the government internalizes the negative impact of the debt more and hence it sells off more shares to reduce the optimal level of debt and so the bankruptcy risk.

### 5 Conclusion

This paper endogeneizes the fraction of shares to be auctioned off when privatizing SOEs. The government faces levered firms, which use their debt strategically to maximize their willingness to pay and so, maximize their probability of winning. Consequently, there is a close correlation between the optimal choice of the fraction of shares to be sold and the level of debt of the winner and hence the risk of bankruptcy.

We have extended our model in various directions. One such extension is found in subsection 4.1. We introduce the possibility for the bidder to extract private benefits. We find that the government is better off selling fewer shares when bidders can extract private benefits since higher private benefits reduce the debt level. In subsection 4.2, we analyze the government’s choice when it favors a domestic bidder over a foreign one. We
find that the government always sells more shares to alleviate the negative impact of the
debt through the bankruptcy risk.

Depending on government preferences between revenues and firms’ profits, different
optimal mechanisms can be derived. The government may benefit by making the number
of shares allocated contingent on the bids. But, if the government does not have this
possibility, and so has to determine the number of shares ex ante, then a simple English
auction is optimal as long as the government has no national preferences. Discriminatory
mechanisms turn out to be optimal in other cases.

Throughout we have ignored the fiscal issue of LBOs. But, debt interest payments
are tax deductible, adding a further advantage to debt financing. We plan to address the
issue of the tax advantage in the context of privatization in our future research.

6 Appendix

6.1 Appendix A1

The differentials of (3) yield:

\[
\frac{dD^*_i}{d\alpha} = \frac{\partial^2 w_i(D_i)}{\partial D_i \partial \alpha} = \frac{\partial^2 w_i(D_i)}{\partial D_i} = \frac{1 - F(D_i)}{-(1 + k - \alpha)f(D_i) - kD_i f'(D_i)} < 0
\]

\[
\frac{dD^*_i}{dk} = \frac{\partial^2 w_i(D_i)}{\partial D_i \partial k} = \frac{\partial^2 w_i(D_i)}{\partial D_i} = \frac{D_if(D_i)}{-(1 + k - \alpha)f(D_i) - kD_i f'(D_i)} < 0
\]

6.2 Appendix A2

The probability of bankruptcy is:
\[ q(\alpha, k) = F(D^*_i) \]

The derivatives are \( \frac{\partial q(\alpha,k)}{\partial \alpha} = f(D^*_i) \frac{dD^*_i}{d\alpha} < 0 \) and \( \frac{\partial q(\alpha,k)}{\partial k} = f(D^*_i) \frac{dD^*_i}{dk} < 0. \)

### 6.3 Appendix A3

Let us begin by deriving the optimal debt level; since the revenues \( y_i \) are drawn independently from a uniform distribution \( F(.) \) on the interval \( Y_i = [0, v_i] \) with density \( f(.) > 0 \), we have:

\[
(1 - \alpha)(1 - F(D^*_i)) - D^*_i k f(D^*_i) = 0
\]

\[
(1 - \alpha) \left(1 - \frac{D^*_i}{v_i}\right) - \frac{D^*_i k}{v_i} = 0
\]

\[
D^*_i = \frac{1 - \alpha}{k + 1 - \alpha v_i}
\]

(Bidder \( i \)'s willingness to pay for \( \alpha \) shares is:

\[
w_i(D^*_i) = \alpha \int_0^{v_i} y_i dF(y_i) + (1 - \alpha) \left(\int_0^{D^*_i} y_i dF(y_i) + \int_{D^*_i}^{v_i} D^*_i dF(y_i)\right) - k \int_0^{D^*_i} y_i dF(y_i)
\]

\[
w_i(D^*_i) = \frac{\alpha v_i}{2} + (1 - \alpha) D^*_i \left(1 - \frac{D^*_i}{2v_i}\right) - k \frac{(D^*_i)^2}{2v_i}
\]

\[
w_i(D^*_i) = \frac{D^*_i [2v_i - D^*_i (1 + k)] + \alpha(D^*_i - v_i)^2}{2v_i}
\]

By replacing \( D^*_i \) by its value in the bidder \( i \)'s willingness to pay, we obtain:
\[ w_i(D_i^*) = \frac{(1-\alpha)v_i}{(k+1-\alpha)v_i} \left[ 2v_i - \frac{(1-\alpha)v_i(1+k)}{k+1-\alpha} \right] + \alpha \left( \frac{(1-\alpha)v_i}{(k+1-\alpha)v_i} - v_i \right)^2 \]

\[ w_i(D_i^*) = \frac{1}{2v_i} \left( v_i^2 \frac{2(1-\alpha)(k+1-\alpha) - (1+k)(1-\alpha)^2}{(k+1-\alpha)^2} + \alpha v_i^2 \left( \frac{k}{k+1-\alpha} \right)^2 \right) \]

\[ w_i(D_i^*) = \frac{2(1-\alpha)(k+1-\alpha) - (1+k)(1-\alpha)^2 + \alpha k^2 v_i}{2(k+1-\alpha)^2} \]

\[ w_i(D_i^*) = \frac{1 - \alpha + k\alpha}{2(k+1-\alpha)v_i} \] (8)

The objective function of the government writes:

\[ \Pi(\alpha) = \int_0^1 w_w(D_w^*(v_w,\alpha), \alpha, v_w) 2g(v_w)[1 - G(v_w)]dv_w \]

\[ + (1 - \alpha) \int_0^1 \left( \int_{D_w^*(v_w,\alpha)}^{v_w} (y_w - D_w^*(v_w,\alpha)) f(y_w)dy_w \right) 2G(v_w)g(v_w)dv_w \] (9)

Since the bidders are symmetric and \(v_i\) are i.i.d. over [0,1] with uniform cumulative \(G(.)\) and density \(g(.)\), the first term writes:

\[ \int_0^1 w_w(D_w^*(v_w,\alpha), \alpha, v_w) 2g(v_w)[1 - G(v_w)]dv_w = \int_0^1 \frac{1 - \alpha + k\alpha}{2(k+1-\alpha)v_w} - v_w 2(1-v_w)dv_w \]

\[ = \frac{1 - \alpha + k\alpha}{6(k - \alpha + 1)} \] (10)

For the second term, let us before make the following calculus:

\[ \int_{D_i^*}^{v_i} (y_i - D_i^*)f(y_i)dy_i = \frac{1}{v_i} \int_{D_i^*}^{v_i} (y_i - D_i^*)dy_i \]

\[ = \frac{v_i}{2} \left( 1 - \frac{D_i^*}{v_i} \right)^2 \] (11)

Hence, the second term writes after replacing \(D_i^*\) by its value:
(1 - \alpha) \int_0^1 \left( \int_{D_w(v_w, \alpha)}^{v_w} (y_w - D_w^*(v_w, \alpha)) f(y_w) dy_w \right) 2G(v_w)g(v_w) dv_w = \frac{k^2(1 - \alpha)}{3(k - \alpha + 1)^2}

Hence, we find:

\Pi(\alpha) = \frac{1 - \alpha + k\alpha}{6(k - \alpha + 1)} + \frac{k^2(1 - \alpha)}{3(k - \alpha + 1)^2} \tag{12}

The FOC is:

\frac{d\Pi(\alpha)}{d\alpha}\bigg|_{\alpha=\alpha^*} = 0 \tag{13}

= \frac{k^2}{6(k - \alpha + 1)^2} - \frac{k^2(-1 + k + \alpha)}{3(k - \alpha + 1)^3}

We derive the maximum \alpha^* = 1 - \frac{k}{3}, since the SOC is satisfied at \alpha^* (\frac{d^2\Pi(\alpha)}{d\alpha^2} = \frac{k^2(1 - \alpha^* - k)}{(1 + k - \alpha^*)^3} < 0).

6.4 Appendix A4

Each bidder’s program is to maximize his willingness to pay with respect to \(D_i\), which is now written:

\[ w_i(D_i) = \int_{D_i}^{v_i} [\tilde{\phi}(y_i - D_i) + \alpha(1 - \tilde{\phi})(y_i - D_i)]dF(y_i) + d_i(D_i) \tag{14} \]

The FOC becomes:
\[(1 - \alpha)(1 - \bar{\phi})(1 - F(D^*_i)) = D^*_i k f(D^*_i) \quad (15)\]

Assuming the SOC is verified, the differential of (15) yields:

\[\frac{dD^*_i}{d\phi} = -\frac{\frac{\partial^2 w_i(D_i)}{\partial D_i \partial \phi}}{\frac{\partial^2 w_i(D_i)}{\partial D_i^2}} = \frac{-(1 - \alpha)(1 - F(D_i))}{-(k + (1 - \alpha)(1 - \phi)) f(D_i) - k D_i f'(D_i)} < 0\]

Following the proof of proposition 3, the result on bankruptcy is straightforward.

The government’s objective function is written:

\[
\Pi(\alpha) = \int_0^1 w_{-w}(D^*_w(v_{-w}, \alpha, \alpha, v_{-w}) 2 g(v_{-w})[1 - G(v_{-w})]dv_{-w} + (1 - \alpha)(1 - \bar{\phi}) \int_0^1 \left( \int_{D^*_w(v_{-w}, \alpha)}^{v_w} (y_w - D^*_w(v_{-w}, \alpha)) f(y_w)dy_w \right) 2 G(v_w)g(v_w)dv_{-w} \quad (16)
\]

Simple manipulations yield:

\[\Pi(\alpha) = \frac{1}{6} \left( 1 + k \left( -1 + \frac{k}{k + (1 - \alpha)(1 - \phi)} \right) \right)
\]

\[+ \frac{k^2(1 - \alpha)(1 - \phi)}{3(k + (1 - \alpha)(1 - \phi))^2} \quad (17)\]

The FOC is:

\[\frac{d\Pi(\alpha)}{d\alpha} = \frac{k^2(1 - \phi)}{6(k + (1 - \alpha)(1 - \phi))^2} + \frac{k^2(1 - \phi)(1 - k - \alpha - \phi(1 - \alpha))}{3(k + (1 - \phi)(1 - \alpha))^3} = 0 \quad (18)\]

We derive the maximum \(\alpha^* = 1 - \frac{k}{3(1 - \phi)}\), since the SOC is satisfied at \(\alpha^* \) \(\frac{d^2\Pi(\alpha)}{d\alpha^2} = \frac{k^2(1 - \phi)^2(1 - k - \alpha^* + \phi(1 - \alpha^*))}{(k + (1 - \alpha^*)(1 - \phi))^4} < 0\). Moreover, we have \(\frac{d\alpha}{d\phi} < 0\).
6.5 Appendix A5

The objective function of the government is now the sum of the price paid, the revenues generated by the winner, and the domestic bidder’s profit. The domestic bidder’s profit is the expected value of its willingness to pay times the probability that it will win the auction. We find that the bidder’s willingness to pay is increasing in $\alpha$:

Since $\frac{\partial w_{-w}(D_{-w}^*,\alpha,s)}{\partial D_{-w}(s,\alpha,s)} = 0$, we have:

$$\frac{dw_{-w}(D_{-w}^*,\alpha,s)}{d\alpha} = \frac{\partial w_{-w}(D_{-w}^*(s,\alpha),\alpha,s)}{\partial \alpha}$$

$$= \int_{D_{-w}}^{v_{-w}} (y_{-w} - D_{-w}) dF(y_{-w}) > 0$$

We deduce that the optimal value of $\alpha$ is greater when the domestic bidder’s profit is a part of the government’s objective function.

References


