On the algebraic estimation of whole two-wheeled vehicles dynamics via High Order Sliding Mode Differentiators

Mohammed El-Habib Dabladji, Dalil Ichalal, Hichem Arioui, Said Mammar

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Abstract— **The present study extends our recent result on the states estimation of two-wheeled vehicles. Based on the estimation of the lateral dynamics with the observer proposed in [1], a second estimator is presented in this paper to reconstruct the longitudinal tire-road forces, engine and braking torques. To this end, the longitudinal dynamics is analyzed in order to recover the tire-road forces without any knowledge of tire parameters. The effectiveness of the proposed approach is shown through simulation results. Concluding remarks wrap up the papier.**

I. INTRODUCTION

The growth of advanced rider assistance systems (ADAS) for motorcycles is an important key issue to answer mortality and seriously wounded of riders worldwide [2]. For standard vehicles, safety systems are developed and operated well in dangerous situations. Unfortunately, these ADAS are no longer suitable for single track vehicles and the direct transposition of the safety systems developed for cars is not obvious.

Several elements may explain the delay in the promotion of safety systems for powered two-wheeled (PTW) vehicles: the low market of two-wheelers and the the investments on R&D which are limited compared to the others vehicles. Moreover, cognitive aspects (users acceptance and ride freedom) are decisive that are often neglected. Finally, complex dynamical aspects leading to a very hard vehicle in term of control and sensitivity [3], [4].

From the control theory point of view, safety issues and the design of ADAS can be seen as a problem of controllers and observers design. Then, our researches aim to the design of passive and active safety systems by using the automatic control theory. These controllers are based on dynamic states which can be measured by mean of sensors and/or observers. These observers are fundamental in perspective of passive or active assistance systems design. In the field of observer design for single track vehicles, there is a few approaches since the application is less attractive. Nevertheless, one can cite the design and implementation of an Extended Kalman Filter (EKF) [5] which takes into account the nonlinear behavior of the system and some stochastic measurement noises. More recently, High Order Sliding Mode technique has been used to estimate rider's torque and the roll angle by using a linearized model of the vehicle [6] with constant longitudinal velocities. Nonlinear observers based on Takagi-Sugeno representation are developed in [7], [8]. All these techniques are based on considering models (linear or nonlinear) of the tire road forces (Pacejka's model [9]) which are empirical models, thus, uncertain.

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In this paper, an approach to estimate the whole dynamics of two-wheeled vehicles is proposed. It is based on algebraic analysis and observation which do not need any model of the tire forces. Thanks to the recent advances in numerical signal differentiation, such as High Order Sliding Mode Differentiators [10], Linear Time Varying differentiator [11] and numerical differentiation technique proposed by M. Fliess in [12], it is possible to estimate asymptotically, in finite time or non asymptotically, respectively, the time derivatives of the measurements. If the states and the unknown inputs of the nonlinear model are algebraically observable, then, it is possible to express the unknown states and inputs as nonlinear functions, only, in term of known measurements and their successive derivatives up to finite orders. Finally, it will be sufficient to use the numerical differentiators to estimate these time-derivatives and, consequently, thanks to the algebraic reformulation of the unknown variables, estimate algebraically the unknown states and unknown inputs. A first step of this technique is developed in [1] for estimating the roll angle, the lateral forces and rider's torque for a two-wheeled vehicle. In the present paper, an extension is proposed to estimate the longitudinal dynamics of the motorcycle, which are required for developing active safety systems based on the reconstruction of mobilized adhesion. In addition, some analysis are given about the tire adhesion and the distribution (ratio) of the braking torque on the front and the rear wheels.

II. PROBLEM STATEMENT

In the context of improving the passive and active safety of motorcycle riders, it seems important to know some parameters or variables in the vehicle dynamics. Some of these variables can easily be measured such as the longitudinal acceleration or the yaw rate. In contrast, other variables are very hard or impossible to measure.

It is well known in motorcycle and bicycle community that longitudinal and lateral forces are very hard to measure or to estimate ; and at the same time, they are seen to be the most important variables that affect the comfort and safety of riders [13].

In this context, the main contribution of this work is to extend the existing works on the estimation of two-wheeled vehicle's lateral dynamics by the estimation of the longitudinal forces. The general scheme of such an observer is presented briefly in figure 1.

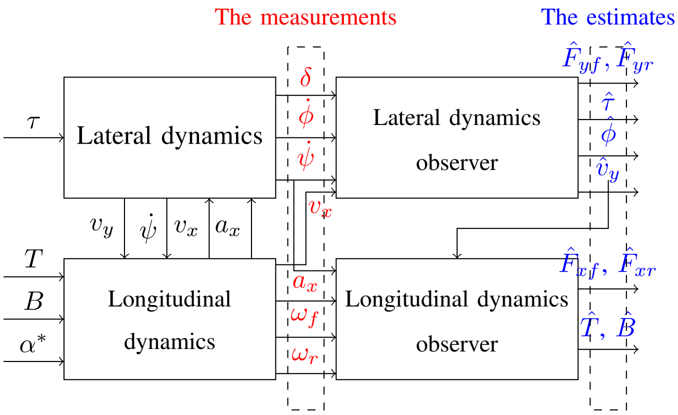

Fig. 1. Overall scheme of the whole two-wheeled vehicles dynamics principle estimation

First, we focus on the estimation of the lateral dynamics. Indeed, the estimation of the lateral dynamics of two-wheeled vehicles has been discussed in several papers [14], [15], [5], [6], [1]. In general, we consider as measurements: the steering angle, the yaw rate, the roll rate and the longitudinal velocity which is seen as a time-varying parameter. The most relevant states (and inputs) to be estimated and which are difficult to measure are the lateral forces, the lateral velocity and the steering torque.

As this first step provided with well estimated dynamics under an acceptable time, we propose in this paper as a second step to estimate the longitudinal forces based on some of the lateral estimates. This paper focuses only on the second. For lateral dynamics estimation, readers can refer to the articles cited above.

The present paper is organized as follows: in the next section, a brief presentation of the lateral dynamics and some of the developed observers is presented. Section IV is devoted to the description of the two-wheeled longitudinal model and the corresponding observer. In section V, we present the obtained simulation results. Finally, we finish by a conclusion and some perspectives.

III. Two-wheeled lateral dynamics description and observation

This section describes briefly the two-wheeled vehicle lateral dynamics model and the corresponding observers.

A. Lateral dynamics

Since observers depend strongly to the accuracy of the used models, the underlying models must be precise. But at the same time, they must be simpler for real-time calculation constraints.

To our knowledge, a good compromise between simplicity and accuracy for the modeling of lateral dynamics of two-wheeled vehicles is Sharp model [16].

Under the following assumptions, we will describe briefly the lateral dynamics model:

- The vehicle is considered as the set of two linked bodies (front and main frame).
- The motorcycle is assumed to be moving on a flat road.
- The rider is considered rigidly linked to the main frame.
- The effect of aerodynamic forces, suspension and pitch motions are neglected.
- From the longitudinal dynamics, only the longitudinal velocity and the longitudinal acceleration are considered affecting the lateral dynamics of the vehicle.

The obtained model involves 4 degrees of freedom (the lateral, the yaw, the lateral and the steering motions).

Thereby, we obtain the following differential equations [16]:

$$\left\{\begin{array}{rcl} F_{yf}+F_{yr} & = & M(\dot{v}_y+v_x\dot{\psi})+M_f k\ddot{\psi}+d_1\ddot{\phi}+M_f e\ddot{\delta} \\ \sum M_z & = & M_f k(\dot{v}_y+v_x\dot{\psi})+a_2\ddot{\phi}+a_3\ddot{\psi}+a_1\ddot{\delta} \\ & - & a_4 v_x\dot{\phi}-d_2 v_x\dot{\delta} \\ \sum M_x & = & d_1\dot{v}_y+b_2\ddot{\phi}+a_2\ddot{\psi}+b_1\ddot{\delta}+b_5 v_x\dot{\psi}+d_3 v_x\dot{\delta} \\ \sum M_s & = & M_f e\dot{v}_y+b_1\ddot{\phi}+a_1\ddot{\psi}+c_1\ddot{\delta}-d_3 v_x\dot{\phi} \\ & + & c_3 v_x\dot{\psi}+K\dot{\delta} \end{array}\right. \tag{1}$$

where:

$$\left\{\begin{array}{rcl} \sum M_z & = & l_f F_{yf}-l_r F_{yr} \\ \sum M_x & = & b_4\sin(\phi)-b_3\sin(\delta) \\ \sum M_s & = & -b_3\sin(\phi)-c_2\sin(\delta)-\eta F_{yf}+\tau \end{array}\right. \tag{2}$$

M_f, M_r and M are the mass of the front, the main frame and the whole vehicle respectively, ϕ is the roll angle, δ is the steering angle, F_{yf} and F_{yr} are the lateral front and rear forces respectively and τ is the torque applied on the handlebar. More details on the motorcycle parameters and expressions are given in appendix A.

The lateral forces F_{yf} and F_{yr} can be modeled in a quasi-static or a dynamic way (by introducing the relaxation length). Moreover, many different quasi-static models are to be found in the literature such as the linear form [16] or the pacejka model [9]. For brevity, this part has not been discussed in this paper.

B. Observer design

Several works can be found in the literature to estimate the lateral dynamics of two-wheeled vehicles. For the estimation of the roll angle and the steering angle, readers can refer to [14], [15], [5]. For the estimation of the lateral tire forces, the only works in this area can be found in [6], [7], [1].

This is not the aim of this paper. Due to lake of space, we have deliberately omitted this part.

Remark 1: In what follows, it is assumed that the lateral velocity is estimated using the observer proposed in [1].

IV. Two-wheeled longitudinal dynamics description and estimation

This section is devoted to the modeling of the longitudinal dynamics. After, we will propose an algorithm to estimate the longitudinal forces. The only restriction for the observation part is to know in addition to the longitudinal acceleration and the rotational speed of wheels, the ratio of braking torque on the two tires during the deceleration phase, and to suppose that during acceleration phase, the engine torque is applied only to rear wheel.

A. *Modeling of longitudinal dynamics*

In this subsection, an analytic model of the longitudinal dynamics is derived from the single-corner model [13], [17].

In addition to the assumptions considered for modeling the lateral dynamics, we consider here that:

- The lateral velocity is well estimated from the first observer.

Now, according to figure 2, the following model is deduced:

$$\left\{\begin{array}{rcl} M(a_x - v_y\dot{\psi}) & = & F_{xf} + F_{xr} \\ i_{fy}\dot{\omega}_f & = & -R_f F_{xf} - B_f \\ i_{ry}\dot{\omega}_r & = & -R_r F_{xr} + E - B_r \end{array}\right. \tag{3}$$

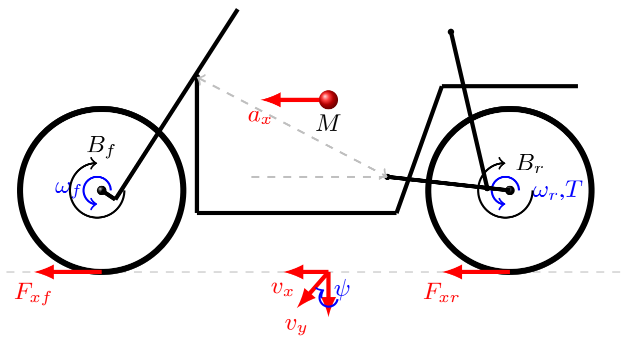

Fig. 2. Geometrical representation of a motorcycle model with longitudinal dynamics

B_f and B_r are the braking torques applied to the front and the rear wheel respectively. E is the engine torque. We suppose that the engine torque is applied only to the rear wheel. The signification of the other parameters of the model are given in appendix A.

As said before, the ratio of braking torque between the two wheels is considered known. We consider $B_f = \alpha B$ and $B_r = (1-\alpha)B$ with $\alpha \in [0,1]$ and B is the new braking torque to distribute between the two wheels. Thus, the model (3) can be writing in the following form:

$$\left\{\begin{array}{rcl} M(a_x - v_y\dot{\psi}) & = & F_{xf} + F_{xr} \\ i_{fy}\dot{\omega}_f & = & -R_f F_{xf} + \alpha T \\ i_{ry}\dot{\omega}_r & = & -R_r F_{xr} + (1-\alpha) T \end{array}\right. \tag{4}$$

where:

$$T = \left\{\begin{array}{cc} E & \text{acceleration phase} \\ -B & \text{braking phase} \end{array}\right. \tag{5}$$

and

$$\alpha = \left\{\begin{array}{cc} 0 & \text{acceleration phase} \\ \alpha^* & \text{braking phase} \end{array}\right. \tag{6}$$

and α^* is ratio of braking torque between the two wheels.

It is easy to measure the longitudinal acceleration thanks to accelerometer, the yaw rate from a gyroscope sensor and the wheels angular speed with optical encoders. In fact, these sensors are available on motorcycles equipped with antilock braking systems (ABS). Thus, the more difficult task consists on the estimation of the longitudinal forces and the braking and engine torques. This is the aim of the next subsection.

B. *Study of detectability*

Before speaking about the estimation of the longitudinal dynamics, let us recall some definitions about the observability and the detectability of systems with unknown inputs.

Definition 1: [18] Consider the following nonlinear system with $x(t)$ is the state vector, $u(t)$ is the known inputs vector, $w(t)$ the unknown inputs (UI) vector and $y(t)$ is the measurements vector:

$$\left\{\begin{array}{rcl} \dot{x}(t) & = & f(x(t), u(t), w(t)) \\ y(t) & = & h(x(t), u(t), w(t)) \end{array}\right. \tag{7}$$

For every initial condition $x(0)$, any known input $u(t)$ and and any couple of UI $(w(t), \bar{w}(t))$, the nonlinear system (7) with two different trajectories $x(t)$ and $\bar{x}(t)$ is called:

- state strongly observable: if $y(t, x(t), u(t), w(t)) = y(t, \bar{x}(t), u(t), \bar{w}(t))$ implies that: $x(t) = \bar{x}(t)$.
- state strongly detectable: if $y(t, x(t), u(t), w(t)) = y(t, \bar{x}(t), u(t), \bar{w}(t))$ implies that: $x(t) \to \bar{x}(t)$ as $t \to \infty$.
- state strongly asydetectable: if $y(t, x(t), u(t), w(t)) \to y(t, \bar{x}(t), u(t), \bar{w}(t))$ as $t \to \infty$ implies that: $x(t) \to \bar{x}(t)$ as $t \to \infty$).

Remark 2: In [18], strong asydetectability has been called strong* detectability.

Definition 1 speaks only about the state observability or detectability. The unknown input observability (detectability) relates to the possibility of reconstruct the UI uniquely infinite-time (asymptotically) having as information the known inputs and outputs.

Definition 2: [19] For every initial condition $x(0)$ and any known input $u(t)$, the nonlinear system (7) is called:

- state and UI strongly observable: if $y(t, x(t), u(t), w(t)) = y(t, \bar{x}(t), u(t), \bar{w}(t))$ implies that: $x(t) = \bar{x}(t)$ and $w(t) = \bar{w}(t)$.
- state and UI strongly detectable: if $y(t, x(t), u(t), w(t)) = y(t, \bar{x}(t), u(t), \bar{w}(t))$ implies that: $x(t) \to \bar{x}(t)$ and $w(t) \to \bar{w}(t)$ as $t \to \infty$.
- state strongly asydetectable: if $y(t, x(t), u(t), w(t)) \to y(t, \bar{x}(t), u(t), \bar{w}(t))$ as $t \to \infty$ implies that: $x(t) \to \bar{x}(t)$ $w(t) \to \bar{w}(t)$ as $t \to \infty$).

Let us check the UI strong observability (detectability or asydetectability) of the model giving by (3).

We consider that the longitudinal dynamics state vector ($[\omega_f(t), \omega_r(t)]^T$) is measurable. Moreover, since a_x and $\dot{\psi}(t)$ are considered measurable and $v_y(t)$ is considered well estimated from the lateral dynamics observer (asymptotically), then we can say that $F_{xf} + F_{xr}$ is also known asymptotically (from (4)). So, the vector $z(t) = [\omega_f(t), \omega_r(t), F_{xf} + F_{xr}]^T$ is known asymptotically.

Now, applying the definition giving for the state and unknown input strong asydetectability. Consider the trajectory of the system (3) and another trajectory:

$$\left\{\begin{array}{rcl} M(\bar{a}_x - \bar{v}_y\dot{\bar{\psi}}) & = & \bar{F}_{xf} + \bar{F}_{xr} \\ i_{fy}\dot{\bar{\omega}}_f & = & -R_f \bar{F}_{xf} - \bar{B}_f \\ i_{ry}\dot{\bar{\omega}}_r & = & -R_r \bar{F}_{xr} + \bar{T} - \bar{B}_r \end{array}\right. \tag{8}$$

and suppose that $z(t) \to \bar{z}(t)$ when $t \to \infty$. Thus, we obtain:

$$\begin{cases} \tilde{F}_{xf} + \tilde{F}_{xr} \to 0 \\ -R_f \tilde{F}_{xf} - \tilde{B}_f \to 0 \\ -R_r \tilde{F}_{xr} + \tilde{T} - \tilde{B}_r \to 0 \end{cases} \tag{9}$$

where: $\tilde{F}_{xf} = F_{xf} - \bar{F}_{xf}$, $\tilde{F}_{xr} = F_{xr} - \bar{F}_{xr}$, $\tilde{T} = T - \bar{T}$, $\tilde{B}_f = B_f - \bar{B}_f$ and $\tilde{B}_r = B_r - \bar{B}_r$

We see that the obtained system has $(\tilde{F}_{xf} + \tilde{F}_{xr}) \to 0$ as unique solution, but $(\tilde{F}_{xf}, \tilde{F}_{xr}) \to (0, 0)$ is not the unique one. Then, the longitudinal forces cannot be detectable with such a model. This motivates our constraints to consider the ratio of braking torque to be known.

In the case where this ratio is known, we obtain the following system of equation:

$$\begin{cases} \tilde{F}_{xf} + \tilde{F}_{xr} \to 0 \\ -R_f \tilde{F}_{xf} + \alpha \tilde{U} \to 0 \\ -R_r \tilde{F}_{xr} + (1-\alpha)\tilde{U} \to 0 \end{cases} \tag{10}$$

Now, we see that the only solution of the above system of equations is $[\tilde{F}_{xf}, \tilde{F}_{xr}, \tilde{U}]^T \to 0$. So, if the ratio of braking torque is known, the longitudinal forces become strongly asydetectable i.e. they will be estimated asymptotically.

C. Longitudinal dynamics reconstruction

Now, we'll continue the design of the cascaded observer by the estimation of the longitudinal forces. We consider that the longitudinal acceleration, the angular velocities of the wheels and the yaw rate are measured. The lateral speed is considered well estimated from the first observer. The diagram of the proposed observer is given in figure 1.

From (4), thanks to the first algebraic equation which is known and since all the state vector $x = [\omega_f, \omega_r]^T$ is measurable. So, as seen before, the system is strongly asydetectable.

From (4), the system can be writing in the following form:

$$\underbrace{\begin{pmatrix} 1 & 1 & 0 \\ -R_f & 0 & \alpha \\ 0 & -R_r & 1-\alpha \end{pmatrix}}_{D(\alpha)} \begin{pmatrix} F_{xf} \\ F_{xr} \\ U \end{pmatrix} = \begin{pmatrix} M(a_x - v_y \dot{\psi}) \\ i_{fy}\dot{\omega}_f \\ i_{ry}\dot{\omega}_r \end{pmatrix} \tag{11}$$

One can notice that the matrix $D(\alpha)$ is nonsingular for all values of $\alpha \in [0, 1]$. The yaw rate and the longitudinal acceleration are given by appropriate sensors. The lateral velocity is obtained from the lateral dynamics observer (figure 1). Hence, to estimate the longitudinal forces, we request the knowledge of only the derivatives of the state vector.

Note that considering the tire forces as unknown inputs is more consistent with real constraints, because they depend on several parameters (tire pressure, tire adhesion, mass transfer between the two tires, etc.). In fact, it is not necessary in this paper to known the tire parameters or dynamics.

Since the rotational tire velocities ω_f and ω_r and their first time derivatives are bounded, several algorithms can be used to estimate the first derivatives like the super-twisting algorithms [10], Linear Time Varying differentiator [11] or numerical differentiation technique proposed by M. Fliess in [12]. In this work, we use the super-twisting algorithms [10].

$$\begin{cases} \dot{v}_{11} &=& v_{12} - \lambda_{11}|v_{11} - \omega_f|^{\frac{1}{2}}\mathrm{sign}(v_{11} - \omega_f) \\ \dot{v}_{12} &=& -\lambda_{12}\mathrm{sign}(v_{12} - \dot{v}_{11}) \\ \dot{v}_{21} &=& v_{22} - \lambda_{21}|v_{21} - \omega_r|^{\frac{1}{2}}\mathrm{sign}(v_{21} - \omega_r) \\ \dot{v}_{22} &=& -\lambda_{22}\mathrm{sign}(v_{22} - \dot{v}_{21}) \end{cases} \tag{12}$$

sign refer to the sign function. λ_{ij} are positive scalars and are chosen according to the limits of the derivatives of ω_f and ω_r (see [10]).

Consider $v = [v_{12}, v_{22}]^T$ the estimates of $[\dot{\omega}_f, \dot{\omega}_r]^T$. From these estimates and the equation (11), we can estimate the longitudinal forces and the engine or braking torque by the following equation:

$$\begin{pmatrix} \hat{F}_{xf} \\ \hat{F}_{xr} \\ \hat{U} \end{pmatrix} = D^{-1}(\alpha) \begin{pmatrix} M(a_x - \hat{v}_y \dot{\psi}) \\ i_{fy}v_{12} \\ i_{ry}v_{22} \end{pmatrix} \tag{13}$$

Thus, we have estimated the longitudinal forces and the engine or braking torque without any knowledge of the tire parameters.

V. Simulation Results

The results shown in this paper refer to a simulation test. The nonlinear model used for the simulation includes the coupled longitudinal and lateral dynamics. The tire forces are modeled by the magic formula of Pacejka. The simulations are carried out for a lane change maneuver.

The simulation results are presented first for the case without measurement noises and with the exact knowledge of the mass and the geometric parameters of wheels. Thus, we obtain figures 3, 4 and 5.

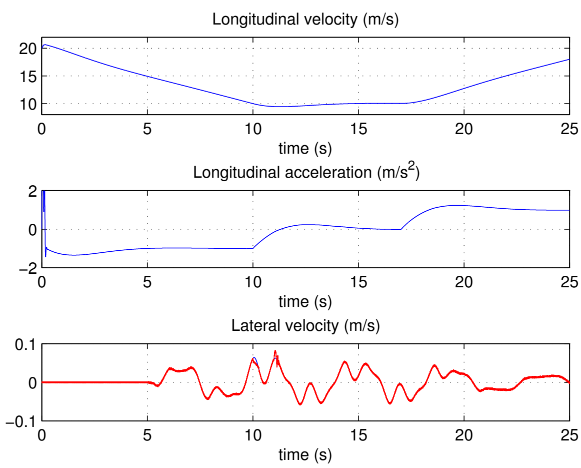

Fig. 3. (From top to bottom) Longitudinal velocity, longitudinal acceleration and Lateral velocity (blue) and its estimate (red)

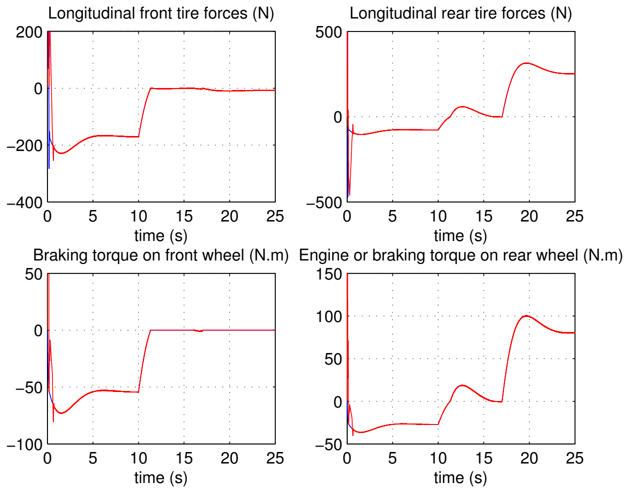

Fig. 4. **Without noises**(Top) Front and rear longitudinal tire forces (blue) and their estimates (red), (bottom) Front and rear engine or braking torques (blue) and their estimates (red)

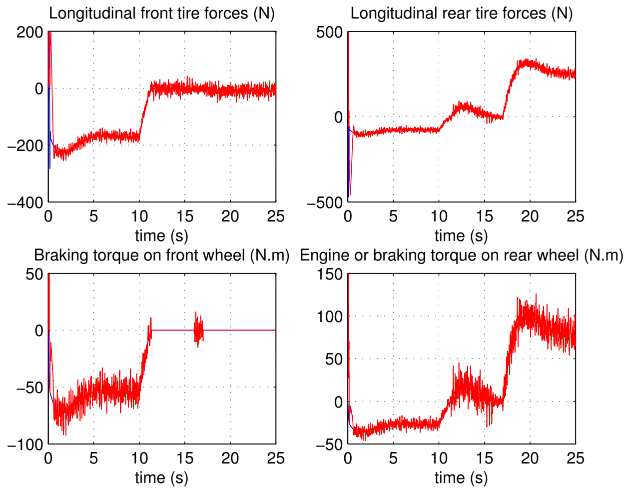

Fig. 6. **With noises** (Top) Longitudinal front and rear tire forces (blue) and their estimates (red), (bellow) Front and rear engine or braking torques (blue) and their estimates (red)

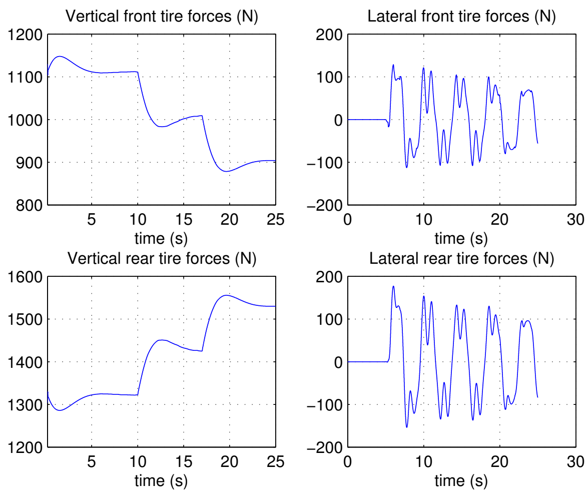

Fig. 5. (Left) Front and rear vertical forces, (right) Front and rear lateral forces

From the figure 3, we notice that the proposed scenario includes three phases. Before $10s$, we are in braking phase and the longitudinal acceleration is about $-1m/s^2$. From $10s$ to $17s$, no braking or engine torque is applied. After $17s$, the vehicle is in an acceleration phase. We see also that the lateral velocity is well estimated from the first observer. From figure 4, we observe that the estimation of the longitudinal tire forces and the engine and braking torques are estimated exactly and in finite time. Figure 5 shows the variations of the lateral and vertical forces for the corresponding scenario.

Now, if the measurements are noisy, the results of simulation are given in figure 6 where each measure is considered affected by a random and centered signal of magnitude 8% of its maximum.

The effect of noises is apparent but the results remain acceptable. Of course, it is well known that the super-twisting algorithm is very sensitive to measurement noises and we have a compromise in the choice of the differentiator gains. If they are chosen sufficiently large, we will have good and fast estimation, but the observer will be very sensitive to noises. And in the other case, the observer will be less sensitive to noises but the estimation of the unknown signals will not be accurate.

In figure 7, we check the situation when the mass M or the geometric parameters of the tires are not known exactly. Let us choose for the observer part: $\hat{M} = 0.9 * M$, $\hat{i}_{fy} = 0.9 * i_{fy}$ and $\hat{i}_{ry} = 1.1 * i_{ry}$. Because the reconstruction of the unknown inputs (the longitudinal forces and torques) is done algebraically without any feedback law, it is obvious that the reconstruction of these inputs is sensitive to the knowledge of these parameters.

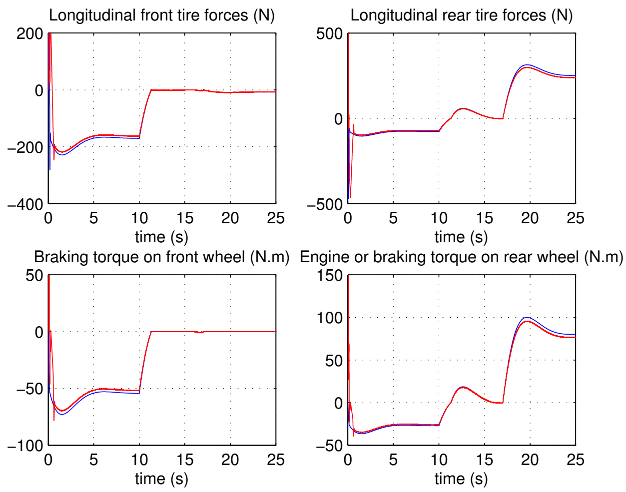

Fig. 7. **Without noises and with errors on the geometric parameters** (Top) Longitudinal front and rear forces (blue) and their estimates (red), (bottom) Front and rear engine or braking torques (blue) and their estimates (red)

Finally, we will study the influence of the variation of the ratio of braking torque α^* on the longitudinal adhesion on each tire.

Recall that the longitudinal (lateral) adhesion μ_x (μ_y) on a tire is equal to the ratio between the longitudinal (lateral) tire force and the vertical force on the same tire. The longitudinal and lateral adhesion must always be less than a friction limit depending on the tire and the conditions of the road. Moreover, the longitudinal and lateral adhesion must satisfy the following inequality [13]:

$$\mu_x^2 + \mu_y^2 \leq \mu_{max} \leq 1 \tag{14}$$

In figure 8, we propose the simulation results of the longitudinal adhesion for different values of braking torque ratio α^*.

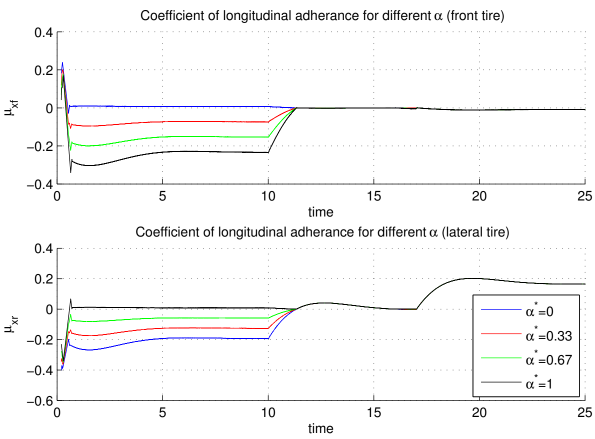

Fig. 8. Front and rear longitudinal adhesion for different values of α^*

From figure 8, we notice that the variation of α^* affects directly the adhesion requested by each tire. This can be seen for future work as a degree of freedom to perform the safety of two-wheeled vehicles in active way.

VI. Conclusion and Perspectives

In this paper, an estimator was proposed to reconstruct the longitudinal dynamics of two-wheeled vehicles. The lateral dynamics were considered estimated from an independent lateral observer. Without the knowledge of the ratio of braking torque, we proved the undetectability of the longitudinal tire forces. When this ratio is known, an estimator was proposed to reconstruct the longitudinal tire forces and engine and braking torques. Simulations were given to show the effectiveness of the proposed approach.

This work associated with previous results on the estimation of two-wheeled lateral dynamics allows us to estimate all tire forces. Moreover, by considering the braking torque's ratio and its effect on the longitudinal adhesion, future works will be devoted to enhance the safety of motorcycles.

TABLE I

Motorcycle dynamic parameters signification	
v_x , v_y	longitudinal and lateral velocities
ϕ , ψ , δ	roll, yaw and steering angles
F_{yf} , F_{yr}	front and rear lateral forces
F_{xf} , F_{xr}	front and rear longitudinal forces
F_{zf} , F_{zr}	front and rear vertical forces
τ	steering torque
M_f , M_r , M	mass of the front frame, the rear frame and the whole motorcycle
K	damper coefficient of the steering mechanism
a_x	Longitudinal acceleration
ω_f , ω_r	Front and rear wheel angular speed
B_f , B_r , E	Front and rear brake and engine torques
R_f , R_r	Front and rear tire radii
i_{fy} , i_{ry}	Front and rear rotational inertia
The other parameters a_i , b_i , c_i and d_i are described in table II	

TABLE II

Motorcycle parameters expressions and numerical values

parameters a_i , b_i , c_i and d_i
$a_1 = M_f ek + I_{fz} \cos\epsilon$
$a_2 = M_f jk - C_{rxz} + (I_{fz} - I_{fx}) \sin\epsilon \cos\epsilon$
$a_3 = M_f k^2 + I_{rz} + I_{fx} \sin^2\epsilon + I_{fz} \cos^2\epsilon$, $a_4 = \frac{i_{fy}}{R_f} + \frac{i_{ry}}{R_r}$
$b_1 = M_f ej + I_{fz} \sin\epsilon$
$b_2 = M_f j^2 + M_r h^2 + I_{rx} + I_{fx} \cos^2\epsilon + I_{fz} \sin^2\epsilon$
$b_3 = \eta Z_f - M_f eg$, $b_4 = (M_f j + M_r h) g$
$b_5 = M_f j + M_r h + \frac{i_{fy}}{R_f} + \frac{i_{ry}}{R_r}$, $c_1 = I_{fz} + M_f e^2$
$c_2 = (\eta Z_f - M_f eg) \sin\epsilon$, $c_3 = M_f e + \frac{i_{fy}}{R_f} \sin\epsilon$
$d_1 = M_f j + M_r h$, $d_2 = \frac{i_{fy}}{R_f} \sin\epsilon$, $d_3 = -\frac{i_{fy}}{R_f} \cos\epsilon$
Parameters a_i , b_i , c_i and d_i contain the geometrical parameters of the motorcycle and their signification are illustrated in [16]
Numerical values
$M_f = 30.65kg$, $M_r = 217.45kg$, $M = M_f + M_r$
$K = 6.77N.s/rad$, $I_{rx} = 31.18kg/m^2$, $I_{rz} = 21.07kg/m^2$
$g = 9.81m/s^2$, $C_{rxz} = 1.74kg/m^2$, $I_{fx} = 1.24kg/m^2$
$I_{fz} = 0.44kg/m^2$, $i_{fy} = 0.72kg/m^2$, $i_{ry} = 1.05kg/m^2$
$a = 0.949m$, $e = 0.024m$, $f = 0.028m$, $h = 0.616m$
$R_f = 0.305m$, $R_r = 0.305m$, $\eta = 0.116m$, $\epsilon = 0.47°$
$l_r = 0.585m$, $l_f = 0.829m$

Appendix

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