Analytical Modeling of Transient Process In Terms of One-Dimensional Problem of Dynamics With Kinematic Action

V Kravets, K Bas, T Kravets, Lyudmila Tokar

Analytical Modeling of Transient Process In Terms of One-Dimensional Problem of Dynamics With Kinematic Action

V.Kravets[1a], K.Bas[1], T.Kravets[2] & L. Tokar[1]

1 – State Higher Educational Institution “National Mining University”, Dnipropetrovsk, Ukraine

2 – Dnipropetrovsk National University of Railway Transport, Dnipropetrovsk, Ukraine

a – prof.w.kravets@gmail.com

Keywords: material system, kinematics action, mathematical model, analytical solution, characteristic equation, dynamic design

ABSTRACT. One-dimensional dynamic design of a component characterized by inertia coefficient, elastic coefficient, and coefficient of energy dispersion. The component is affected by external action in the form of time-independent initial kinematic disturbances and varying ones. Mathematical model of component dynamics as well as a new form of analytical representation of transient in terms of one-dimensional problem of kinematic effect is provided. Dynamic design of a component is being carried out according to a theory of modal control.

Introduction. Analytical modeling is the essential stage of technical system dynamic design followed by computational and full-scale experiment [0, 0]. Analytical modeling of dynamic systems is based upon traditional mathematical methods of solutions of differential equation systems [0], theory of modal control [0], root-locus technique [0], and root-locus method [0].

Free motion dynamics of one-dimensional mechanical system experiences analytical study in a work by Kravets [0]; forced motion dynamics in terms of external dynamic effect was considered in a work by Kravets [0]. The paper models forced motion dynamics in terms of external kinematic effect.

Formulation of the problem. Fig.1 demonstrates dynamic scheme of one-dimensional mechanical system.

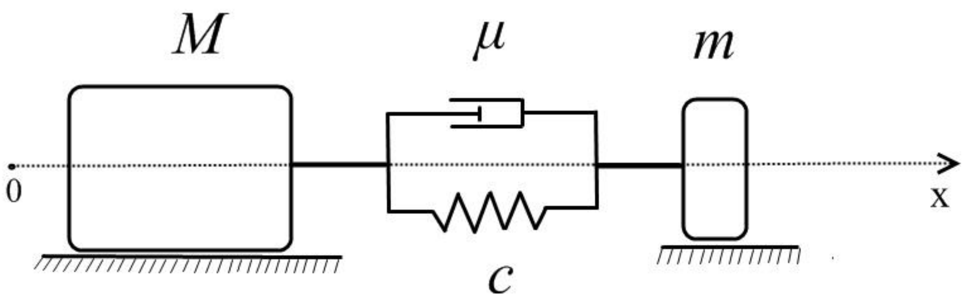

Fig. 1. Dynamic scheme of kinematic effect problem

Here M and m are interacting masses, c is coefficient of elasticity, μ is damping coefficient.

It is assumed that $M >> m$. m mass is finite and specified. $x(t)$ motion of m mass cannot effect $a(t)$ motion of M mass. Notion of M mass is supposed as specified function of $a(t)$ time. For example: $a(t) = V_{0t}$ where V_0 is M mass velocity.

It is required to develop $x(t)$ analytical solution modeling stable transient and determining steady motion of m mass depending upon such running design parameters of mechanical system as μ and c.

Mathematical model. Continuous dynamic model with single degree of freedom is described with the help of following matrix differential equation:

$$\begin{Vmatrix} \dot{x_1} \\ \dot{x_2} \end{Vmatrix} = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} \begin{Vmatrix} x_1 \\ x_2 \end{Vmatrix} + \begin{Vmatrix} f_1(t) \\ f_2(t) \end{Vmatrix} \tag{1}$$

where $x_1 = \dot{x}(t)$; $x_2 = x(t)$.

For mechanical system under consideration, the equation coefficients are determined as follows:

$$a_{11} = -\frac{\mu}{m},\ a_{12} = -\frac{c}{m}, \tag{2}$$
$$a_{21} = 1,\ a_{22} = 0\ .$$

Power function for kinematic effect is identified in the form of:

$$f_1(t) = \frac{\mu}{m}\dot{a}(t) + \frac{c}{m}a(t) \tag{3}$$
$$f_2(t) = 0\ .$$

Analytical solution. Following normalized form for analytical solution x(t) determining motion of mass m is as follows:

$$x(t) = \frac{e^{\lambda_1 t}}{\lambda_1 - \lambda_2}\begin{vmatrix} \lambda_2 - & \dot{x_0} \\ 1 - & x_0 \end{vmatrix} + \frac{e^{\lambda_1 t}}{\lambda_1 - \lambda_2}\sum_1^\infty \frac{1}{\lambda_1{}^n}\,\mathrm{f}_{(0)}{}^{(\mathrm{n}-1)} - \frac{1}{\lambda_1 - \lambda_2}\sum_1^\infty \frac{1}{\lambda_1{}^n}\mathrm{f}_{(\mathrm{t})}{}^{(\mathrm{n}-1)} \tag{4}$$
$$+\frac{e^{\lambda_2 t}}{\lambda_2 - \lambda_1}\begin{vmatrix} \lambda_1 - & \dot{x_0} \\ 1 - & x_0 \end{vmatrix} + \frac{e^{\lambda_2 t}}{\lambda_2 - \lambda_1}\sum_1^\infty \frac{1}{\lambda_2{}^n}\,\mathrm{f}_{(0)}{}^{(\mathrm{n}-1)} - \frac{1}{\lambda_1 - \lambda_2}\sum_1^\infty \frac{1}{\lambda_2{}^n}\,\mathrm{f}_{(\mathrm{t})}{}^{(\mathrm{n}-1)}.$$

Here analytical solution is represented in the form of dependence on the roots of characteristic equation: λ_1, λ_2; specified initial disturbances $\dot{x}_0$ x_0; specified external power effect within the initial time period *f*(0) and current one *f(t).*

Analytical modeling. If external kinematic effect is specified as: $a(t) = V_0 t$ then considering $\dot{a}(t) = V_0$ that both function and its derivatives are:

$$f(t) = \frac{\mu}{m}V_0 + \frac{c}{m}V_0 t\ ,\ f(0) = \frac{\mu}{m}V_0\ , \tag{5}$$
$$\dot{f}\,(t) = \frac{c}{m}V_0\ ,\ \dot{f}\,(0) = \frac{c}{m}V_0\ ,$$
$$\ddot{f}\,(t) = 0\ ,\ a(t).$$

Hence:

$$\sum_1^\infty \frac{1}{\lambda_1^n} f_{(0)}^{(n-1)} = -V_0;\ \sum_1^\infty \frac{1}{\lambda_2^n} f_{(0)}^{(n-1)} = -V_0; \tag{6}$$
$$\sum_1^\infty \frac{1}{\lambda_1^n} f_{(t)}^{(n-1)} = V_0(\lambda_2 t - 1);\ \sum_1^\infty \frac{1}{\lambda_2^n} f_{(t)}^{(n-1)} = V_0(\lambda_1 t - 1).$$

Substituting the results into general formula and performing simple transformations we obtain laconic analytical representation of transient:

$$x(t) = \frac{\lambda_1 e^{\lambda_2 t} - \lambda_2 e^{\lambda_1 t}}{\lambda_1 - \lambda_2} x_0 + \frac{e^{\lambda_2 t} - e^{\lambda_1 t}}{\lambda_1 - \lambda_2} (V_0 - \dot{x}_0) + V_0 t \,. \qquad (7)$$

In case of complex roots of characteristic equation: $\lambda_{1,2} = \alpha \pm i\beta$ transient is modeled by means of following function:

$$x(t) = e^{\alpha t} \left[\begin{vmatrix} \alpha - \dot{x}_0 \\ 1 - x_0 \end{vmatrix} \frac{\sin \beta t}{\beta} + x_0 \cos \beta t \right] - e^{\alpha t} V_0 \frac{\sin \beta t}{\beta} + V_0 t \,. \qquad (8)$$

According to the given different forms of transient records, following particular cases of root distribution are being modeled:

1. λ_1, $\lambda_2 = 0$;
2. $\lambda_1 = \lambda_2 = \alpha$, $\beta = 0$;
3. $\lambda_{1,2} = \pm i\beta$, $\alpha = 0$.

In the context of particular case one, transient is modeled as:

$$x(t) = \frac{e^{\lambda t}}{\lambda} (\dot{x}_0 - V0) + \frac{1}{\lambda} (V0 - \dot{x}_0 + \lambda x0) + V0t. \qquad (9)$$

In the context of particular case two assuming $\lambda_1 \to \lambda_2$ or $\lambda_1 - \lambda_2 \to 0$, i.e. $\Delta\lambda \to 0$ and considering that:

$$\lim_{\Delta\lambda \to 0} \frac{1 - e^{\Delta\lambda t}}{\Delta\lambda} = -t, \qquad (10)$$

We obtain following transient:

$$x(t) = e\lambda t[(\dot{x}_0 - \lambda x_0)t + x_0 - V_0 t] + V_0 t \qquad (11)$$

Assuming that $\beta \to 0$, i.e. $\lambda_1 = \lambda_2 = \alpha$ and considering that:

$$\lim_{\beta \to 0} \frac{\sin \beta t}{\beta} = t, \lim_{\beta \to 0} \cos \beta t = 1 \qquad (12)$$

We obtain transient in its equivalent record:

$$x(t) = e^{\alpha t} \left[\begin{vmatrix} \alpha - \dot{x}_0 \\ 1 - x_0 \end{vmatrix} t + x_0 - V_0 t \right] + V_0 t. \qquad (13)$$

In the context of case three transient is described with the help of following time function:

$$x(t)=\left[\begin{vmatrix} 0- \dot{x}_0 \\ 1-x_0 \end{vmatrix} - V_0\right] \frac{\sin\beta t}{\beta} + x_0 \cos\beta t + V_0 t. \quad (14)$$

Analytical design. Dynamic design of mechanical systems is to select running design parameters depending upon required transient quality: aperiodic transient or vibration one; degree of stability; oscillation frequency and amplitude; control time etc. Transient performance depends on distribution of characteristic equation roots within complex plane. Adequate distribution of characteristic equation roots is achieved by selection of running parameters of mechanical system. For linear dynamic systems analytical selection is possible.

Characteristic equation of one-dimensional dynamic system is:

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = 0 \quad (15)$$

Roots of characteristic equations depend on coefficients of differential equations as follows:

$$\lambda_1 + \lambda_2 = a_{11} + a_{22}\ , \lambda_1 \cdot \lambda_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}. \quad (16)$$

For the involved mechanical system running parameters are directly determined by the formulas:

$$\mu = -m(\lambda_1 + \lambda_2),\ c = m \cdot \lambda_1 \cdot \lambda_2\ . \quad (17)$$

In terms of complex roots we obtain:

$$\mu = -2\, m\alpha,\ c = m(\alpha^2 + \beta^2), \quad (18)$$

where α is degree of stability; and β is factor of natural frequency.

In the context of particular case one we determine:

$$\mu = -2m\lambda_1, \text{c=0}, \quad (19)$$

i.e. elastic element is not available in the mechanical system.

In the context of particular case two we determine:

$$\mu = -2\, m\lambda, c = m\lambda^2 \quad (20)$$

or

$$\mu = -2\, m\alpha, c = m\alpha^2 \quad (21)$$

In the context of particular case three we determine:

$$\mu = 0, c = m\beta^2, \tag{22}$$

i.e. damping component is not available in the mechanical system.

Summary. New record of analytical solutions of linear differential equations in harmonic form is proposed. The form is applied for analytical modeling and design of one-dimensional dynamic system in terms of kinematic effect. Qualitatively different forms of transients within one-dimensional mechanical system as well adequate running design parameters of elastic and damping elements have been obtained.

References

[1] Khachaturov, A.A. 1976. Dynamics of a road-railcar-driver system (in Russian). – Moscow: Mechanical Engineering, 535 P.

[2] Hubka, W. 1987. Theory of technical systems (translation from German language).– Moscow: Mir, 208 P.

[3] Smirnov, V.I. 1974. A course of higher mathematics (in Russian).– Moscow: Nauka V.2, 656 P.

[4] Kuzovkov, N.T. 1976. Modal control and monitoring facilities (in Russian). – Moscow: Mechanical Engineering, P. 184.

[5] Udermann, E.T. 1972. Root-locus method in the theory of automatic systems (in Russian). – Moscow: Nauka, 448 P.

[6] Kravets, V.V. 1978. Dynamics of solid bodies system in the context of complex control (in Russian). – Kyiv: Applied Mechanics, Issue 7, P. 125-128.

[7] Kravets, V.V.., Bas K.M., Kravets, Vl.V. 2012. Dynamic design of the simplest vehicle component (in Russian). – Sevastopol: Messenger of SebNTU, Issue 135, P. 188-191.

[8] Kravets, V.V.., Bas K.M., Kravets, Vl.V., Burov, V.S. 2014. Analytical method of the simplest vehicle component dynamic design in terms of external effect (in Russian). – Sevastopol: Scientific Messenger of the First Ukrainian Marine Institute, Issue 1, P. 79-82.