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HAL Id: hal-01301646
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Gini Index for Evaluating Bus Reliability Performances for Operators and Riders

Neila Bhouri
Université Paris-Est ; IFSTTAR/COSYS/GRETTIA,
14-20 Boulevard Newton Cité Descartes, Champs sur Marne
F-77447 Marne la Vallée, Cedex 2 - France
Phone: +33 1 81 66 86 89
Email: neila.bhouri@ifsttar.fr

Maurice Aron
Université Paris-Est; IFSTTAR/COSYS/GRETTIA,
14-20 Boulevard Newton Cité Descartes, Champs sur Marne
F-77447 Marne la Vallée, Cedex 2 - France
Phone: +33 1 81 66 86 87
Email: maurice.aron@ifsttar.fr

Gérard Scemama
Université Paris-Est ; IFSTTAR/COSYS/GRETTIA,
14-20 Boulevard Newton
Cité Descartes, Champs sur Marne
F-77447 Marne la Vallée, Cedex 2 - France
Email: gerard.scemama@ifsttar.fr

Word count: 5662
Nr of Figures: 3
Nr of Tables: 2
Submission date: August, 1, 2015
Revised submission date: 
ABSTRACT

Reliability of bus travel time is an objective of major concern for bus operators and users. However, due to traffic conditions and variability in bus demand, deviations from schedules are unavoidable, leading to an overall decrease in level of service and capacity. The ability to accurately and effectively analyze various performance measures is fundamental to determining how well the bus service is adhering to its service standards. Understanding and developing methods to assess transit operations performance is not only valuable in identifying potential problem areas along a route; it is also constructive in proposing effective strategies to improve service reliability. A state-of-the-art and a state-of-practice studies of the service regularity measures for high-frequency buses, we propose in this paper the use of the Lorenz curve as a tool for the analysis of bus regularity. We show that the Lorenz curve, if drawn for the scheduled headways, allows analysis of the regularity of the waiting time of all riders of the bus line and is a good measure from the users’ point of view. The Lorenz curve drawn for the ratio of the observed to the scheduled headway allows analysis of the regularity of the bus service with regard to the scheduled headways and is therefore an effective measure, mainly from the operator’s point of view.

1. INTRODUCTION

Reliability of bus travel time is an objective of major concern for the different stakeholders of the public transport (PT) system: service provider, customers and the community. According to the service provider, namely the operator of the PT system, the main interest is to provide the service in a way to satisfy the demand, respecting the operations’ constraints. However, irregularities in travel time imply bus bunching deteriorating the regularity of operations. For users, the first criterion that appears to interest them is the regularity/reliability of travel time. The ‘regularity’ criterion is a contributing factor in the identification of the route to adopt (1). Finally, for the community or public service authority, the interest is the attractiveness of the bus line it invests in. A regular bus service has a direct positive impact on how passengers perceive availability and timeliness of service but headway irregularity discourages use of public transit (2).

Due to traffic conditions and variability in bus demand, deviations from schedules are unavoidable leading to an overall decrease in level of service and capacity. The ability to accurately and effectively analyze various performance measures is fundamental to determining how well the bus is adhering to its service standards. Understanding and developing methods to assess performance of transit operations is not only valuable in identifying potential problem areas along a route, it is also constructive in proposing effective strategies to improve service reliability.

The state-of-the-art of reliability indicators (3, 4) shows many different indicators. However, the measures that are in use or those developed in theory for bus reliability are usually unsatisfactory for service regularity measures of high-frequency buses. Generally, they are not expressed on a normalized scale and therefore cannot be used to compare one route with another. Furthermore, for the most part, they are not immediately or intuitively understandable for senior management or non-expert external stakeholders.

Henderson et al. (5) proposed a Regularity index based on the Gini ratio which is independent of the headways length. This key performance index was described as difficult to understand and to use (6). The Gini index used in economic studies to measure the inequality of revenues and health among the population (7) is based on the Lorenz curve (8). We show in this paper that the Gini index although a good measure of bus line regularity is not sufficient for its analysis. Therefore, the Lorenz curve is a useful tool for this analysis. We show also, that when the Lorenz curve is drawn for the scheduled headways, it is a relevant indicator for the equity of the waiting time. But it may not be an interesting
indicator for the operator who establishes the timetable not on the criterion of equality between users but rather to satisfy the total demand. The operator is more interested by the respect of the scheduled headways given by the timetable. We propose in this paper the use of a Lorenz curve based on the ratio between the observed and scheduled headways for the analysis of the regularity of buses.

The following section of the paper gives a literature review of the state-of-the-art and the state-of-practice of the indicators used by the operators to assess the PT reliability. Section 3 explains the Gini ratio and the Lorenz curve. In section 4 we give the demonstration showing that the Lorenz curve based on the scheduled headways is equivalent to the Lorenz curve based on the riders’ waiting time if the headways are established so as to have the same number of riders in each bus. We also demonstrate that the Lorenz curve based on the ratio of observed to scheduled headways is a good indicator of the adherence of buses to scheduled headways. In section 5, we give first results for different key performance indicators, and after, the Lorenz curves of different buses, drawn for the scheduled and the ratio of observed to scheduled headways. We show how the analysis of these curves results in a better understanding of the regularity of the PT line from the point of view of riders and operators. Finally, section 6 gives the conclusion and perspectives.

2. LITERATURE REVIEW OF METHODS USED TO ASSESS PT RELIABILITY

There are different methods to assess the operating quality of PT service. Different indicators are available in the literature. For each of these indicators, one can use different statistics and different reporting methods.

As given by the “Transit Capacity and Quality of Service Manual” (6), the most common measures of reliability used by transit operators are:

a) On-time performance indicators measure the degree to which vehicles arrive at the scheduled times,

b) Headway adherence indicators measure the consistency or “evenness” of the interval between transit vehicles,

c) Missed trips, and

d) Distance traveled between mechanical breakdowns.

The first two indicators are time-based and the last two ones are distance-based service quality indicators. The focus in this paper is on time-based quality indicators: on-time performance refers to “Adherence to timetable”. It is generally measured for “low frequency” routes through a punctuality indicator. The headway adherence indicator is used when vehicles run at frequent intervals (less than 10 minutes).

These indicators are also used by European quality certification institutions such as CEN, AFNOR and AENOR. In Europe generally, regularity/punctuality is often calculated according to the European “quality of service” norm (EN 13 816), defined in the years 2000: the percentage of passengers affected by services that do not respect the timetable within an interval of 2 or 3 minutes of the announced time; and the percentage of passengers affected by late arrival services of no greater than 3 or 5 minutes and early arrival of 1 minute (measured at 59 seconds). The minimum percentage required by the norm is 80%. I can also be 90% or even 95% depending on the decisions of the local authority. The values differ depending on the type of lines or routes. The approach towards certification enables requirements to evolve as improvements are made. The results are of course highly dependent on the urban context, the traffic situation, and the structural improvements carried out on the line or route.
These same indicators are used by the majority of members of the International Bus Benchmarking Group (IBBG) as given in (9). IBBG is a program of international benchmarking of bus operations and PT. It is made up of a consortium of twelve international bus organizations, to provide a forum for a number of medium sized and large bus organizations from different parts of the world to share experiences, compare performance, identify best practices and learn from one another (10, 11). As summarized in (12), the majority of IBBG members are using either Headway Adherence or the Timetable Regularity; only “London buses” is using Excess Waiting Time (EWT).

**Headway Adherence (HA).** The definition of the HA and the way to measure it vary according to the authors:

- (6) uses $\text{CoV}_D$, a kind of Coefficient of Variation of headways- in order to measure the HA:

$$\text{CoV}_D = \frac{\text{The standard deviation of headway deviations}}{\text{the average of scheduled headways}}$$

(1)

The "Headways Deviations" in Equation 1, are the deviations of observed headways compared to scheduled headways. A low $\text{CoV}_D$ corresponds to a high HA; it takes into account all headway deviations.

- According to (9) and (13), the HA is the percentage of headways that deviate no more than a specific amount from the scheduled interval. This specific amount is a percentage of the headway.

**The Timetable Regularity**, sometimes referred to as “Wait Assessment”, is computed as the service regularity but where the specific amount is no longer a percentage but an absolute number of minutes.

**Waiting time:** for frequent service, passengers are assumed to go to the stop without expectations of boarding a particular bus at a particular time. Passengers’ waiting time depends on the scheduled headways and the regularity of service. The average wait increases as service regularity decreases. The observed average waiting time formula given by Equation 3 is due to Welding (13) and has been reformulated by (15). Equation 4 is proposed by (16).

$$\text{AWT (Observed)} = \frac{\sum_{i=1}^{N} h_i^2}{2 \sum_{i=1}^{N} h_i}$$

(3)

$$\text{AWT (Observed)} = \frac{H}{2} (1 + \text{CoV}_O^2)$$

(4)

Where $h_i$ is the $i^{th}$ observed headway, $H$ is the mean observed headway, $\text{CoV}_O$ is the coefficient of variation of the observed headways and $N$ is the number of headways.

Planners at London Transit use a measure, mathematically independent of the headway, called the Excess Waiting Time (EWT). EWT is based on the average passenger waiting time. The EWT is defined as the difference between the real average passenger waiting time and the scheduled average passenger waiting time.

$$\text{AWT (Scheduled)} = \frac{H_S}{2} (1 + \text{CoV}_S^2)$$

(5)

Where $H_S$ is the average scheduled headway and $\text{CoV}_S$ is the Coefficient of Variation of scheduled headways.
\[ EWT = AWT \text{ (Observed)} - AWT \text{ (Scheduled)} \]  

(6)  

Other metrics are often given which are extreme values, the minimum and maximum values of measures and the coefficient of variation (CoV), the standard deviation (STD) and the percentage outside of a "comfort zone". STD and CoV expressions indicate the spread of travel time around some expected value; CoV is an interesting metric because it is dimensionless. Golshani (17) proposed the standard deviation of headway, as well as an irregularity index, which gives an indication of long gaps between vehicles. This irregularity index is defined in the same way as the Average Waiting Time AWT (Equation 3). It is defined as the ratio of the mean square headway to the mean headway squared.

\[ IR = \frac{1}{N} \sum_{i=1}^{N} (h_i^2) \left( \frac{1}{N} \sum_{i=1}^{N} h_i \right)^{-2} \]  

(7)  

Literature review of PT reliability statistics can be summarized by the table given in (18).

The measures that are in use or those developed in theory are usually unsatisfactory for service regularity measures of high-frequency buses because they are not expressed on a normalized scale and therefore cannot be used to compare one route with another.

The index presented by Henderson et al. (5) is useful to evaluate the bus headway regularity. It is a good indicator from the users’ point of view, especially for high-frequency buses where users do not plan their arrival time, the most important criterion is therefore the headway regularity. From the operators’ point of view however, even for high-frequency buses, the regularity of buses is not the main concern. Operators may plan nearly regular headways or nearly regular headways on small time intervals. Establishing a timetable is not an easy target and depends on many factors such as the number of passengers, the layover time, the availability of the bus driver, etc. Planning a timetable with regular intervals, even for short headway bus lines is not the optimal solution from the point of view of the operator, as there are many other constraints which the bus operator has to face. From their point of view, a regular bus is the bus which respects the timetable. For this purpose, a good evaluating index is the one which assesses the bus service with regard to the timetable. We propose in this paper a Gini index based on the ratio between the observed and scheduled headways.

### 3. GINI INDEX

The Gini index, also called Gini coefficient or ratio, is used in economic studies mainly to measure the inequality of incomes and health among the population (7). It is a measure of statistical dispersion intended to represent the income distribution of a nation’s residents. The Gini coefficient is a relative measure. It is possible for the Gini coefficient of a developing country to rise (due to increasing inequality of income) while the number of people in absolute poverty decreases.

The Gini coefficient is usually defined mathematically based on the Lorenz curve. A Lorenz curve plots the cumulative percentages of total income received against the cumulative number of the population, starting with the poorest individual or household (see Figure 1, red colors). The Gini coefficient can be thought of as the ratio of the area that lies between the line of equality and the Lorenz curve (the red dotted surface marked A on Figure 1) over the total area under the line of equality (the pink surface marked B on Figure 1); i.e., the Gini coefficient GC = A / (A + B). Thus a Gini index of zero expresses perfect equality, where all values are the same (for example, where everyone has the same income). A Gini coefficient of one (or 100%) expresses maximal inequality.
among values (for example, where only one person has all the income or consumption, and all others have none).

\[ GC = A / (A + B) \]. Since \( A + B = 0.5 \), the Gini index is \( GC = 2A \), or \( GC = 1 - 2B \).

If the Lorenz curve is represented by the function \( Y = L(X) \), the value of \( B \) can be found with integration:

\[
B = \int_0^1 L(X) dX
\] (8)

The Gini is an inequality index. As we are interested in bus regularity, we use the regularity index that we call “Gini”, rather than the Gini index (GC)

\[
Gini = 1 - GC = 2 * B = 2 * \int_0^1 L(X) dX
\] (9)

We can show that Equation 9 is equivalent to:

\[
Gini = cov(X, F(X)) \cdot \frac{2}{X}
\] (10)

Where \( cov \) is the covariance function, \( F(X) \) is the cumulative distribution function, \( \bar{X} \) is the mean value of \( X \).

In our paper "Gini" will be computed for 3 different \( X \) vectors: (1) \( X \) is the vector of scheduled headways (then the index is called Gini_S). (2) \( X \) is the vector of observed headways (the index is called Gini_O). (3) Finally \( X \) is the vector of Headway Ratios \( HR = (HR_1, HR_2, \ldots HR_n, \ldots HR_N) \):

\[
HR_n = \frac{The \ nth \ observed \ headway}{The \ nth \ scheduled \ headway} = \frac{h_i^0}{h_i^S}
\] (11)

In this latter case the index is called Gini_R.

Replacing \( X \) in equation 10, the Gini index is given by

\[
Gini = \frac{2 \sum_{i=1}^{N} (h_i - H).i}{N^2.H}
\] (11)

Where, \( h_i \) is the headway (scheduled for Gini_S, observed for Gini_O, and Headway Ratios for Gini_R), \( N \) is the number of headway observations, \( H \) is the mean value, \( i \) is the rank of the headway or of the headway ratio.

4. SCHEDULED HEADWAY REGULARITY INDEX AND WAITING TIME

Let us consider the Lorenz curve based on the scheduled headways. The horizontal axis represents the cumulative proportion of buses, sorted from buses with the shortest headways to buses with the longest. The vertical axis represents the cumulative proportion of the total headways of the individual buses as they are arrayed on the horizontal axis. The diagonal line is the function that describes perfectly regular service with equal headways for all buses.
We show that the Lorenz curve based on the scheduled headways is identical to the Lorenz curve plotting the waiting time when the timetable is established to get the same number of users in each bus (except for the passengers of the first bus of the day, for which no headway is attached) and is therefore a good indicator from the riders’ perspective.

Let us assume, for the sake of simplicity, that the scheduled headways between the consecutive bus number i - 1 and i are independent of the bus stop on the bus route; this means that there is one unique headway $h_i$ for bus i at all the bus stops. Assuming that the bus operator establishes the scheduled headways so that all buses take the same number of passengers ($N_p$), equal to the forecasted traffic demand for the time corresponding to the headway and that passengers go to the stop randomly without expectations of boarding a particular bus. The cumulative percentage of headways or of buses (on the Y-axis of the Lorenz curve) is thus equal to the cumulative percentage of passengers taking these buses. The mean waiting time of a passenger is equal to half of the headway; the total waiting time of the passengers of each bus $i$ is equal to the number of passengers by bus multiplied by half of the headway (Equation 10).

\[ w_i = N_p \cdot \frac{h_i}{2} \]  

(10)

Let us sort the buses according to their headway (beginning by the bus with the smallest headway); the waiting time for all passengers of buses 1 to i are given by Equation 12:

\[ W_{[1,i]} = \sum_{j=1}^{i} N_p \cdot \frac{h_j}{2} = \frac{N_p}{2} \sum_{j=1}^{i} h_j \]  

(12)

Where [1,i] is the interval from 1 to i. The total waiting time for the whole day on the bus line, is:

\[ W_r = \frac{N_p}{2} \sum_{j=1}^{N} h_j \]  

(13)

The cumulative percentages of the waiting time spent up to the $i^{th}$ bus, related to the total waiting time, is:

\[ W_{[1,i]} / W_r = \frac{N_p}{2} \sum_{j=1}^{i} h_j / \frac{N_p}{2} \sum_{j=1}^{N} h_j = \frac{\sum_{j=1}^{i} h_j}{\sum_{j=1}^{N} h_j} \]  

(14)

The Lorenz curve plotting $W_{[1,i]} / W_r$, the cumulative percentages of waiting time against the cumulative percentages of the total population waiting for buses, starting with the smallest waiting time, is thus identical to the Lorenz curve plotting the cumulative percentages of scheduled headways. Thus the Gini_S index represents exactly the waiting time equity.

5. GINI INDEX FOR THE COMPARISON OF THE REGULARITY OF BUSES

When operators establish their timetables they are well aware of the irregularity of scheduled headways; so they do not need to analyze it. They are more interested by the adherence of the observed headways to the scheduled ones; that adherence can be analyzed thanks to the Lorenz curve based on the ratios {observed/scheduled} headways; the adherence is assessed by the value of the corresponding Gini index, named Gini_R. The Gini_R is appropriate when the public transport company controls the bus headways. It is less appropriate when the company controls the schedules. As other indicators, the Gini_R must be used carefully by operators; its interest is to prioritize the actions to highlight the more severe problems.
Remark. The denominators of the ratios used in the Gini_R (respectively the CoV_D) are the scheduled headways (respectively the average scheduled headway). When increasing the number of buses, for example, multiplying by two the bus frequency, the scheduled headways are then divided by two (this can be the case when two bus lines have a common trunk and there are twice as many buses in the common trunk than at the ends of the lines). Assuming the same distribution of the headway deviations, for both cases the ratios used in the Gini_R and the CoV_D are multiplied by two; this indicates that the same absolute deviation costs twice as much relative to the headway. This has a sense when assuming that the doubled frequency is due to twice as many users. However, for comparing two (or more) lines with different frequencies, it is better to have the same index value, whatever the frequency; this is obtained by replacing the observed headway $h_i$ by its relative value $h_i/N_i$ and multiplying the ratios used in the Gini_R by $N_i$.

For instance $N_0$ is the minimum number of observations in any one of the lines. This value of $N_0$ implies that the numerator of the indexes remains positive.

### 6. RESULTS

Table 1 gives the values of different indicators for the scheduled (Gini_S), observed (Gini_O) and the ratio observed to scheduled (Gini_R) headways. In a longer paper, one can give results for more indicators such as Headway Adherence, Excess Waiting Time, etc. On Table 2, we give the bus line number, ranked for each indicator from the smallest to the greatest value of the indicator.

**Table 1: Indicators value for fifteen different bus lines**

<table>
<thead>
<tr>
<th>Line</th>
<th>Mean_S</th>
<th>Mean_O</th>
<th>Gini_S</th>
<th>Gini_O</th>
<th>Gini_R</th>
<th>STD_S</th>
<th>STD_O</th>
<th>COV_S</th>
<th>COV_O</th>
<th>Wait_S</th>
<th>Wait_O</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>8.2</td>
<td>9.94</td>
<td>0.81</td>
<td>0.7</td>
<td>0.77</td>
<td>3.26</td>
<td>5.25</td>
<td>0.4</td>
<td>0.53</td>
<td>4.75</td>
<td>6.36</td>
</tr>
<tr>
<td>L2</td>
<td>8.95</td>
<td>8.27</td>
<td>0.83</td>
<td>0.71</td>
<td>0.76</td>
<td>2.88</td>
<td>4.39</td>
<td>0.32</td>
<td>0.53</td>
<td>4.94</td>
<td>5.3</td>
</tr>
<tr>
<td>L3</td>
<td>9.02</td>
<td>8.72</td>
<td>0.8</td>
<td>0.68</td>
<td>0.74</td>
<td>3.4</td>
<td>5.21</td>
<td>0.38</td>
<td>0.6</td>
<td>5.15</td>
<td>5.92</td>
</tr>
<tr>
<td>L4</td>
<td>10.47</td>
<td>11.18</td>
<td>0.77</td>
<td>0.72</td>
<td>0.76</td>
<td>4.33</td>
<td>5.58</td>
<td>0.41</td>
<td>0.6</td>
<td>6.13</td>
<td>6.98</td>
</tr>
<tr>
<td>L5</td>
<td>10.88</td>
<td>11.6</td>
<td>0.74</td>
<td>0.7</td>
<td>0.78</td>
<td>5.11</td>
<td>6.26</td>
<td>0.47</td>
<td>0.53</td>
<td>6.64</td>
<td>7.56</td>
</tr>
<tr>
<td>L6</td>
<td>11.39</td>
<td>10.68</td>
<td>0.89</td>
<td>0.89</td>
<td>0.88</td>
<td>2.94</td>
<td>2.27</td>
<td>0.26</td>
<td>0.21</td>
<td>6.07</td>
<td>5.58</td>
</tr>
<tr>
<td>L7</td>
<td>12.52</td>
<td>10.76</td>
<td>0.85</td>
<td>0.89</td>
<td>0.91</td>
<td>3.43</td>
<td>2.23</td>
<td>0.27</td>
<td>0.21</td>
<td>6.73</td>
<td>5.61</td>
</tr>
<tr>
<td>L8</td>
<td>12.8</td>
<td>14.8</td>
<td>0.84</td>
<td>0.97</td>
<td>0.94</td>
<td>4.01</td>
<td>1.1</td>
<td>0.31</td>
<td>0.07</td>
<td>7.03</td>
<td>7.44</td>
</tr>
<tr>
<td>L9</td>
<td>13.79</td>
<td>10.14</td>
<td>0.88</td>
<td>0.84</td>
<td>0.85</td>
<td>4.6</td>
<td>3.12</td>
<td>0.33</td>
<td>0.31</td>
<td>7.66</td>
<td>5.55</td>
</tr>
<tr>
<td>L10</td>
<td>13.92</td>
<td>14.42</td>
<td>0.85</td>
<td>0.75</td>
<td>0.79</td>
<td>4.35</td>
<td>6.56</td>
<td>0.31</td>
<td>0.46</td>
<td>7.64</td>
<td>8.7</td>
</tr>
<tr>
<td>L11</td>
<td>14.33</td>
<td>14.45</td>
<td>0.9</td>
<td>0.79</td>
<td>0.81</td>
<td>2.84</td>
<td>5.4</td>
<td>0.2</td>
<td>0.37</td>
<td>7.45</td>
<td>8.23</td>
</tr>
<tr>
<td>L12</td>
<td>14.44</td>
<td>15.6</td>
<td>0.85</td>
<td>0.73</td>
<td>0.81</td>
<td>5.02</td>
<td>7.91</td>
<td>0.35</td>
<td>0.51</td>
<td>8.09</td>
<td>9.81</td>
</tr>
<tr>
<td>L13</td>
<td>16.71</td>
<td>16.82</td>
<td>0.84</td>
<td>0.81</td>
<td>0.93</td>
<td>5.23</td>
<td>5.61</td>
<td>0.31</td>
<td>0.33</td>
<td>9.18</td>
<td>9.35</td>
</tr>
<tr>
<td>L14</td>
<td>19.78</td>
<td>19.00</td>
<td>0.95</td>
<td>0.89</td>
<td>0.92</td>
<td>1.66</td>
<td>3.92</td>
<td>0.08</td>
<td>0.21</td>
<td>9.96</td>
<td>9.91</td>
</tr>
<tr>
<td>L15</td>
<td>21.41</td>
<td>21.67</td>
<td>0.94</td>
<td>0.99</td>
<td>0.99</td>
<td>3.26</td>
<td>0.52</td>
<td>0.15</td>
<td>0.02</td>
<td>10.95</td>
<td>10.84</td>
</tr>
</tbody>
</table>

Where *_S: stands for Scheduled; *_O: Observed and *_R for the ratio between observed and scheduled headway. STD: Standard Deviation, COV: Coefficient of Variation, Wait: Waiting Time (Equation 4).
Table 2: Bus line number, ranked for each indicator from the smallest to the greatest value of the indicator

<table>
<thead>
<tr>
<th>Gini_S</th>
<th>Gini_O</th>
<th>Gini_R</th>
<th>STD_S</th>
<th>STD_O</th>
<th>COV_S</th>
<th>COV_O</th>
<th>Wait_S</th>
<th>Wait_O</th>
</tr>
</thead>
<tbody>
<tr>
<td>L5</td>
<td>L3</td>
<td>L3</td>
<td>L14</td>
<td>L15</td>
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One can notice on Table 2, that the rank of the lines is different for each indicator, which means that each one of these indicators gives an evaluation from a different point of view of the bus line. The analysis of the regularity of the bus line on the basis of these indicators is not an easy task. The advantage of the Gini method is the possibility of analysis given by the Lorenz curve and not only the use of the Gini-coefficient.

Figure 1: Lorenz curves based on scheduled headways of three different bus lines.

Lorenz curves are plotted for the scheduled headways for three different bus lines on Figure 1. As explained in Section 4, these curves indicate the riders’ waiting time equity (Equation 14). The Lorenz curve of bus L3 is the farthest from the bisector, one can conclude that bus L3 presents more inequity compared to the two other buses: Gini_S indexes are of 0.854 for L1, 0.840 for L2 and 0.744 for L3. Thirty percent of headways of bus L3 correspond to about 14% of the total waiting time, while it represents 25% for bus L1. As explained previously, the Lorenz curve is given in a normalized scale. Therefore, the Lorenz curves on Figure 1, based on the scheduled headways, allows for the comparison of the equity of all users of the bus line. But, in order to compare the theoretical waiting times between the two bus lines, one has to multiply the normalized waiting time represented on the...
Y-axes by the duration of the bus service in the day, then divide the result, for each line, by two times
the number of the bus frequency on the line.

One can also draw the Lorenz curve of the observed headway. But the signification of observed and
scheduled headway curves is different. The observed headway curves mix both the equity of
scheduled headways and the respect of buses of these scheduled headways. It is not useful to draw the
observed headways Lorenz curves, except for highlighting, if any, the difference between scheduled
and real headways. Note a drawback in the comparison between the Lorenz curves of scheduled and
observed headways: a lack of difference between both curves can be misleading, since the scheduled
and observed headways are independently sorted (the bus corresponding to the $i^{th}$ shortest scheduled
headway is not necessarily the bus corresponding to the $i^{th}$ shortest observed headway).

The analysis of Figure 1 is interesting from the riders’ point of view but not from the operator’s point
of view. Having equity of scheduled headways on a bus line is not the first concern of a bus operator.
As previously explained, the bus operator establishes the timetable in order to satisfy the demand,
while taking into account all other constraints. As the number of passengers is not equal at all times of
the day, it is normal to have inequality of headways. For the bus operator, the diagnosis of the respect
of the scheduled headways is more relevant than the equality of scheduled headways.

Figure 2 shows two Lorenz curves, the first one corresponds to the scheduled headways and the
second corresponds to the Lorenz curve based on the ratios of observed to scheduled headway for the
same bus line. Figure 3 gives the Lorenz curve based on the ratios for three different bus lines. The
numerator and denominator of each ratio address the same bus, so the inconvenience highlighted for
the first Lorenz curves and Gini index disappears.

As one can see on Figure 2, the equity in reliability of the bus line is better than the equity in
scheduled headway. The Lorenz curve of the ratio of observed to scheduled headway is closer to the
bisector than the Lorenz curve of the scheduled headway.

The scheduled headway curve shows two main parts: a first approximately straight line from the
origin ($x=0; y=0$) up to nearly $x=78\%$ of headways (corresponding to $y=65\%$ of the cumulative
headways) with a slope of about 0.83, and a second one for the last 22% of headways with a slope of
1.59. This means that the line has two modes, possibly a rush period with short headways
corresponding to a large number (78%) of scheduled buses, and longer headways in 22% of buses
(possibly off-peak). The Lorenz curve corresponding to the ratio shows also two periods, for 70% of
cases the slope is of 0.93% and for 30% of cases the slope is of 1.17. This means that 70% of headways were shorter than scheduled - in this case the observed headway is on average the scheduled one multiplied by 0.93. This also means that 30% of headways are larger than scheduled (on average 1.59 times the scheduled one). But this does not provide information about the absolute length of the delay nor about the time of occurrence. It is possible to have one bus with a larger headway than scheduled, followed by another bus with a shorter headway than scheduled.

On Figure 3, the Lorenz curve of the ratios is drawn for three different lines. This allows analyzing the equity for each one of the bus lines. The comparison (in the sense of headway adherence) on a same graphic of the reliability of the different lines is correct only if the average ratios are the same for all lines. For a whole day and a given line, although headways shorter and larger than scheduled compensate, the average ratio, \[ \frac{1}{N} \sum_{j=1}^{N} \left( \frac{h_{observed}}{h_{scheduled}} \right) \] is not necessarily equal to 1, the formula not being linear. However on checking, it was found that this average ratio is very close to the value 1 for all the bus lines, thus very close to each other, therefore the comparison is right. As one can see from the figure, bus line L2 is more regular than bus line L1, which in turn is more regular than bus line L3. The Gini index is of: Gini_S(L1)=0.85, Gini_S(L2)=0.89 and Gini_S(L3)= 0.7.

The line L2 has three different parts, which can be approximated by three straight lines:

(1) the first one has a low slope of 0.2 (the straight line begins at x=0 and y=0 and ends at x=13%, y=2.6% ). This means that 13 % of observed headways are shorter than the scheduled headways; in this part the average ratio, equal to the slope of 0.2%, means that the observed headways are five times less than the scheduled ones, possibly indicating bus bunching.

(2) a larger part of the curve may be represented by a second straight line from x=13%, to x=93%, its slope is of (90% - 2.6%)/(93% - 13%)=1.09, very close to the value 1. This means that 80% of observed headways are very close to scheduled ones (slope 1.09) and 7% of observed headways are greater than the scheduled ones.

(3) a last small straight line with a slope of (100% - 90%)/(100% - 93%) =1.42, which means that 7% of observed headways are greater than the scheduled ones, with an average interval equal to 1.42 times the scheduled headway. This concerns operators as well as riders.

Line L1 has two distinguishable parts: for a large percentage of buses, the observed headways are smaller than the scheduled ones; for a smaller percentage of buses they are greater. The “convexity” of the Lorenz curve of L3 shows a continuous evolution of the slope, from a slope of 0.4 at the beginning, for headways shorter than scheduled, to a final slope (when x and y are equal to 1) of 1.75,
indicating observed headways 1.75 times larger than scheduled, this also concerns operators as well as riders.

The Lorenz curve based on headway ratios drawn for each bus stop of the bus line may be relevant information for operators as it shows the regularity of the bus (with respect to headway adherence) depending on the rank of the stop in the line or on its position in the city, its size, its strategic importance in the network, etc.. This allows analyzing the reliability as a function of the cartography of the line (i.e. central part versus periphery part, etc.).

For lines where multiple routes or branches come together to form one trunk service along major arterial routes, it is useful to draw the Lorenz curve and to compute the Gini index separately for each branch.

7. CONCLUSION

We present in this paper a state-of-the-art and a state-of-practice studies of the service regularity measures of high-frequency buses. The majority of measures that are in use, or those developed in theory are unsatisfactory because they are not expressed on a normalized scale and therefore cannot be used to compare one route with another. The other shortcomings of service regularity measures is that they are not immediately or intuitively understandable for non-expert stakeholders. Also as shown in this paper (Tables 1 and 2), each key performance gives a point of view on the regularity and does not allow a real analysis of the regularity of the bus line.

We propose in this paper the use of the Lorenz curve as a tool for the analysis of two kinds of regularity of buses. The Lorenz curve based on the scheduled headways evaluates the regularity of the intervals. We show that, if the bus company built the timetable so as to satisfy the same number of users for each bus, the Lorenz curve based on scheduled headways allows the analysis of the regularity of the waiting time of all riders. This concerns mainly riders, but also operators, constrained to propose quality service for all end-users. The main concern of the operator is the empirical adherence in the established timetable. The Lorenz curve based on the ratios of the observed to the scheduled headways allows for analysis of the regularity of a line with respect to scheduled headways; it quantifies the occurrences of bus bunching and of headways much larger than scheduled, which concerns mainly operators, but also riders. Plotting on the same graph the Lorenz curves of different lines results in the comparison of their reliability. It is therefore an effective measure from the operator’s point of view.

REFERENCES


