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Non-linear vibrations of shells: A literature review from 2003 to 2013

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The present literature review focuses on geometrically non linear free and forced vibrations of shells made of traditional and advanced materials. Flat and imperfect plates and membranes are excluded. Closed shells and curved panels made of isotropic, laminated composite, piezoelectric, functionally graded and hyperelastic materials are reviewed and great attention is given to non linear vibrations of shells subjected to normal and in plane excitations. Theoretical, numerical and experimental studies dealing with particular dynamical problems involving parametric vibrations, stability, dynamic buckling, non stationary vibrations and chaotic vibrations are also addressed. Moreover, several original aspects of non linear vibrations of shells and panels, including (i) fluid structure interactions, (ii) geometric imperfections, (iii) effect of geometry and boundary conditions, (iv) thermal loads, (v) electrical loads and (vi) reduced order models and their accuracy including perturbation techniques, proper orthogonal decomposition, non linear normal modes and meshless methods are reviewed in depth.

1. Introduction

Shell structures are used in aircraft, spacecraft, rockets, cars, computers, submarines, boats, storage tanks and the roofs of buildings. In the last decade, the continuous development of material science and engineering, along with the increasing demand to produce light weight structures, has led to the use of advanced materials (laminated composites and functionally graded materials (FGMs)) in designing shell structures. Among diverse fields of application, aerospace and aeronautical applications are particularly challenging as they involve fluid structure interactions and the use of new materials with less known properties. Therefore, models should be developed that take into account non linear effects, such as large structural deflections, and predictions of the structural responses to large amplitude excitations.

In case of linear vibrations of closed (circumferentially complete) circular cylindrical shells subjected to radial periodic excitations, the mode shape of the shell is a standing wave directly excited by the external excitation, which presents a number n of nodal diameters. This is known as the *driven mode*. For large amplitude vibrations of shells there is an intrinsic one to one internal resonance between the driven mode and the orthogonal mode having the same shape and natural frequency as the driven mode, but rotated by $\pi/2n$, known as the *companion mode*. This mode interaction yields a traveling wave response in the circumferential direction of the shell that appears even for vibration amplitudes smaller than the shell thickness. In fact, the presence of a traveling wave response near resonance has a fundamental difference with respect to linear vibrations. Another important feature of closed circular cylindrical shells in large amplitude vibrations is the dynamic axisymmetric contraction, which has been addressed both theoretically and experimentally. In particular, during large amplitude vibrations, the shell presents an inward axisymmetric dynamic contraction with twice the excitation frequency to guarantee the in plane quasi inextensibility of the shell. Without this axisymmetric contraction, the shell would increase its length along the circumference significantly, which is against shell mechanics. In fact, shells bend more easily than they stretch.

Shell structures subjected to in plane loads lose stability and buckle after a certain threshold. In fact, in the presence of static in plane loads, instability occurs through pitchfork bifurcation, whereas in the case of periodic in plane loads the so called parametric instability occurs through period doubling bifurcation in the frequency neighborhood of twice the natural frequencies of the bending mode. In the latter case, and in contrast with non linear vibrations of shells due to radial harmonic excitation, the instability may arise even for small excitation amplitudes and much below the static buckling load.

Shell structures that are coupled to a liquid have lower natural frequencies as a consequence of the added virtual mass of the liquid itself. In closed circular cylindrical shells, the effect of added virtual mass is lower for the asymmetric modes than for axisymmetric modes. Therefore, the non linearity displayed by thin closed circular shells is enhanced by the presence of dense fluid. Moreover, circular cylindrical shells supported or clamped at both ends and conveying liquid lose stability via divergence, which is a static pitchfork bifurcation of the equilibrium, when the flow speed reaches a critical value. The divergence is strongly subcritical, becoming supercritical for high flow velocities. In this case, the system has two or more stable solutions related to divergence much before pitchfork bifurcation occurs. In other words, if the shell is perturbed from the initial configuration, it may have severe deformations resulting in failure much below the critical velocity that can be predicted by linear analysis.

Complicated dynamics and chaotic motions are other interesting features that have been widely reported for shell structures. These complicated motions may occur for shells subjected to large amplitude excitations, at specific excitation frequencies and force amplitudes.

Shell structures generally do not follow a certain scenario to become chaotic, since the routes to chaos depend on all system parameters. These complex behaviors are usually analyzed using bifurcation diagrams, Poincaré maps, phase plane plots, time responses and spectra, Lyapunov exponents, Lyapunov spectrum and fractal dimensions.

In contrast with closed circular cylindrical shells, curved panels (i.e. open shells, with single and double curvature) do not always exhibit an internal resonance, unless for a specific shallowness and thickness ratios, and do not have axisymmetric dynamic contraction. The peculiar aspects of panels that make the non linear analysis of these structures interesting are their asymmetric nature of oscillation with respect to the initial undeformed middle surface, the softening non linear behavior that turns into hardening for large amplitude vibrations and the diverse internal resonance conditions that may occur for different geometries.

The present paper aims to review the existing literature on non linear vibrations of shell structures from 2003 to 2013. Flat and imperfect plates and membranes are excluded. Literature reviews of the earlier works on large amplitude vibrations of shells before 2003 include those of Leissa [1], Evensen [2], Kubenko and Koval'chuk [3,4], Amabili and Païdoussis [5] and Moussaoui and Benamar [6]. In fact, an extensive literature review until 2003 on the non linear dynamics of shells *in vacuo*, filled with or surrounded by quiescent and flowing fluids, has been provided in Ref. [5] and will not be repeated here. However, to elaborate the new research carried out on non linear vibrations of shell type structures, it is necessary to refer to some fundamental references, briefly. The monograph of Amabili [7] has also provided a literature review on non linear vibrations and dynamic stability of shells till 2008 and has shown that many questions in this area still remain unanswered. Therefore, the present literature review focuses on geometrically non linear free and forced vibrations of closed and open shells made from traditional and advanced materials with and without fluid structure interaction. Studies dealing with particular dynamical problems involving parametric vibrations, stability, dynamic buckling, non stationary vibrations and chaotic vibrations are profoundly addressed. Moreover, several original aspects of non linear vibrations of shells and panels, including geometric imperfections, effect of geometry and boundary conditions, thermal loads, electrical loads and reduced order models including perturbation techniques, proper orthogonal decomposition, non linear normal modes, natural modes and meshless methods, are fully reviewed. Works dealing with experiments are also discussed.

The reviews are generally organized chronologically, according to the publication date, and are structured as follows: non linear theories of shell structures are discussed in Section 2. In Section 3, non linear vibrations of circumferentially complete shells are given. In order to better elaborate the review, Section 3 has been divided into four subsections depending on the type of loads applied on the shell. In particular, in Section 3.1, free and forced vibrations of closed isotropic shells due to radial harmonic excitation are studied; in Section 3.2, dynamics and stability of closed shells subjected to axial loads are discussed, and in Section 3.3 the existing literature of shells conveying fluid are given. Section 3.3 has been divided into two subsections, Section 3.3.1 deals with dynamical problems of shells filled with still fluid and Section 3.3.2 discusses the stability and dynamics of shells subjected to flowing fluid. In Section 3.4, papers dealing with non linear vibrations of closed isotropic shells subjected to non stationary, thermal and electrical forces are reviewed.

In Section 4, non linear free and forced vibrations of curved panels are discussed. Depending on the geometry of the panel, this section has been divided into 2 subsections: Section 4.1 discusses cylindrical and doubly curved panels and Section 4.2 discusses spherical caps and panels of other geometries including conical and ellipsoidal shapes.

Non linear vibrations of shells made of advanced materials are discussed in Sections 5 and 6. In particular, sandwich and

laminated composite shells, panels and fluid structure interaction in composite shells are reviewed in Sections 5.1, 5.2 and 5.3, respectively, and literature on non linear vibrations of FGM shells and panels can be found in Sections 6.1 and 6.2, respectively. Section 7 is dedicated to non linear vibrations of hyperelastic shells and, finally, papers dealing with experiments are listed in Section 8.

2. Non-linear theories of shells

The classical theories of non linear shell mechanics, suitable for thin shells, include those of Donnell [8], Novozhilov [9], Sanders [10], Koiter [11] and Flügge-Lur'e-Byrne [12] that are all based on the Kirchhoff-Love hypotheses. In particular, Donnell's non linear shallow shell theory neglects in-plane inertia and gives accurate results only for very thin shells. In this theory, in-plane displacements are infinitesimal, while the transverse displacement is of the order of shell thickness. Moreover, the non linear terms are retained only in the transverse displacement (the von Kármán type non linear terms, in analogy to the treatment of plates); however, other effects, such as the non linearities involving in-plane displacements, are neglected. The Donnell theory, without the shallow shell hypothesis, retains in-plane inertia and is more accurate than Donnell's non linear shallow shell theory; however, it still keeps non linear terms of only the von Kármán type. The Sanders theory is a more refined non linear shell theory developed by Sanders [10] originally in tensorial form. The same theory was also derived independently by Koiter [11] around the same period, leading to the designation of this theory as the Sanders-Koiter non linear shell theory. This theory is suitable for finite deformations with small strain and moderately small rotations. Therefore, in the Sanders-Koiter theory the in-plane displacements are not infinitesimal, and in the strain-displacement relations, non linear terms that depend on both in-plane and transverse displacements exist. In both the Donnell and the Sanders-Koiter theories the changes of curvature and torsion of the middle surface are assumed to be linear. Moreover, the Sanders-Koiter theory gives accurate results for vibration amplitudes significantly larger than the shell thickness, whereas Donnell's theory is accurate only for very thin shells and modes of high circumferential wave number.

The general non linear theories of shells developed by Novozhilov [9] and the Flügge-Lur'e-Byrne theory [12] are very similar and only differ in terms of change in curvature and torsion. In both of these theories, non linearities can be retained in changes in curvature and torsion [7]. Moreover, the thinness assumption is delayed in the derivations of both theories and the strain-displacement relations include non linearities in both transverse and in-plane displacements. All of these classical theories are obtained by neglecting transverse shear deformation and rotary inertia, and thus, can give inaccurate results for moderately thick or laminated anisotropic shells. In order to overcome this limitation, shear deformation theories have been introduced. These theories can be classified as the first order and higher order shear deformation theories; in the first category, a shear correction factor is required for the equilibrium since a uniform shear strain is assumed through the shell thickness. Higher order shear deformation theories overcome this limitation since a realistic shear stress distribution through the shell thickness is assumed, which also satisfies the condition of zero shear stresses at both the top and bottom shell surfaces. These theories have been completely discussed in the books of Amabili [7], Reddy [13] and Carrera et al. [14]. Moreover, linear shear deformable and zigzag (layer wise constant shear angle) theories have been profoundly reviewed by Carrera [15,16] and Reddy and Arciniega [17].

The non linear first order shear deformation theory was primarily proposed by Reddy and Chandrashekhara [18] and was based on Sanders-Koiter non linear terms. Five independent variables—three displacements and two rotations—were used to describe the shell deformations. Non linear first order shear deformation theory of shells based on von Kármán type non linear terms and its finite element formulations is also given in the study by Reddy [13]. Librescu [19] developed a non linear shell theory by expanding the shell displacements with cubic terms in the transverse coordinate. Dennis and Palazotto [20] and Palazotto and Dennis [21] extended Reddy's linear higher order shear deformation shell theory to non linear deformation by introducing the von Kármán type non linear terms. The non linear higher order shell theory of sandwich anisotropic shells with transversely compressible core was developed by Hohe and Librescu [22]. In their theory, the Kirchhoff-Love hypotheses were adopted for the face sheets and a second/third order power series expansion was considered for the core displacements. Chaudhuri [23] proposed a non linear zigzag theory for the non linear finite element analysis of doubly curved shells by considering the full non linear strain-displacement relations for the five independent variables of the shell. Amabili and Reddy [24] developed a new theory for closed and open shells that, in contrast with previous non linear shell theories, was derived with consistency retaining rotary inertia, shear deformation and non linearities in both in-plane and transverse displacements. The new theory has shown superiorities over the existing non linear shear deformation theories in predicting large amplitude vibrations of deep and moderately thick laminated circular cylindrical shells [25] and curved panels [26]. Recently, Amabili [27] extended the theory of [24] by adding the thickness stretching effect and taking into account the geometric imperfections. In this new theory, six independent variables were used to describe shell deformation. In particular, in [27] a uniform transverse normal strain is assumed through the shell thickness. This theory has been extended to third order transverse normal strain by Amabili [28]. The advantages of shell theories retaining transverse normal stress and strain are using the full three-dimensional constitutive equations and they are particularly suitable for soft materials, like rubbers and biological materials, where very large deformations are achieved and accompanied by large thickness reduction. Parisch [29] and Sansour [30] developed independently shell theories that introduce quadratic assumption of shell displacement over shell thickness. Moreover, an accurate linear shell theory that takes into account the thickness variation was developed by Carrera et al. [31] and Ferreira et al. [32]. Tensor based geometrically non linear theories of shells are quite abundant in the literature [33–45]. Eremeyev and Pietraszkiewicz [33] developed a general non linear theory for shells taking into account the phase transitions of materials. In their theory shell displacements were expressed by work averaged translations and rotations of the shell cross sections. Moreover, all shell relations were found from the variational principle of the stationary potential energy. Opoka and Pietraszkiewicz [34] consistently obtained a complete boundary value problem for the non linear theory of thin elastic shells expressed thoroughly in terms of stress resultants and bending of the shell reference surface. Arciniega and Reddy [35] developed an improved first order shear deformation theory with seven independent variables under the Lagrangian framework and presented the finite element formulations that could be used to study the non linear analysis of a broad range of shell geometries made of different types of materials including isotropic, laminated composite and FGMs. Opoka and Pietraszkiewicz [36] modified their previous work [34] by formulating a new version of the Lagrangian theory of thin shells in terms of displacements of the reference surface and based on the principle of virtual work. Berdichevsky [37] proposed a non linear

theory for sandwich shells based on the asymptotic variational method. Shen et al. [38] developed a modified Koiter shell theory and discussed the role of metric and curvature tensors in constructing non linear elastic shell models of the Koiter type. Pietraszkiewicz [39] reviewed some equivalent expressions of the bending tensor in the non linear shell theory and indicated that the one proposed by Shen et al. [38] was not exact. The Koiter shell theory has also been discussed from the perspective of non linear three dimensional elasticity by Steigmann [40].

Non linear shell theories that were proposed using the Cosserat continuum can be found in Refs. [41–45]. The basic idea of this variant of continuum mechanics model is the independence of translations and rotations, and therefore by analogy the independence of forces and moments. The non linear theory of micropolar shells has been given in Ref. [41], and the local symmetry group for the dynamical exact theory of shells is established in Ref. [42]. A short review of Cosserat theories for shell structures was carried out by Altenbach et al. [44], and the non linear shell theory in nanoscale was presented by Altenbach and Ermeyev [45]. Moreover, the non linear strain gradient theory of shells was presented by Lazopoulos and Lazopoulos [46].

3. Non-linear vibrations of closed isotropic shells

This section is divided into four parts, depending on the type of excitation: (i) free and forced vibrations of shells due to radial excitation; (ii) dynamics and stability of shells subjected to axial loads; (iii) fluid shell interaction; and (iv) forced vibrations of shells subjected to other types of loading conditions.

3.1. Free and forced vibrations of shells due to radial excitation

The history of thin circular cylindrical shells subjected to large amplitude excitations and discussions made by different researchers on the softening nature of the response has been well explained in the review paper of Amabili and Païdoussis [5]. However, it is worth having a brief review on how the correct behavior of shells can be numerically modeled. In fact, to accurately predict the large amplitude behavior of circular cylindrical shells, the mode shapes should be expanded by sufficient asymmetric and axisymmetric modes. The original idea of mode expansions of shell flexural displacement involving the driven and companion modes plus an axisymmetric term is attributed to Evensen [2,47,48], Dowell and Ventres [49] and Atluri [50]. In particular, the axisymmetric term considered by Evensen was a squared sine in the axial coordinate with the amplitude related to the amplitude of linear terms in order to satisfy the continuity of the shell circumferential displacement, while Refs. [49,50] only considered the first axisymmetric mode of linear vibrations in order to satisfy exactly the out of plane boundary conditions and the in plane boundary conditions on average. Although the effect of adding an axisymmetric term in the mode expansion was found to be significant, discrepancies still existed between theory and experiments since experimental observations of Evensen [51] and Olson [52] illustrated softening type non linearity whereas the numerical results of Dowell and Venters [49] and Atluri [50] predicted hardening non linearity. Varadan et al. [53] showed that the hardening type results obtained in Refs. [49,50] are due to the choice of the axisymmetric term. Amabili [54,55] elaborated the effect of axisymmetric terms and showed that in order to correctly predict the softening behavior of circular cylindrical shells, at least the first two axisymmetric modes (axisymmetric modes with an even number of longitudinal half waves do not give any contribution) should be retained in the mode expansion in addition to the driven and companion modes.

In 2003, Amabili [56,57] extended his previous works on non linear forced vibrations of simply supported circular cylindrical

shells subjected to concentrated harmonic force. In particular, in Ref. [56], Donnell's non linear shallow shell theory was used and the non linear frequency response curves were obtained using the Galerkin method considering several expansions, including 16 or more natural modes of the shell that included sufficient asymmetric and axisymmetric modes. Moreover, a code based on the arc length continuation technique was utilized to perform bifurcation analysis. Several interesting non linear phenomena were observed including softening type non linearity, different types of traveling wave response, amplitude modulated response and modal interactions. In order to validate the numerical simulations and to understand the physical behavior of the shell, experiments were also carried out following a stepped sine technique and by spanning the excitation frequency up and down in very small increments in the frequency neighborhood of the shell's fundamental frequency. In the experimental tests the response of the shell was measured by using two accelerometers and the excitation was provided by an electrodynamic exciter connected to the shell by a stinger glued in a position close to the middle of the shell. The time responses were measured using a Difa Scadas II front end connected to a workstation with the software CADA X of LMS for signal processing. The same front end was used to generate the excitation and its closed loop control was used to keep the value of the excitation force constant for any excitation frequency, during the measurements of non linear response.

Amabili [57] compared the numerical results obtained from Donnell's non linear theory (with and without in plane inertia) with Sanders Koiter, Flügge Lur'e Byrne and Novozhilov's theories following a Lagrangian approach. Different expansions involving 14–48 generalized coordinates, associated with natural modes of simply supported shells, were used and it was shown that the results from Sanders Koiter, Flügge Lur'e Byrne and Novozhilov are practically coincident. Moreover, a small difference was observed between the previous three theories and Donnell's theory with in plane inertia. On the other hand, Donnell's non linear shallow shell theory proved to be the least accurate among the five theories compared. Comparisons between these theories in predicting the non linear response of a circular cylindrical shell are shown in Fig. 1.

The internal resonances of undamped and unloaded circular cylindrical shells were investigated by Popov [58,59] by means of Hamiltonian dynamics. The analysis was based on Donnell's shell theory and the Rayleigh Ritz technique was used to discretize the equations of motion and to study non linear systems that had two and three degrees of freedom, respectively. Avramov [60] and Avramov et al. [61] used the method of multiple scales to study the forced vibrations of circular cylindrical shells with two internal resonances and found hardening behavior for standing and traveling wave responses of the shell. They used Donnell's non linear theory and an expansion for the transverse displacement that consisted of only the fundamental driven and companion modes plus an axisymmetric term in the form of a squared sine in the axial coordinate. In fact, the source of the reported hardening non linearity could be due to the inappropriate expansions considered for the flexural displacement.

To study non linear forced vibrations of simply supported shells, Gonçalves et al. [62] presented a low dimensional model, combining a perturbation technique with the proper orthogonal decomposition (POD) method. In their analysis, the POD method was used to investigate the relative importance of each mode obtained by the perturbation solution on the non linear response and the total energy of the system. Touzé et al. [63] used the non linear normal modes method to predict the response of circular cylindrical shells subjected to harmonic excitation and compared the accuracy of different truncations obtained using linear and non linear modes. They showed that a reduced order model comprising one or two

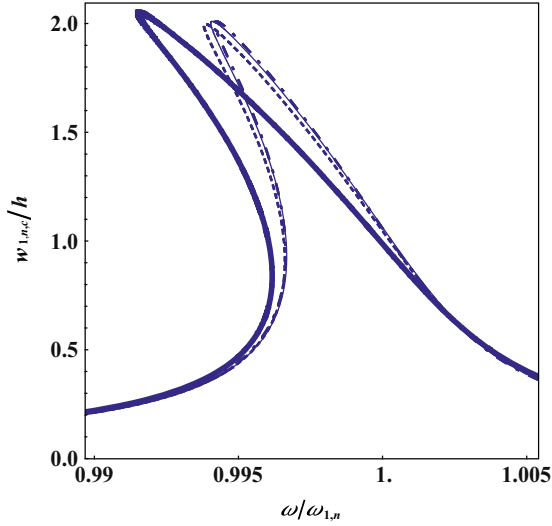


Fig. 1. Non-linear frequency response curve for the fundamental mode of a perfect shell subjected to radial harmonic load. $\omega_{1,n}$ is the fundamental frequency, ω is the excitation frequency, $w_{1,n,c}$ is the amplitude of the driven mode, h denotes the thickness. Donnell's shallow shell theory without in-plane inertia, Donnell's theory with in-plane inertia, Sanders-Koiter theory, Flugge-Lu're-Byrne theory (coincident with Novozhilov). From Amabili [57].

non linear normal modes gives perfect results for vibration amplitudes lower than or equal to the thickness of the shell. However, non linear normal modes obtained by perturbation techniques, as other perturbation techniques, lose accuracy when the vibration amplitude becomes larger.

Using a Lagrangian approach and the method of harmonic balance, Rougui et al. [64] investigated the non linear free and forced vibrations of simply supported cylindrical shells. A simple approximate transverse displacement expansion including the driven mode and the axisymmetric modes (neglecting the companion modes and using the squared sin axisymmetric term) was used for the minimization of the energy functional and it was shown that the shell exhibits a softening behavior. The method of harmonic balance was also successfully used by Avramov [65,66] in obtaining the non linear normal modes and traveling waves of simply supported circular cylindrical shells.

Non linear free and forced vibrations of rotating shells were investigated by Chen and Dai [67], Wang et al. [68-70], Liu and Chu [71] and Chen [72]. In particular, in Refs. [67,72] the method of multiple scales was used to study the non linear vibrations of rotary truncated conical shells with simply supported and clamped boundary conditions, and it was found that the shell exhibits hardening behavior. Using Donnell's non linear shallow shell theory, Wang et al. [68,69] used different multi dimensional models and included the Coriolis force and damping effects in the equations of motion. To obtain the frequency response curves, the equations of motion were solved by direct time integration in [68] and the method of harmonic balance in [69]. Wang et al. [70] also studied the non linear vibrations of cantilevered shells subjected to a moving concentrated harmonic force along the shell using the harmonic balance method. Liu and Chu [71] used a single mode Galerkin approximation together with the method of multiple scales to obtain the frequency response curves of cantilevered rotating thin circular cylindrical shells. They used the eigen functions of a clamped free beam to discretize the equations of motion. In all these studies, Refs. [67-72], the effect of axisymmetric modes was neglected and therefore hardening behavior was obtained for the shells under investigation.

Kurylov and Amabili [73] compared the large amplitude response of isotropic shells obtained using Chebyshev polynomials, power

polynomials and trigonometric functions for simply supported and clamped boundary conditions. The Sanders Koiter non linear shell theory was used and it was found that trigonometric functions are very efficient for simply supported boundary conditions while they require a large number of degrees of freedom for other boundary conditions, thereby decreasing the computational efficiency. It was also found that Chebyshev polynomials have the advantage of requiring similar degrees of freedom for any boundary condition whereas ordinary power polynomials present ill conditioned mass and stiffness matrices in the linear problem, which can become problematic in non linear analysis. The same authors [74] also thoroughly investigated the non linear vibrations of cantilevered circular cylindrical shells using the Sanders Koiter non linear shell theory and expanded the shell displacements in terms of a harmonic function for the circumferential variable and Chebyshev polynomials for the longitudinal variable. A novel technique to reduce the number of degrees of freedom by introducing artificial axisymmetric modes in the expansions of the displacements was used and different expansions involving 18-52 generalized coordinates were considered to study the convergence of the solution.

Non linear vibrations of cantilevered shells were also studied by Zhang et al. [75]. The shell was assumed to be in high speed milling process and was subjected to a harmonic transverse cutting force. Considering a two to one internal resonance between the first two asymmetric modes of the shell and neglecting the companion and axisymmetric modes, the method of multiple scales was used to obtain the modulation equations and the fourth order Runge-Kutta algorithm was utilized to solve the modulation equations and to obtain the bifurcation diagrams.

Non linear free vibrations of pre stressed circular cylindrical shells having combinations of clamped, free and simply supported boundary conditions, resting on an elastic foundation, were investigated by Bakhtiari Nejad and Bideleh [76]. The non linear Sanders Koiter theory was used and a minimal single mode approximation with the Lindstedt Poincaré perturbation technique was utilized to obtain the backbone curves that showed erroneous hardening behavior.

The effect of geometric imperfections on the large amplitude response of circular cylindrical shells was studied in Refs. [56,57,61,74]. In particular, Amabili [56,57] discussed the effect of three different sets of imperfections on the fundamental mode of a simply supported shell, and showed that asymmetric imperfections having the same shape of the fundamental mode have a large effect on the non linear response of the shell. He showed that, due to the presence of small imperfection, the trend of non linearity is not affected while the traveling wave response is largely modified since the natural frequencies of the driven and companion modes do not coincide anymore. The initial geometric imperfection having the shape of the driven and companion modes of the shell was also taken into account by Avramov et al. [61]. They found that the non linear normal modes of the shell are mainly determined by shell imperfections. Kurylov and Amabili [74] studied the effect of imperfection on large amplitude vibrations of clamped free shells in depth and found that unlike other types of boundary conditions, geometric imperfections introducing an ovalization of the shell play an important role in non linear response, increasing significantly the softening behavior of the shell.

Chaotic vibrations of circular cylindrical shells were studied in Refs. [77-82]. In particular, using the Galerkin approach and the fourth order Runge-Kutta method, Krysko et al. [77] showed that the chaotic regions of a circular cylindrical shell depend greatly on the loading angle when the shell is subjected to transversal sinusoidal load. Moreover, they investigated the local and global stability loss of closed shells and found that stiff stability loss is associated with a Hopf bifurcation and transition to chaotic and harmonic motions. Krysko et al. [78,79] illustrated that increasing the amplitude of the external load in the presence of various

temperature intensities can yield qualitative changes in the dynamics of the shell and can result in transition from chaotic behavior to harmonic and vice versa. In a series of papers, Awrejcewicz et al. [80], Krysko et al. [81] and Awrejcewicz et al. [82] discussed comprehensively the possible routes to chaos in circular cylindrical shells. In particular, in the first part [80], the practical convergence of diverse numerical methods including Galerkin, finite difference method, Ritz and different Runge Kutta techniques in predicting the chaotic behavior of shells was discussed. In the second part [81], it was shown that closed circular cylindrical shells may exhibit the modified Pomeau Manneville scenario and closed shells reinforced by flexible non compressed ribs may exhibit the modified Ruelle Takens New house Feigenbaum scenario to chaotic vibrations, and in the third part [82] the Lyapunov exponents were calculated.

Non linear vibrations of closed spherical shells were studied thoroughly by Arafat and Nayfeh [83]. The possibility of activation of two to one internal resonance between different vibration modes of the shell was investigated, and the method of multiple scales was used to obtain primary resonance responses and bifurcations due to an external axisymmetric excitation.

3.2. Dynamics and stability of shells subjected to axial loads

The parametric instability and the post critical behavior of simply supported circular cylindrical shells subjected to periodic axial loads were studied by Pellicano and Amabili [84]. Donnell's non linear shallow shell theory was used and the equations of motion were obtained using the Galerkin method and discretizing the governing partial differential equations with a relatively large number of linear modes, including axisymmetric and asymmetric modes. To perform bifurcation analysis, a pseudo arc length continuation scheme was utilized and the role of parametric resonance in the structural

collapse of shells was confirmed. It was shown that, when the excitation frequency is close to twice the natural frequency of an asymmetric mode, a period doubling bifurcation takes place, resulting in quite large vibration amplitudes. Fig. 2(a c) shows the response of a thin perfect shell under periodic axial load. It can be clearly observed that dynamic instability occurs much below the static buckling load as a result of period doubling bifurcation. It can be seen that before the bifurcation point, only axisymmetric modes contribute to the vibration of the shell.

The parametric resonance of circular cylindrical shells has also been discussed by Popov [85], who considered a simple model of the shell with two degrees of freedom. In a series of papers, Gonçalves and del Prado [86,87] and Gonçalves et al. [88 90] studied the dynamic instability of axially excited simply supported, perfect circular cylindrical shells using Donnell's non linear shallow shell theory and following the Galerkin approach. In particular, modal interactions between non linear vibration modes with equal or nearly equal natural frequencies and their influence on parametric and snap through instability were studied in [86], and suitable reduced order models, which could reproduce the shell response accurately, were obtained in [87]. Transient and steady state stability boundaries of an isotropic shell were obtained in [88] and special attention was devoted to the determination of critical loading conditions using a numerical continuation technique and the Runge Kutta method. Moreover, the complexity of the basins of attraction of low frequency vibration responses and the escape from the potential well as a result of combined static and periodic axial loads were clarified in [89]. In Ref. [90], the same authors focused more on the use of basins of attraction as a measure of reliability and safety of the shell and commented that, due to the fractal nature of the basin boundaries and as a result of the sub critical behavior, the shell response is very sensitive to initial conditions and uncertainties in system parameters.

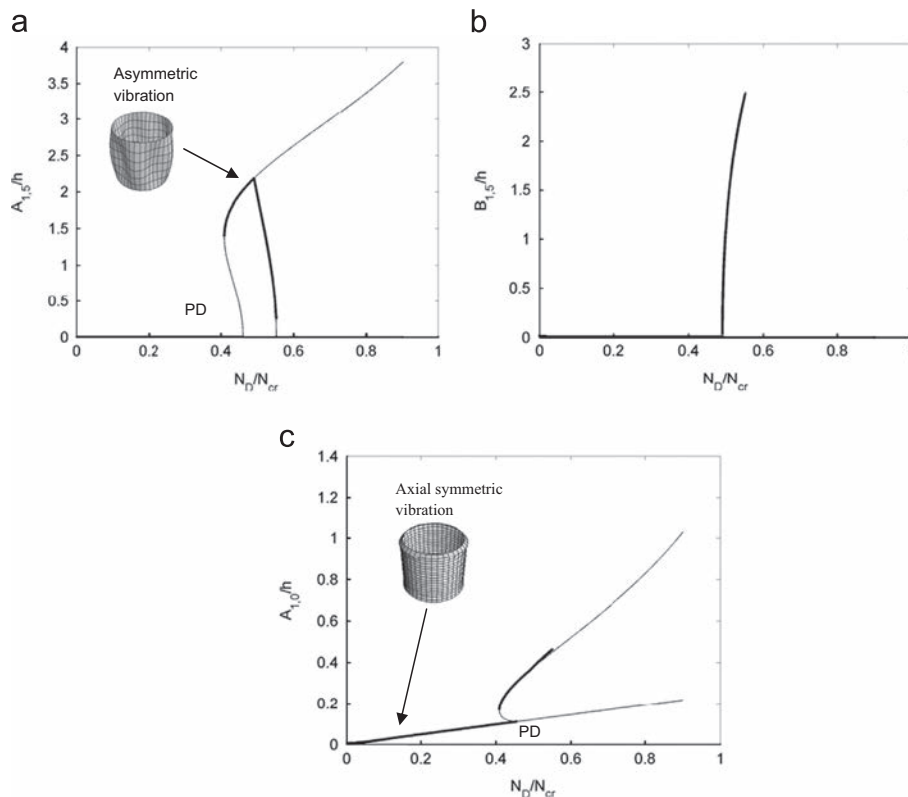


Fig. 2. Dynamic bifurcation of an isotropic thin circular cylindrical shell under periodic axial load when the excitation frequency is close to twice the fundamental frequency. (a) Amplitude of the driven mode/ h ; (b) amplitude of the companion mode/ h ; (c) amplitude of the first, axisymmetric mode/ h . h Denotes the thickness, N_D is the dynamic load, N_{cr} is the critical static buckling load and PD is period-doubling bifurcation. From Pellicano and Amabili [84].

Kochurov and Avramov [91] investigated the non linear modes and traveling waves of parametrically excited cylindrical shells. They used Donnell's non linear shallow shell theory and considered an eight degree of freedom model consisting of six asymmetric modes, a squared sine term to describe the asymmetry of displacements with respect to the shell middle surface and a constant term that did not depend on the circumferential coordinate. They mentioned that the constant term is added to describe the radial displacement of the points of the end sections of the shell or in other words to assume that the end sections can "breathe" during vibrations. Moreover, using the harmonic balance method, they found that non linear normal modes of vibration correspond to standing waves, and the behavior of non linear normal modes and traveling waves is qualitatively similar. The same authors [92] also studied the parametric resonance of circular cylindrical shells with a combination of resonance. A six degree of freedom model was considered. The modulation equations were derived for two types of combination of resonances by employing the method of multiple scales and it was shown that, depending on the combination of resonance, the shell exhibits either softening or hardening behavior.

By considering a two degree of freedom model based on Donnell's non linear shallow shell theory, Gonçalves et al. [93] studied the global dynamics of isotropic shells due to a combination of static and harmonic axial loads. By plotting Poincaré sections, bifurcation diagrams and basins of attractions, sub critical and super critical bifurcation scenarios were investigated and periodic, quasi periodic and chaotic motions were obtained.

Parametric stability of rotating truncated conical shells was investigated by Chen and Dai [67] and Chen [72]. The method of harmonic balance was used to obtain the regions of instability. Moreover, by considering a single degree of freedom model, Chen [72] used the Melnikov method to study the homoclinic and sub harmonic bifurcation of the shell, and the method of multiple scales to obtain the frequency amplitude curves.

Theory and experiments for non linear dynamics of circular cylindrical shells connected to a top mass and under base excitation were conducted by Pellicano and Avramov [94] and Pellicano [95,96]. In particular, in Ref. [94] dynamics of a shell carrying a rigid disk on top and clamped at the bottom was investigated using the Sanders Koiter non linear theory and solutions were obtained using a perturbation technique and direct time integration of equations of motion. Moreover, linear experimental modal analysis was performed and the damping ratios obtained from the experiments were used to conduct the non linear analysis. In Ref. [95], non linear vibrations were studied via a Lagrangian approach based on the Sanders Koiter theory, and Chebyshev polynomials were used to discretize the energy functional. The experimental analysis carried out in Ref. [95] was similar to that in Ref. [94] and included linear modal analysis. In Ref. [96], Pellicano extended his previous works by modifying the experimental analysis and obtaining a non linear numerical model that included the shell shaker interaction. In particular, large amplitude vibrations were studied experimentally following a stepped sine testing procedure and it was observed that there is a strong energy transfer mechanism from low frequency axisymmetric and asymmetric modes resulting in non linear interactions between these modes. Moreover, a saturation phenomenon regarding the vibration of the top disk was observed, associated with the violent shell vibration. The numerical simulations in Ref. [96] clarified the experimental results, and the shell model was similar to Ref. [95]. Fig. 3(a c) shows the experimental results obtained by Pellicano [96] on a circular cylindrical shell with top mass subjected to base excitation. Strong interaction between the shaker and the shell disk system can be observed from the figures. Moreover, it can be seen that the acceleration on the shell is surprisingly huge. Parametric vibrations of shells with geometric imperfections have

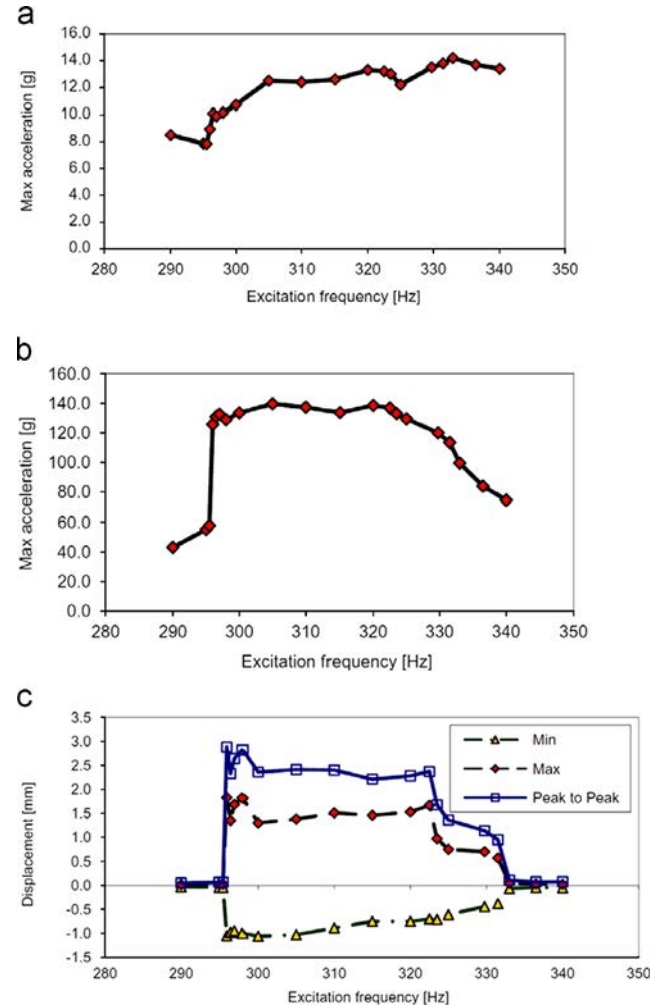


Fig. 3. Experimental results for the large-amplitude vibrations of a circular cylindrical shell with top mass subjected to base excitation. (a) Base excitation amplitude (acceleration [g]); (b) top disk amplitude (acceleration [g]); (c) maximum, minimum and the peak-to-peak of the shell response at the mid-span. From Pellicano [96].

been investigated in Refs. [97–102]. Kubenko and Koval'chuk [97] analyzed the effect of initial imperfections on the stability and dynamics of circular cylindrical shells. In particular, they reviewed 193 papers in this field and studied the influence of initial axisymmetric and non axisymmetric deflections on the parametric vibrations of imperfect shells. They suggested that, although numerous studies can be found addressing the dynamic problems of circular cylindrical shells, the effect of imperfections is an important issue that should be further elaborated.

Catellani et al. [98] developed a multi mode approach based on the shell theories of Donnell and Sanders and analyzed the correlation of parametric instability with shell collapse, taking into account the effect of geometric imperfections. Pellicano and Amabili [99] extended their previous study [84] by deeply analyzing the effect of geometric imperfection on post critical buckling, non linear vibrations and chaotic oscillations of shells subjected to combined static and periodic axial loads. To classify the non linear dynamics, they extracted the correlation dimension, calculated the Lyapunov exponents and depicted the Poincaré sections and bifurcation diagrams. Pellicano [100] also investigated the sensitivity to geometric imperfections in the dynamic stability analysis of strongly compressed circular cylindrical shells. He used the Sanders Koiter non linear theory and showed that the role of imperfections becomes important in case of shells subjected to combined axial compression

and periodic loads since dynamic instabilities interact with the static potential well. The effect of initial geometric imperfections on the parametric vibrations of circular cylindrical shells was also investigated by Kochurov and Avramov [101]. In their analysis, Donnell's non linear shallow shell theory was used and an expansion of transverse displacement with eight degrees of freedom including six asymmetric modes (driven and companion modes), one axisymmetric mode and a constant term was utilized to obtain the equations of motion via the Galerkin approach. Then, by assuming that the frequencies corresponding to the axisymmetric and the constant term are much higher than the asymmetric modes, the time derivatives of those modes were set to zero and a system of six second order ordinary differential equations plus two algebraic equations was solved by the method of multiple scales and a continuation algorithm. The effect of geometric imperfections on the dynamic instability of cylindrical shells carrying a top mass and subjected to base excitation was investigated by Mallon et al. [102]. Based on Donnell's shallow shell theory and considering a multi mode expansion, they obtained a semi analytical model that was verified by finite element simulations in case of linear vibration and buckling. Moreover, depending on the type of initial imperfection, they found periodic, quasi periodic and chaotic post critical behaviors for the shells under investigation.

3.3. Fluid shell interaction

This subsection is divided into two sections: (i) fluid structure interaction for shells containing still fluid; and (ii) fluid structure interaction for shells subjected to flowing fluid. It is worth noting that papers addressing the non linear response of shells containing still fluid also discuss the non linear vibrations of empty shells. Therefore, a reader interested in Sections 3.1 and 3.2 should also refer to Section 3.3.1.

3.3.1. Fluid structure interaction for shells containing still fluid

In the studies reviewed in this section, the contained fluid is assumed to be incompressible and inviscid. In fact, liquid filled shells vibrating in the low frequency range satisfy very well the incompressible assumption. Moreover, non linear effects in the dynamic pressure and in the boundary conditions at the fluid structure interface, and the shell pre stress due to fluid weight are neglected. Therefore, to determine the hydrodynamic pressure exerted by the fluid on the shell, the fluid motion has been described by the velocity potential, which satisfies the Laplace equation in cylindrical coordinates.

Non linear free vibrations of fluid filled circular cylindrical shells were investigated in Refs. [103–108]. In particular, Kubenko et al. [103] used the asymptotic Krylov Bogolyubov Mitropol'skii averaging method to study the sub harmonic responses of shells fully filled with a perfect incompressible liquid. Circumferential traveling waves in carrier shells filled with liquid were investigated by Koval'chuk and Fillin [104]. The same authors [105] also studied the flexural vibrations of initially bent, partially filled circular cylindrical shells. Kubenko et al. [106] extended their previous work [103] and studied non linear free vibrations of fluid filled shells with internal resonance using the same averaging technique. Diverse internal resonances of simply supported and clamped shells filled with fluid were investigated by Koval'chuk and Kruk [107]. Silva et al. [108] studied the non linear free vibration response of fluid filled circular cylindrical shells following a qualitatively accurate reduced order model obtained via the POD method. By comparing the backbone curves of empty and fluid filled shells, they showed that the internal fluid yields a stronger softening behavior. It should be noted that, in all these papers the governing equations of motion were obtained using Donnell's non linear shallow shell theory.

Amabili [56] performed a full series of comparisons between numerical and experimental results for large amplitude vibrations of fluid filled shells subjected to radial excitation. Fig. 4 (a and b) shows the comparison between the experimental and numerical frequency response curves of a water filled shell with the number of circumferential waves equal to 10. The presence of one to one internal resonance between driven and companion modes appear in both theoretical and experimental curves. Amabili [57] compared the responses of different non linear theories in the case of fluid filled shells and showed that, although Donnell's non linear shallow shell theory is the least accurate theory for empty shells, it gives sufficient accurate results for water filled shells. This was because the in plane inertia, which is

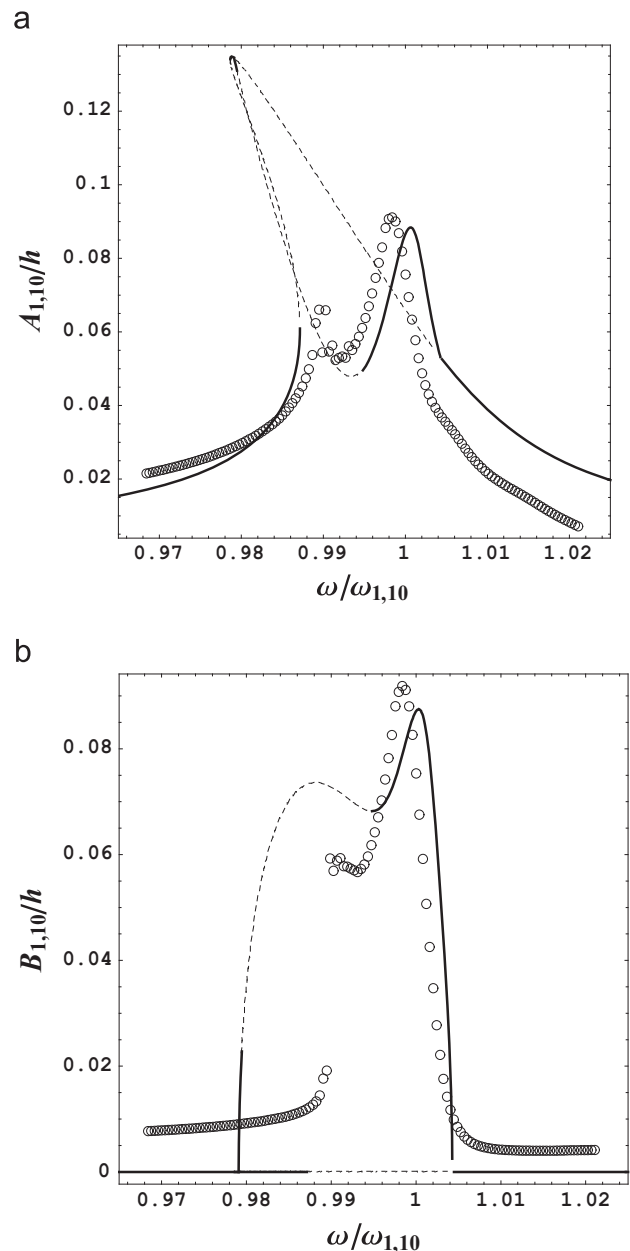


Fig. 4. Comparison between experimental and theoretical frequency–response curves for a water-filled shell ($n = 10, m = 1$); force 3 N and is applied in the radial direction, ω is the excitation frequency. Experimental data; — stable solution; - - - unstable solutions. (a) Amplitude of the driven mode/h; (b) amplitude of the companion mode/h. h Denotes the thickness of the shell. From Amabili [56].

neglected in Donnell's shallow shell theory, is less important for a water filled shell that has a large radial inertia owing to the presence of internal liquid.

The chaotic vibrations of partially filled shells were investigated by Koval'chuk and Kruk [109] using a model that had only two degrees of freedom. Karagiozis et al. [110] developed two numerical models based on Donnell's non linear theory with and without in plane inertia to study non linear vibrations of clamped fluid filled shells. In a series of papers Koval'chuk and Kruk [111,112], Koval'chuk et al. [113] and Kubenko et al. [114] discussed the problem of non linear forced vibrations of completely filled simply supported circular cylindrical shells using Donnell's non linear shallow shell theory and the Krylov Bogolyubov Mitropol'skii averaging technique. Particularly, in Refs. [111,112,114] a three degree of freedom model was used that included the driven and companion modes plus a quartic sine term as a function of longitudinal coordinate to describe the axisymmetric contraction of the shell, whereas in Ref. [113] a five degree of freedom model that contained four asymmetric modes was considered to study the non linear interaction of different flexural modes. The quartic sine term was used in the expansion of the transverse displacement instead of the well known squared sine, to satisfy the moment resultant at the edges of the shell exactly.

Reduced order models based on POD and the non linear normal modes method, for studying large amplitude vibrations of fluid filled shells subjected to radial harmonic excitation, have been obtained in Refs. [108,115-118]. In particular, Amabili et al. [115,116] compared the Galerkin and POD models of a water filled cylindrical shell for moderate and large amplitude vibrations. In Ref. [115] it was shown that the POD technique allows a drastic reduction in the computational effort. Moreover, it was illustrated that models with only three degrees of freedom based on POD have the same level of accuracy as models built by the Galerkin technique with sixteen degrees of freedom. In Ref. [116], it was found that more proper orthogonal modes are necessary to reach energy convergence for chaotic dynamics and it is important to extract the modes using a time series of complex responses (quasi periodic or chaotic). Touzé and Amabili [117] used the non linear normal modes method and built a reduced order model with only two degrees of freedom that showed accurate results for vibration amplitudes up to twice the thickness of the water filled shell. The detailed comparison between POD and non linear normal modes method in studying non linear vibrations of fluid filled shells was given by Amabili and Touzé [118]. Silva et al. [108] combined a perturbation technique and the POD method and assessed the quality of the proposed method by studying the convergence of frequency amplitude curves, bifurcation diagrams and time responses.

Parametric vibrations of shells conveying fluid were investigated in Refs. [84,88,99,108,119]. Pellicano and Amabili [84] showed that compared to empty shells, the presence of liquid changes the dynamic stability bounds interestingly. Gonçalves et al. [88] discussed the significance of the transient response, in studying the dynamic buckling of fluid filled shells. Pellicano and Amabili [99] confirmed that the presence of fluid provides a safety effect on the onset of instability that occurs via period doubling bifurcation. In the same paper, they discussed the chaotic response of fluid filled imperfect shells under periodic axial loads. Parametric instability boundaries and chaotic vibrations of fluid filled cylindrical shells were also studied by Silva et al. [108]. The same authors [119] also discussed the dynamic instability and snap through buckling of partially filled shells.

3.3.2. Fluid structure interaction for shells subjected to flowing fluid

Systematic research on the non linear dynamics of shells conveying fluid was conducted by Paidoussis and it is synthesized in his monograph [120].

Theory and experiments for the dynamic stability of circular cylindrical shells subjected to incompressible subsonic liquid and air flow have been reported by Karagiozis et al. [121-125]. In the theoretical part of these studies, the shell was assumed to be in contact with inviscid fluid, and the fluid structure interaction was described by the potential flow theory. Experiments on non linear dynamics of clamped shells subjected to axial flow were described in Ref. [121] and its visual experimental evidence was provided in Ref. [122]. A subcritical non linear softening behavior was reported for shells subjected to internal and external flows. Three different types of experimental tests were carried out on fully clamped shells in Ref. [123], which included experiments on (i) elastomer shells in annular airflow; (ii) elastomer shells with internal airflow; and (iii) aluminum or plastic cylindrical shells with internal water flow. It was found that the interaction between the shell and the fully developed flow gives rise to instabilities in the form of static or dynamic divergence at sufficiently high flow velocities. In Ref. [124], the effect of varying the thickness to radius and length to radius ratios of the shell on non linear stability was discussed and it was found that an increase of the circumferential wave number, for shells with the same thickness to radius ratio, enhances the subcritical behavior of the shell. Karagiozis et al. [125] elaborated the experiments performed in Ref. [123] and compared the experimental results with numerical ones obtained for simply supported and fully clamped shells in details. Shells with internal and annular fluid flows were studied and it was found that the system loses stability by static, strongly subcritical divergence. In order to obtain the numerical results, Donnell's non linear shell theory was used and the Galerkin approach was employed to obtain the equations of motion, which were solved using a pseudo arc length continuation and collocation method and Gear's backward differentiation formula. For fully clamped shells, models having seven (4 asymmetric modes, 3 axisymmetric modes), eight (4 asymmetric modes, 4 axisymmetric modes) and eleven (8 asymmetric modes, 3 axisymmetric modes) degrees of freedom were considered, whereas for the simply supported shell the numerical model had eighteen degrees of freedom (12 asymmetric modes, 6 axisymmetric modes).

The effect of imperfections on the non linear stability of shells containing fluid flow was investigated by Amabili et al. [126] using a refined model. A Lagrangian approach based on (i) Donnell's theory retaining in plane inertia and (ii) the Sanders Koiter theory was utilized retaining 51 degrees of freedom in the discretization and differently from previous works, the effect of fluid viscosity was taken into account by using the time averaged Navier Stokes equations. Non classical boundary conditions were used to simulate the conditions of experimental tests in a water tunnel and satisfactory comparisons of numerical and experimental results were conducted. It was shown that asymmetric geometric imperfections with the same number of circumferential waves as the mode associated with instability play a significant role, transforming the pitchfork bifurcation at divergence to a folding (saddle node) bifurcation. Good agreement was shown with the available experimental results for divergence of aluminum shells conveying water. Moreover, it was found that axisymmetric and asymmetric imperfections with a number of circumferential waves that is not a multiple of the number of waves at instability play a small role.

The combined effect of geometric imperfections and fluid flow on the non linear vibrations and stability of shells was investigated by del Prado et al. [127]. The behavior of the thin walled shell was modeled by Donnell's non linear shallow shell theory and the shell was assumed to be subjected to a static uniform compressive axial pre load plus a harmonic axial load. A low dimensional model was obtained using the Galerkin method and the numerical solutions were found using a Runge Kutta scheme. It was shown that the parametric instability regions, bifurcations and basins of attraction are affected by the initial geometric imperfection and flow velocity.

The effect of fluid viscosity was also retained by Karagiozis et al. [128] in studying the non linear vibrations of harmonically excited circular cylindrical shells conveying water flow. A detailed analysis was carried out using a pseudo arc length continuation method at different flow velocities and for (i) fixed excitation amplitude and variable excitation frequency and (ii) fixed excitation frequency and variable excitation amplitude. Moreover, in order to classify the system dynamics, direct time integration of equations of motion was performed to obtain bifurcation diagrams of Poincaré maps, the maximum Lyapunov exponent and the Lyapunov dimension. Periodic, quasi periodic, sub harmonic and chaotic responses were detected, depending on the flow velocity and amplitude of the harmonic excitation. Fig. 5(a and b) shows the non linear forced response of the driven and companion modes of a simply supported shell subjected to external harmonic excitation and different flow velocities. It is interesting to see that the softening behavior is enhanced by increasing the flow velocity.

By neglecting the effect of fluid viscosity and considering the potential flow model, non linear forced vibrations and stability of

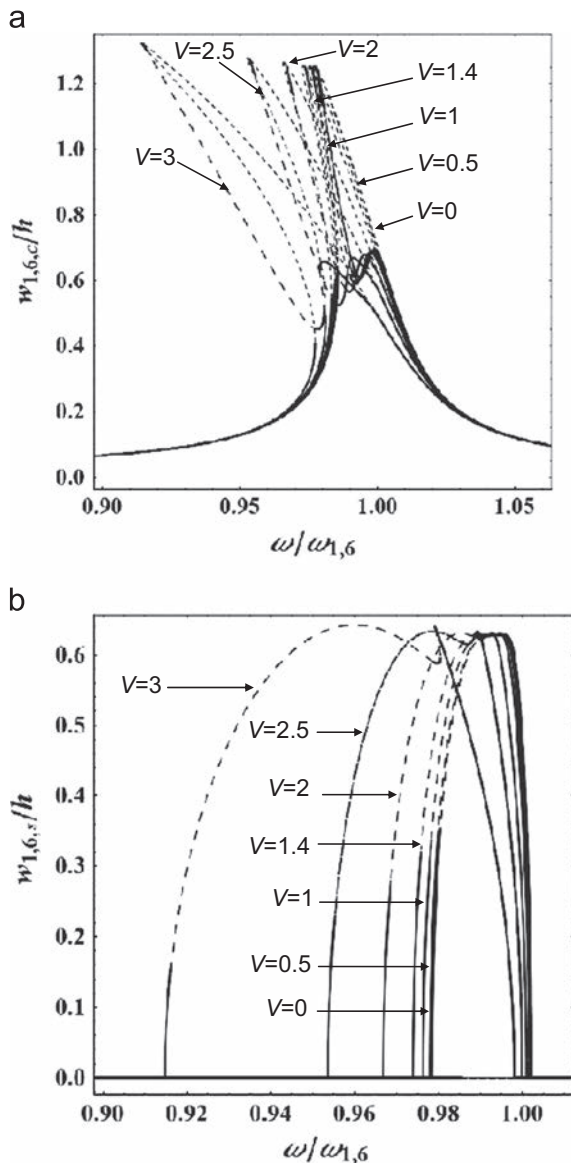


Fig. 5. Effect of flow velocity on non-linear frequency–response curves. Force 0.0165N. (a) Amplitude of the driven mode/h; (b) amplitude of the companion mode/h. h Denotes the thickness of the shell and V is the flow velocity. From Karagiozis et al. [128].

shells interacting with fluid flow were investigated in Refs. [129–132]. Koval'chuk [129] used Donnell's non linear theory together with the Galerkin approach and the Krylov Bogolyubov Mitropol'skii averaging technique to study the non linear vibrations of the shell, neglecting the effect of axisymmetric modes. The same theory and solution methodology was used by Koval'chuk and Kruk [130]. However, in their analysis, the numerical model had six degrees of freedom, which included four asymmetric modes plus two axisymmetric modes. The axisymmetric modes were described as quartic sine terms. Kubenko et al. [131] extended the previous works of Refs. [129,130] by showing the mathematical procedure for the Krylov Bogolyubov Mitropol'skii method in studying multi mode non linear free, forced and parametrically excited vibrations of shells in contact with flowing fluid. Kubenko et al. [132] also studied the vibrations of cylindrical shells interacting with a fluid flow and subjected to external periodic pressure with slowly varying frequency.

Non linear dynamics of cantilevered circular cylindrical shells subjected to flowing fluid was investigated by Paak et al. [133]. The non linear model of the shell was based on the Flügge theory retaining non linear terms due to mid surface stretching, and the fluid model was based on the potential flow theory. The unsteady interaction and asymptotic dynamics of a viscous fluid with an elastic shell were also examined by Chueshov and Ryzhkova [134] using the linearized Navier–Stokes equations and Donnell's non linear shallow shell theory.

Non linear vibration of circular cylindrical shells in supersonic flow and non linear flutter instability of shells is an important subject. Little publications can be found in the last decade on non linear studies dealing with isotropic shells, whereas publications concerning linear studies or non linear analysis of non isotropic shells are wider (see Section 5.3). The only publications available since 2003 are those by Kurilov and Mikhlin [135] (even if the spelling of both name and surname is different, the first author is the same author of Refs. [73,74]) and Jansen [136]. Donnell's non linear shallow shell theory was used in both works. However, in Ref. [135] a pseudo arc length continuation scheme was used to solve the equations of motion, whereas in Ref. [136] the non linear response was obtained following a semi analytic perturbation method. Earlier investigations in this field were carried out by Amabili and Pellicano [137,138] and new results based on this approach were published in Ref. [5].

3.4. Forced vibrations of shells subjected to other types of loading conditions

The non linear response of circular cylindrical shells subjected to dynamic transient loading was investigated by Sansour [139] using the finite element method. A time integration scheme was proposed and formulated for a shell with seven parameters and with arbitrary geometric and material non linearities, which conserved energy, momentum and angular momentum.

Non linear vibrations of isotropic circular cylindrical shells in thermal environments were investigated in Refs. [78,79,140,141]. Awrejcewicz and Krysko [140] proposed an approach based on finite difference and a relaxation method to solve the coupled thermoelastic problem of a flexible shallow shell. Shu et al. [141] studied the coupled thermoelastic vibrations of circular cylindrical shells in large deflection. In their analysis, the non linear terms were only retained in the strain displacement relation in the longitudinal direction, and a single mode approximation was used to study the effect of temperature on the vibration amplitude.

Dynamic buckling of imperfect shells under short term impulsive loads was reviewed by Kubenko and Koval'chuk [97]. Moreover, large amplitude vibrations of cylindrical shells subjected to non stationary loads (e.g. impulse) were investigated in Refs. [142–148]. Shul'ga and Bogdanov [142] developed a numerical method based on

the integro differential method of constructing difference schemes to find the response of conical shells with different taper angles subjected to an impulsive load. The same authors [143] proposed a numerical scheme based on the finite difference method to study the dynamic stress strain state of reinforced circular cylindrical shells subjected to different initial deflections. Vibrations of discretely ribbed, reinforced shells were also investigated by Meish and Kairov [144] based on the Timoshenko shell theory and using a finite difference scheme. In a series of papers, Lugovoi et al. [145–147] used the same theory and extended the work of Ref. [144] by solving the problem via a new technique based on the Richardson extrapolation method in [145], studying the dynamics of compound cylindrical shells with spherical caps in [146], and investigating the dynamic behavior of cylindrical shells in an elastic medium in [147]. Moreover, the stability of vibration mode transformation between extensional and flexural modes, and two to one internal resonance in thin walled circular cylindrical shells subjected to large initial radial velocity or impulse was investigated by Shi et al. [148].

Large amplitude vibrations of shells subjected to electrical loads were investigated in Refs. [149–152]. In particular, using the three dimensional equations of non linear electroelasticity, Yang et al [149] derived a single degree of freedom model, which was solved by the method of harmonic balance to obtain the non linear frequency amplitude behavior of the electric current near resonance. Karnaukhov and Tkachenko [150] studied the vibrations of an infinitely long thin cylindrical shell made of a non linear elastic piezoceramic material subjected to periodic electric force. They used the method of harmonic balance to plot frequency response curves and time responses, for different levels of electric loading. The non linear vibration response of a thick walled spherical shell subjected to mechanical pressure and electric field was investigated by Yong et al. [151]. They assumed that the dielectric elastomer is isotropic and neo Hookean hyperelastic, and found the critical voltage for various loading conditions. Moreover, they found that the spherical shell may undergo quasi periodic motion while it is subjected to sinusoidal electrical load. The same authors [152] also studied the non linear vibrations of dielectric elastomer circular cylindrical shells modeled by neo Hookean material, when subjected to a periodic voltage. They discussed the influence of thickness and boundary conditions on the critical voltage and stated that by applying a voltage greater than the critical value, the shell will be destroyed. Moreover, to obtain periodic solutions, they used a shooting method.

Dong et al. [153] studied non linear modal coupling and strain growth phenomenon in spherical shells subjected to blast loading. Using the LS DYNA finite element software, they explained that the strain growth in the non axisymmetric response of closed spherical shells is due to the non linear coupling between the radial breathing mode of the shell and its composite modes.

Non linear vibrations and dynamic stability of viscoelastic circular cylindrical shells subjected to a dynamic load (i.e. ramp) irregularly distributed along the surface of the shell were investigated in a series of papers by Eshmatov [154–156]. In all these papers, the equations of motion were derived based on the Timoshenko theory and taking into account shear deformation and rotary inertia. Moreover, by assuming simply supported boundary conditions and by applying the Galerkin approach, the equations of motion were reduced to a set of non linear integro differential equations, which were solved via a quadrature technique.

4. Non-linear vibrations of isotropic panels

This subsection is divided into two sections: (i) cylindrical and doubly curved panels; (ii) spherical caps, and panels of other geometries.

4.1. Cylindrical and doubly curved panels

Non linear vibrations of cylindrical and doubly curved panels (shallow shells) with rectangular base were investigated in depth by Amabili [157–161]. Experiments on large amplitude vibrations of circular cylindrical panels were performed in Ref. [157]. The experimental boundary conditions that were considered allowed zero axial, radial and circumferential displacements on the curved edges and zero axial and radial displacements on the straight edges. A stepped sine testing technique was performed in the frequency neighborhood of the first three natural frequencies of the tested cylindrical panel and frequency response curves were obtained that showed strong softening behavior. Amabili [158,159] extended his work by addressing the non linear vibrations and internal resonances of simply supported cylindrical [158] and doubly curved panels [159], numerically. The shell theories of Donnell and Novozhilov were used and a Lagrangian approach was utilized to obtain the equations of motion. Moreover, the bifurcation analysis was performed by using a pseudo arc length continuation and collocation scheme and in order to check the convergence of the solution, different multi dimensional models were considered. It was found that differently from closed circular cylindrical shells that show weak softening behavior, shallow cylindrical and spherical shells show a softening type non linearity that may turn to hardening for large amplitude vibrations. Moreover, it was shown that hyperbolic paraboloidal shells (cases with negative curvature ratios) show hardening type non linearity. Fig. 6 shows the trend of non linearity for different types of shallow shells. Comparisons between experimental results of circular cylindrical panels and simulations taking into account the geometric imperfections were performed by Amabili [160]. Donnell's non linear theory was used and the equations of motion were obtained via a Lagrangian approach. For experiments, two thin cylindrical panels were tested for several excitation amplitudes to characterize the non linearity. The dimensions of the two panels were chosen in order to have fundamental modes with one and two circumferential half waves, respectively. Amabili [161] also discussed the effect of different boundary conditions on the trend of non linearity of circular cylindrical panels.

Finite element studies for large amplitude vibrations of cylindrical panels can be found in Refs. [162–168]. Sansour et al. [162] proposed

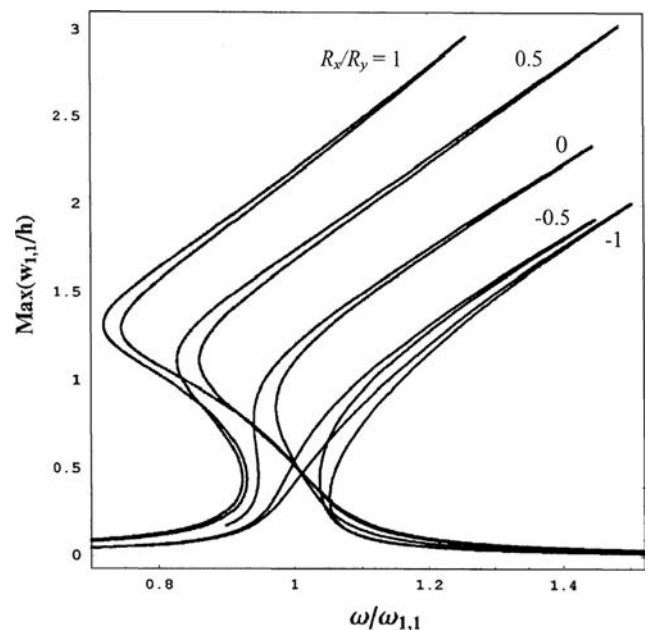


Fig. 6. Effect of curvature ratio (R_x/R_y) on the trend of non-linearity in doubly curved panels. $\omega_{1,1}$ is the fundamental frequency, ω is the excitation frequency and $w_{1,1}$ is the fundamental generalized coordinate. From Amabili [159].

a post processed Galerkin finite element based on hierarchical interpolations to study the non linear response of cylindrical and spherical panels subjected to transient dynamic loads. Ribeiro [163] used first order shear deformation theory and p version finite element with hierarchic basis functions to study non linear free vibrations of moderately thick isotropic doubly curved shallow shells. Shells with different boundary conditions, curvature ratios and thicknesses were studied and it was found that in order to reach accurate results, the p version finite element requires fewer degrees of freedom than the h version finite element method. de Faria [164] assessed the dynamic response of cylindrical panels by studying the effect of moving mass, moving force and load velocity. The equations of motion that were obtained by the finite element method in [164] were non linear only in the velocity of the moving mass, and a perturbation technique was proposed to convert the non linear equations to a series of linear ordinary differential equations. Finite element formulations for large amplitude vibrations of isotropic shallow panels were suggested by Przekop et al. [165]. Non linear first order shear deformation theory was used and it was found that even for moderate deflections, higher modes contribution can be quite large. Ribeiro [166,167] used the same theory and solution methodology of Ref. [163] to illustrate the internal resonances that occur between non linear modes and to study the non linear forced vibrations of shallow cylindrical panels, respectively. In particular, in Ref. [166] the harmonic balance method with a constant term and three harmonics was used to transform the equations of motion into frequency domain equations, which were then solved using a continuation method to obtain the backbone curves, whereas in Ref. [167] a shooting method in conjunction with the Newton method was utilized to solve the equations of motion and to obtain the non linear frequency response curves. Similar finite element formulations were presented by Ribeiro et al. [168] to study non linear vibrations of deep cylindrical panels.

Touzé et al. [63] used non linear normal modes method to study geometrically non linear forced vibrations of simply supported hyperbolic paraboloidal shells, and cylindrical panels with simply supported, in plane free edges. They showed that a reduction to a single non linear normal mode is possible, showing very accurate results for moderate amplitude vibrations. Alijani et al. [169] discussed the accuracy of the method of multiple scales for non linear vibrations of doubly curved panels. Donnell's non linear shallow shell theory was used and the Galerkin method was applied to obtain the governing equations of motion. In order to perform a perturbation analysis and to study the primary resonance response, a quadratic relation was assumed between the excitation frequency and the fundamental frequency of the panel. It was shown that, although in case of hardening non linearity (shallow shells with large radii of curvature) the results resemble those found by numerical integration or continuation methods, in case of softening non linearity (e.g. spherical panels) the solution breaks down as the amplitude becomes greater than the thickness.

Non linear vibrations of shallow shells having a complex shape have been investigated in Refs. [170-175] based on the R functions mesh free technique. The R functions method is a powerful tool to obtain discretization of two dimensional and three dimensional domains of complex shape, and can be applied to moving boundaries. Kurpa et al. [170] obtained a single degree of freedom model for different sets of boundary conditions and presented the backbone curves. The results obtained by the R functions method were in good agreement with the results previously obtained by Amabili [158,159] for simply supported panels and experimental results performed on a cylindrical panel with complex boundary conditions. In particular, the complex panel had symmetrical squared grooves on the straight edges of the panel. Breslavsky and Avramov [171] studied a similar problem by using the R functions method together with a combination of non linear normal modes and multiple scales method to obtain the

backbone curves of a complex cylindrical panel with symmetrical squared grooves on the curved edges. The same problem was also studied by Breslavsky and Avramov [172] using a combination of R functions and the harmonic balance method. The non linear vibrations of complex base shallow shells submerged in a fluid were investigated by Breslavsky et al. [173,174]. The interaction of the shell with the fluid was described by a hyper singular integral equation, which was solved by the boundary element method, and the method of normal modes was used to study the large amplitude vibrations. Pilgun and Amabili [175] performed a novel two step Lagrangian approach to study non linear forced vibrations of cylindrical panels with complex shape and clamped edges. A linear analysis was conducted in the first step, to identify natural frequencies and corresponding natural modes of the panel using Chebyshev polynomials as trial functions. Then, the natural modes obtained from the first step were used as the basis for non linear displacements. Both Donnell and Sanders Koiter theories were used and the bifurcation analysis was carried out using a pseudo arc length continuation and collocation technique.

Chaotic vibrations and complex non linear dynamics of panels have received considerable research attention in the literature. Sansour et al. [176] constructed an energy momentum integration scheme in the framework of the finite element method to study the chaotic motion of panels. Nagai et al. [177] studied the effect of concentrated mass on the chaotic vibrations of shallow cylindrical panels. They used Donnell's non linear theory and the Galerkin approach to obtain the governing equations of motion. The chaotic responses of the shell were examined by Poincaré maps, Lyapunov exponents and Lyapunov dimension following direct time integration of the equations of motion using a Runge Kutta method. It was illustrated that, once the concentrated mass is increased, the maximum Lyapunov exponent decreases gradually. Amabili [159,161] used Gear's backward differentiation formula (BDF) to perform bifurcation analysis on multi modal numerical models and to obtain the bifurcation diagrams, Poincaré maps, Lyapunov exponents and Lyapunov dimension of a spherical and a cylindrical panel, respectively. He found that panels exhibit interesting non linear phenomena such as sub harmonic response, period doubling bifurcation, quasi periodic and chaotic behaviors when they are subjected to large amplitude excitations. Krysko et al. [178] used the Ritz method in conjunction with the fourth order Runge Kutta time integration technique to investigate the chaotic response of spherical panels subjected to sign changing transversal harmonic load. Awrejcewicz et al. [179] studied the transition from regular to chaotic motion in spherical panels with non homogenous thickness. The chaotic vibrations of shallow panels were experimentally investigated by Nagai et al. [180]. The tested panel was simply supported on all edges, and was subjected to gravity and periodic lateral excitation. Two types of chaotic responses were identified that were bifurcated from a sub harmonic resonance response of half order and an ultra sub harmonic resonance response of two third order corresponding to the fundamental mode of vibration. Wang et al. [181] studied the chaotic response of cylindrical reticulated panels by considering a single degree of freedom model and using the Melnikov method. Awrejcewicz et al. [182] studied the chaotic dynamics of cylindrical panels with infinite length subjected to harmonic loads. They found classical routes to chaos including Feigenbaum and Ruell Takens Newhouse scenarios. Maruyama et al. [183] extended the work of Nagai et al. [180], and provided detailed numerical and experimental results for modal interactions of doubly curved shallow panels. The contribution of modes of vibration in the chaotic response was discussed in Ref. [183]. Additional experimental and numerical analyses were performed by Nagai et al. [184] to identify chaotic regions of cylindrical panels with a combination of clamped and simply supported boundary conditions and subjected to lateral periodic acceleration. Krysko et al. [81] observed Feigenbaum scenario associated with

non smooth stability loss for cylindrical panels subjected to uniformly distributed transversal harmonic load. In the same paper, the Ruelle Takens Newhouse route to chaos was found for spherical panels.

Non linear vibrations of viscoelastic panels were investigated in Refs. [185–188]. Khudayarov and Bandurin [185] proposed a numerical algorithm to solve the integro differential equations of viscoelastic panels in contact with gas flow. Eshmatov and Khodzhaev [186,187] studied the effect of concentrated masses on the non linear vibrations and dynamic stability of viscoelastic cylindrical panels, respectively. The Koltunov Rzhansyn singular kernel with three rheological parameters was used for the viscoelastic model and it was observed that an increase in the concentrated mass leads to a sharp drop in the frequency and amplitude of the panel. The effect of damping on the non stationary vibrations of a viscoelastic simply supported cylindrical panel with distributed piezoelectric sensors and actuators was investigated by Karnaukhov and Tkachenko [188]. An integral viscoelastic model was used to describe the viscoelastic behavior of the material, and the effect of viscosity and geometric non linearity on the resonant vibrations of the cylindrical panel was discussed.

4.2. Spherical caps and panels of other geometries

Spherical caps are segments of spherical shells with circular base described in spherical coordinates. Non linear dynamics of shallow spherical caps subjected to radial dynamic edge loading (in the form of step loading) was investigated by Odeh [189]. He found that snap through buckling is possible under peripheral dynamic loading condition and the amount of snapping is considerable when the cap has an opening around the apex.

In a series of papers, Thomas et al. [190], Touzé and Thomas [191], Thomas et al. [192] and Camier et al. [193] investigated in depth the non linear vibrations of free edge spherical caps. In particular, Thomas et al. [190] used Donnell's shallow shell theory and a set of eigen modes obtained from the associated linear problem to calculate all coefficients of the non linear quadratic and cubic terms that appear in the governing equations of motion and discussed the non linear modal interaction rules. Moreover, by deriving a three degree of freedom model and using the method of multiple scales, a specific mode coupling due to 1:1:2 internal resonance between two companion modes and an axisymmetric mode was studied. Touzé and Thomas [191] used the non linear normal modes method to study the effect of geometry on the trend of non linearity. They found that, similar to closed circular cylindrical shells, axisymmetric modes play an important role in the non linear behavior of purely asymmetric modes. Moreover, it was observed that the behavior of asymmetric modes is generally hardening, and it may become softening for certain cases after a 2:1 internal resonance between an asymmetric and an axisymmetric mode. It was also shown that, the behavior of axisymmetric modes may change from hardening to softening for small values of aspect ratios, since the eigen frequencies of the axisymmetric modes depend greatly on the curvature of the spherical cap. Thomas et al. [192] performed experimental tests to validate the occurrence of 1:1:2 internal resonance in free edge spherical caps. The particular internal resonance was observed between the first axisymmetric mode and the sixth asymmetric modes. A coil magnet non contact exciter was used to excite the shell and three transducers, two accelerometers and a laser Vibrometer were used to measure the time evolution of the three involved modes. Particularly, the shell was excited at the center so that only the axisymmetric mode was directly excited and the laser Vibrometer beam was pointing at the center to measure the time response of the axisymmetric mode only. Experimental frequency response curves were obtained for different force levels and quantitative

differences were found between the experimental and theoretical results owing to geometric imperfections that were not included in the numerical model. In order to find consistent experimental and numerical results, Camier et al. [193] improved the previous theoretical models of Refs. [190–192] by taking into account the effect of geometric imperfections. They found that, although good agreement can be seen between the linear experimental and numerical results, there are still discrepancies between the non linear results. In fact, it was observed that the theory over predicts the instability regions, even though it correctly predicts the trend of non linearity. A very possible reason of the discrepancy was mentioned to be the thickness variation or inhomogeneous material property of the tested shell, which was still being neglected in the numerical model.

Non linear vibrations of corrugated spherical shallow panels were investigated by Yuan and Liu [194] and Yuan [195]. Green's function method was used to reduce the problem to a single degree of freedom model with quadratic and cubic non linearities and the solution was found using the Galerkin method. Wang and Song [196] studied non linear free vibrations of uniformly heated bimetallic shallow spherical caps. The Kantorovich averaging method was used to convert the governing partial differential equations to ordinary differential equations and a shooting method was used to obtain the numerical solutions. It was found that the non linear fundamental frequency decreases with temperature increase and increases once the shell height is increased.

Chaotic vibrations of spherical caps can be found in Refs. [197–199]. Soliman and Gonçalves [197] studied the complex axisymmetric dynamics of clamped spherical caps under large harmonic loads. They showed that steady state instabilities including jumps to resonance, sub harmonic response, period doubling bifurcation and cascades of period doubling bifurcations to chaos may occur by varying different parameters of the system. Moreover, they found that a sudden pulse of excitation may lead to the transient failure of the shell. Krysko et al. [198] studied the complex dynamics of spherical caps subjected to transverse distributed sign changeable load and analyzed the scenarios of transition from regular to chaotic motions.

The non linear theory of spherical caps was also used for modeling percussive instruments and loudspeakers. Chaigne et al. [199] discussed the nature of non linear coupling and chaotic response that leads to particular sounds of gongs and cymbals, by calculating the Lyapunov exponents. Bilbao [200] used the non linear shell theory and a finite difference scheme to discuss the sound synthesized by gongs due to large amplitude vibrations. Quaegebeur et al. [201] used a time domain formulation to predict the non linear sound radiation pattern of a prototype loudspeaker that was modeled as a truncated cone with a spherical cap, during large amplitude vibrations.

Non linear free and forced vibrations of shallow conical panels can be rarely found in the literature. The non linear behavior of conical panels under the combined action of peripheral moments and transverse loads was studied by Zhao et al. [202] based on a perturbation method. Thermoelastically coupled non linear axisymmetric vibrations of thin conical panels were investigated by Wang and Dai [203] by applying the Galerkin technique and by assuming similar shape functions for the fundamental mode, thermal bending moment and thermal in plane force resultant. In Ref. [203], a three degree of freedom model was obtained that was solved using the method of multiple scales. A perturbation method was also used in Ref. [204] to obtain the non linear natural frequencies of variable thickness conical shells.

Chaotic vibrations of shallow conical panels can be found in Refs. [81,178,179,205]. Krysko et al. [81,178] found the Feigenbaum scenario for conical panels subjected to large harmonic excitations. Awrejcewicz et al. [179] discussed the routes to chaos of conical

panels with non homogenous thickness. Wang et al. [205] used the Melnikov method to discuss the chaotic response of a conical lattice shell modeled as a single degree of freedom system.

Non linear forced vibrations of discretely reinforced ellipsoidal panels under impulsive loads were investigated by Meish [206]. Non linear first order shear deformation theory was used and a finite difference scheme was utilized to obtain the numerical solutions. The same theory and solution methodology was used by Meish and Maiborodina [207,208] to study non axisymmetric vibrations of ellipsoidal [207] and reinforced ellipsoidal panels [208] subjected to non stationary loads.

5. Non-linear vibrations of composite shells

This section is divided into three subsections: (i) closed composite shells; (ii) composite panels; (iii) fluid structure interaction in composite shells. It must be noted that papers dealing with vibrations of anisotropic shells generally treat vibrations of single layer isotropic shells as specific cases. Therefore, a reader interested in Section 3 should also refer to Section 5.

5.1. Closed composite shells

Toorani [209] and Toorani and Lakis [210] studied non linear free vibrations of composite circular cylindrical shells based on a finite element method and a shear deformation theory. In a series of papers, Jansen [211-215] used Donnell's non linear shell theory and a semi analytic method for studying non linear free and forced vibrations of orthotropic and composite shells. In particular, Jansen [211] presented two types of solutions with different levels of accuracy and complexity. In the first type, which he denoted as "simplified analysis", a number of asymmetric and axisymmetric modes are used to build reduced order models via the Galerkin approach. The obtained reduced order models are then solved either by an averaging technique or by numerical time integration. In the second type of analysis, referred to as the "extended analysis", the boundary conditions of the shell are satisfied exactly and the Fourier decomposition method is used to eliminate the dependence of the solution on the circumferential coordinate. Finally, the resulting boundary value problem for ordinary differential equations is solved numerically by means of a parallel shooting method while a perturbation method is used to assess the influence of large amplitude vibrations. Jansen [211] used these methods to obtain non linear frequency response curves of isotropic and orthotropic shells and to discuss the reasons of discrepancies between the results presented by different researchers. Jansen [212,213] used both simplified and extended methods to study the effect of geometric imperfections and static loading on the large amplitude vibrations of composite shells. He found that, for unloaded shells, asymmetric imperfections mostly influence the linear vibration behavior, while their effect on large amplitude vibration is small. Moreover, he showed that certain axisymmetric imperfections reduce linear frequencies and result in a less softening non linearity, whereas static compressive axial loads yield stronger softening behavior. In Ref. [214], the so called extended analysis was elaborated and non linear vibrations of laminated shells were studied. Jansen [215] also used the semi analytic method based on the perturbation expansion (extended analysis) to study the effect of different boundary conditions on the non linear vibrations and flutter of composite shells. These works were then extended by Rahman et al. [216] who proposed a finite element perturbation method for studying non linear free vibrations of composite circular cylindrical shells. By comparing the results of this new method with the so called simplified and extended analysis

of Refs. [211,214], it was shown that the finite element method may result in a weaker softening behavior.

Non linear vibrations of symmetrically laminated composite shells were investigated by Amabili and Reddy [24] and Amabili [25,217] based on a new higher order shear deformation theory that takes into account non linear terms involving both the normal and in plane displacements. In particular, Amabili and Reddy [24] found that the conventional higher order shear deformation theory with von Kármán type non linearities gives inaccurate results for vibration amplitudes of about twice the shell thickness. Amabili [25] compared the non linear frequency response curves of simply supported shells obtained by (i) higher order shear deformation theory with von Kármán type non linear terms, (ii) Novozhilov shell theory, and (iii) the new theory of Amabili Reddy. He indicated that the conventional higher order shear deformation theory only gives accurate results for modes with high circumferential wave numbers, the Novozhilov theory gives good results for thin laminated shells and the new theory of Amabili and Reddy is preferable, if non linear vibrations of very thick shells are of interest. The comparison between these theories in predicting the non linear forced vibration responses of a thick composite shell with 0/90/90/0 lay up is shown in Fig. 7(a d). The rich non linear dynamics and internal resonances of the type 1:2 between asymmetric and axisymmetric modes for a symmetric cross ply circular cylindrical shell were studied in Ref. [217]. Amabili [218] extended his previous works and studied the non linear vibrations of angle ply circular cylindrical shells by taking into account the possibility of skewed modes (modes with nodal lines not parallel to the longitudinal axis). He found an increased non linearity for a 0/30 lay up shell with respect to an identical symmetric cross ply 0/90/0 laminated shell. Moreover, Amabili [219] developed a new technique for obtaining reduced order models of angle ply circular cylindrical shells by using natural modes of vibration. In Refs. [24,25,217-219], the equations of motion were obtained by the Lagrangian approach and were solved using a pseudo arc length continuation and collocation technique. Shen [220] performed boundary layer analysis for non linear free vibrations of anisotropic shear deformable shells and obtained hardening type non linearity for different stacking sequences. The equations of motion were obtained based on the conventional higher order shear deformation theory with von Kármán type non linearities and a singular perturbation technique was employed to solve the equations. The axisymmetric modes were neglected in the mode expansions.

Dynamic instability and buckling of composite shells were investigated by Jansen [221], Kubenko and Koval'chuk [222], Mallon [223], Mallon et al. [224], Bepalova and Urusova [225], Rahman et al. [226] and Shariyat [227,228]. Jansen [221] used Donnell's non linear shell theory and combined the Lagrangian approach with numerical time integration to study the parametric vibrations of laminated circular cylindrical shells with simply supported boundary conditions. A two mode imperfection consisting of an axisymmetric and an asymmetric mode was taken into account and the shell was statically under axial compression, radial pressure and torsion. Dynamic buckling of isotropic and anisotropic shells under axial step loading was also simulated in Ref. [221]. Experiments on dynamic stability of composite shells were performed by Kubenko and Koval'chuk [222]. The mode shapes of shells subjected to periodic excitation were studied and the effect of steady excitation on the vibration response of three layer glass/fiber reinforced plastic shells was investigated. Theory and experiments on dynamic instability of orthotropic cylindrical shells with top mass under base excitation were presented by Mallon [223] and Mallon et al. [224]. Donnell's shell theory was used and the numerical approach was based on a semi analytic technique that took into account the shell shaker interaction. The experimental results confirmed the observations of the semi analytical analysis and showed that the dynamic stability

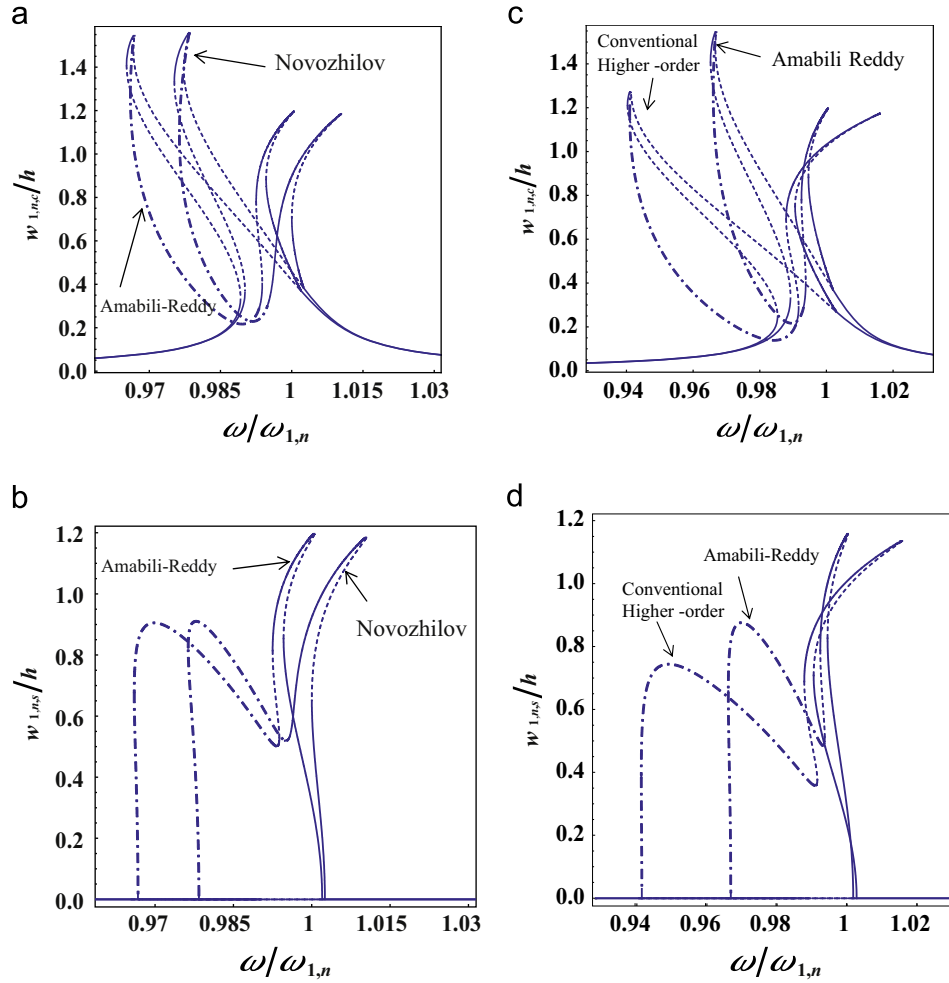


Fig. 7. Comparison between the forced non-linear responses of the Amabili-Reddy theory with classical theories, for a thick composite shell with $n = 2$. (a) Comparison between the amplitude of the driven mode of Novozhilov and Amabili-Reddy theories; (b) comparison between the amplitude of the companion mode of Novozhilov and Amabili-Reddy theories; (c) comparison between the amplitude of the driven mode of the conventional higher-order shear deformation and Amabili-Reddy theories; (d) comparison between the amplitude of the companion mode of the conventional higher-order shear deformation and Amabili-Reddy theories. —, stable periodic solution; - - -, quasi-periodic stable solution; - · -, unstable periodic solution. From Amabili [25].

analysis of these types of structures should be focused near a resonance of relatively low frequency, corresponding to an axisymmetric suspension mode, dominated by axial vibrations of the coupled system. Bepalova and Urusova [225] proposed a numerical approach to obtain the dynamic instability domains of compound shells. Rahman et al. [226] implemented a finite element perturbation method similar to [216] in order to perform a single mode dynamic buckling analysis for unstiffened and ring stiffened composite cylindrical shells. Shariyat [227,228] proposed a double superposition global local formulation inherently suitable for non linear analysis of imperfect composite and sandwich shells. In particular, in Ref. [227], viscoelastic properties of the shell were also taken into account and it was revealed that the relaxation behavior of the viscoelastic material may decrease the dynamic buckling load. In Ref. [228] it was shown that the proposed theoretical approach satisfies all the kinematic and interlaminar stress continuity conditions of composite and sandwich shells at layer interfaces. In both papers, non linear terms in transverse and in plane displacements were included in the strain displacement relations of the shell.

Non linear vibrations of sandwich shells were also investigated in Refs. [229–232]. Lugovoi et al. [229] studied the forced non stationary vibrations of sandwich circular cylindrical shells with cross ribbed cores using a finite difference scheme. Dogan and Vaicaitis [230] used Donnell's shell theory to predict the non linear response of a simply supported double wall sandwich

cylindrical shell subjected to random excitation. Non linear spring dashpot models were used to characterize the behavior of the soft core of the sandwich shell. Zarutskii et al. [231] performed an extensive review on non linear vibrations of sandwich shells under moving and impulsive loads. Mohammadi and Sedaghati [232] focused on the damping characteristics of three layered sandwich cylindrical shells with thin and thick viscoelastic layers using a semi analytic finite element approach.

The effects of geometrical non linearity, rotary inertia, thermal loads and piezoelectric layers on the wave propagation of sandwich shells were discussed by Dong and Wang [233,234]. Kozlov et al. [235] studied the effect of piezoelectric layers on the active damping of forced thermo mechanical resonance vibrations of viscoelastic shells. In their paper, the effect of non linearity was due to the coupling of electromechanical and thermal fields.

5.2. Composite panels

Finite element formulations for non linear free vibrations of composite panels can be extensively found in the literature. Toorani [209] used a hybrid finite element approach and obtained hardening behavior for fully clamped deep orthotropic cylindrical panels. Lee et al. [236] used Donnell's theory to propose a thin laminated doubly curved panel element for the multi mode non linear free vibrations and random response of curved panels. In this paper, although

formulations were presented for composite panels, an isotropic cylindrical panel was considered for the numerical simulations. Swamy Naidu and Sinha [237] and Nanda and Bandyopadhyay [238,239] used the non linear first order shear deformation theory and an eight noded isoparametric quadrilateral element to study non linear free vibrations of doubly curved panels with simply supported boundary conditions. In particular, the effects of moisture concentration and temperature rise on the non linear response of cylindrical, spherical and hyperbolic paraboloidal panels were discussed in Ref. [237], the non linear to linear fundamental frequency of cylindrical panels with cut out was obtained in Ref. [238], and backbone curves of spherical panels with cut out were presented in Ref. [239]. Panda and Singh [240-242] used higher order shear deformation theory and the finite element method to obtain non linear fundamental frequency of cross ply and angle ply doubly curved panels with simply supported and clamped boundary conditions. A nine noded isoparametric quadrilateral element was used and the transverse and in plane displacement non linear terms were retained in the strain displacement relations. In detail, non linear fundamental frequency of cylindrical and hyperbolic paraboloidal panels was presented in Ref. [240], spherical panels were studied in Ref. [241] and thermally post buckled spherical panels were examined in Ref. [242]. Moreover, it was shown in Refs. [241,242] that thick spherical panels exhibit hardening type behavior irrespective of the type of boundary conditions, geometry and material properties. A combination of finite element and Lindstedt Poincaré perturbation method was used by Hashemian and Jam [243] to study the vibrations of symmetrically laminated cylindrical panels. Nanda [244] and Nanda and Pradyumna [245] used a combination of finite element and direct iteration method to obtain the non linear fundamental frequency of laminated cylindrical and spherical panels. Piezoelectric and thermal effects were discussed in Ref. [244], whereas hygrothermal and geometric imperfection effects were investigated in Ref. [245]. In both papers, the finite element formulations were based on higher order shear deformation theory with von Kármán type non linear terms and panels had simply supported and clamped boundary conditions. Panda and Singh [246] used the same theory and finite element formulation of Ref. [242] to obtain non linear fundamental frequency of thermally post buckled doubly curved panels. Mohammadai and Sedaghati [247] discussed the effects of different boundary conditions, small and large displacements, core thickness ratio and electric field intensity on the non linear vibration and damping characteristics of sandwich cylindrical panels with a constrained electrorheological fluid. Finite element formulations similar to Refs. [242,246] were used by Panda and Singh [248] to obtain the non linear fundamental frequency of laminated spherical panels with embedded shape memory alloy fibers.

Non linear free vibrations of composite panels were also investigated with solution techniques other than the finite element method. Kurpa [249] used the first order shear deformation theory and the R functions method to obtain the backbone curves of simply supported multi layer cylindrical panels with complex bases. The effect of variable thickness was introduced in Ref. [250]. Yazdi [251] used Donnell's non linear shallow shell theory and Galerkin single mode approximation together with the homotopy perturbation method to obtain the backbone curves of cross ply doubly curved panels.

Non linear forced vibrations of composite panels subjected to transverse harmonic loads were investigated in Refs. [26,252-256]. Experiments on non linear vibrations of graphite/epoxy cylindrical panels were carried out by Amabili et al. [252]. A plain weave fabrics and a three layer 90/0/90 panel were tested and the trend of non linearity was characterized following a stepped sine technique and by increasing and decreasing the excitation frequency in very small steps in the frequency neighborhood of the lowest natural frequencies. The results showed strong softening behavior.

Adam [253,254] studied non linear flexural vibrations of multi layered doubly curved panels based on a first order layer wise theory. In Ref. [253] the effect of initial imperfections was discussed and it was shown that, depending on the amplitude of the imperfection, a thick laminated panel may exhibit hardening or softening type non linearity, or even a combination of both. The effects of initial curvature and transverse shear flexibility on the non linear frequency amplitude response of panels composed of thick layers were examined in Ref. [254]. Abe et al. [255] used the first order shear deformation theory to study the one to one internal resonance between the second and third asymmetric vibration modes of fully clamped anti symmetric angle ply doubly curved panels. A three degree of freedom model was obtained by applying the Galerkin method and was solved by a shooting technique. Ribeiro [256] discussed the influence of in plane membrane inertia and shear deformation on non linear forced vibrations of clamped cylindrical panels. Non linear first order shear deformation theory was used and the p version finite element method with hierarchic basis functions was employed to obtain the non linear governing equations that were then integrated by using Newmark's technique. Alijani and Amabili [26] used the new theory of Amabili and Reddy [24] to study the rich non linear dynamics and diverse internal resonances of thick and deep laminated circular cylindrical panels. In particular, 1:1, 2:1, 3:1 and 1:1:2 internal resonances were found between different modes of laminated panels and quasi periodic, intermittent and chaotic responses were observed in the vicinity of the internal resonances. Moreover, it was illustrated that laminates with stronger couplings between in plane and shear strains exhibit stronger softening and/or hardening behavior.

Non linear vibrations of curved panels subjected to transverse non periodic dynamic loads were investigated by Shul'ga and Meish [257], Lentzen et al. [258], Ribeiro and Jansen [259], Nanda and Bandyopadhyay [260], Li et al. [261], Nanda [244] and Nanda and Pradyumna [245]. Shul'ga and Meish [257] used the first order shear deformation theory and a finite difference scheme to obtain the dynamic response of three layer spherical and ellipsoidal panels to impulsive loads. Lentzen et al. [258] discussed the non linear dynamic response of clamped semicircular panels with piezoelectric layers subjected to line loads using the finite element method. Ribeiro and Jansen [259] used the non linear first order shear deformation theory and the p version finite element method to study the influence of temperature and fiber orientation on the dynamics of fully clamped panels subjected to uniformly distributed impulse. They found that fiber orientations have a strong effect on the thermo mechanical behavior of panels and the response of thick panels is less influenced by the changes of temperature compared to thin ones. Fig. 8(a-c) shows the effect of fiber orientation on a symmetrically laminated circular cylindrical panel subjected to a uniformly distributed rectangular impulse. Nanda and Bandyopadhyay [260] investigated the non linear transient response of cylindrical and spherical panels with cut out using the finite element method. Non linear first order shear deformation theory was used and the responses of the panel to step and half sine loads were obtained. The non linear response of shallow sandwich panels subjected to blast loading was obtained by Li et al. [261] using a higher order shear deformation theory suitable for panels with compressible core. Nanda [244] and Nanda and Pradyumna [245] used the Newmark average acceleration method with the modified Newton Raphson iteration scheme to perform a transient analysis on piezo laminated and imperfect composite panels, respectively.

Dynamic buckling of composite panels was investigated by Fu et al. [262] and Shariyat [227,228]. Fu et al. [262] used the non linear first order shear deformation theory to study the dynamic buckling of damaged thick spherical panels under impact load. Shariyat [227,228] provided various examples for the dynamic buckling of perfect, imperfect, thick, relatively thin, multilayered

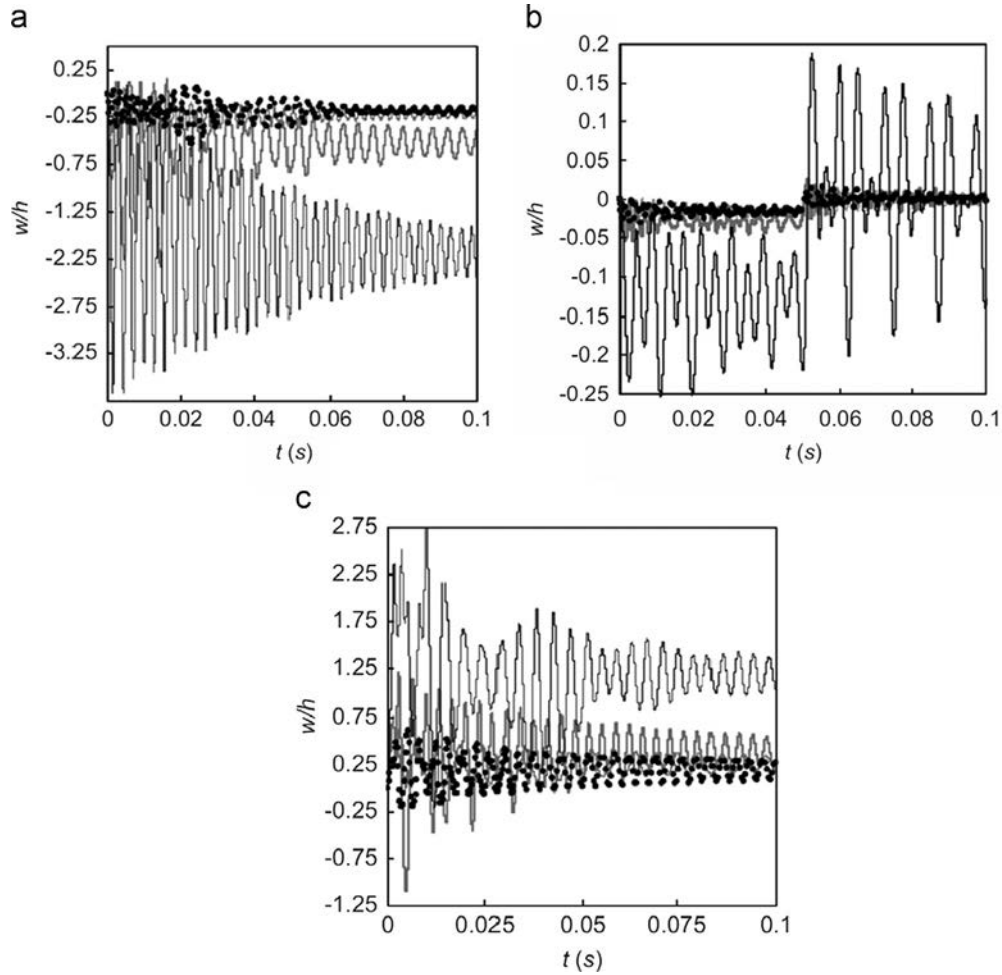


Fig. 8. Transverse displacement of the central point of a $[0/\theta] - \theta/2\theta$ s circular cylindrical panel subjected to a rectangular impulse uniformly distributed along the panel. (a) $\Delta T_0 = -50$ K; (b) $\Delta T_0 = 0$ K; (c) $\Delta T_0 = 50$ K; — $\theta = 0$; - - $\theta = 30$; ··· $\theta = 45$; —· $\theta = 60$; ΔT_0 is the average between the top and bottom surface temperatures of the panel, and θ is the orientation angle. From Ribeiro and Jansen [259].

or sandwich panels with stiff and flexible cores under suddenly applied thermal and mechanical loads.

Dynamic instability of laminated cylindrical and spherical panels with piezoelectric layers subjected to in plane periodic load and electric load was examined by Pradyumna and Gupta [263] using the finite element method. The theoretical formulation was based on the non linear first order shear deformation theory. It was found that the effect of the location of piezoelectric layers is significant at higher vibration amplitudes and marginal at lower amplitudes. Active damping of geometrically non linear vibrations of laminated curved panels with piezoelectric layers was studied by Sarangi and Ray [264,265], Karnaukhov et al. [266] and Shivakumar et al. [267].

5.3. Fluid structure interaction in composite shells

Fluid structure interaction of composite shells and panels is rarely investigated in the literature. The effects of material orthotropy on the non linear vibrations and dynamic instability of circular cylindrical shells in contact with flowing fluid were investigated by del Prado et al. [268]. Donnell's non linear shallow shell theory was used and the fluid was assumed to be non viscous and incompressible. The equations of motion were obtained by the Galerkin approach and were solved by a Runge Kutta technique. An eight degree of freedom model was considered that included the driven, companion, gyroscopic (i.e. the two modes with two longitudinal half waves) and four axisymmetric

modes. Lakiza [269] studied the non linear vibrations of fluid filled glass fiber reinforced cylindrical shells subjected to radial two frequency excitation. Non linear stability of composite shells in subsonic air flow under radial harmonic excitation was investigated by Li and Yao [270]. Their analysis was based on a rough single mode Galerkin approximation and the method of multiple scales.

Geometrically non linear finite element formulation for aero thermoelastic analysis of piezo laminated cylindrical panels was given by Oh and Lee [271]. A multi field layer wise theory was used to study the supersonic flutter characteristics of the composite panel. It was found that the critical aerodynamic pressure of piezo laminated cylindrical panels increases with the increase of surface temperature. Shin et al. [272] used the finite element method to study the non linear flutter of aero thermally buckled laminated cylindrical panels. An arc length method together with a non linear iteration scheme was used to solve the equations of motion. The flutter boundaries of various damped composite panels with different damping treatments, including constrained and sandwiched viscoelastic layers, were investigated by varying the thermal loading.

Besides common structural applications, composite shell theories are often used to model arteries conveying flow in biomechanics. As a first attempt to study the non linear response of human aortic segments, Amabili et al. [273] used the laminated circular cylindrical shell model and the non linear Sanders Koiter shell theory to study the non linear stability of an aortic segment

under steady flow conditions. The aorta was modeled as a three layer composite shell, representing the tunica intima, media and adventitia, and the fluid model contained the unsteady effects of linear potential flow theory and the steady viscous effects obtained from the time averaged Navier Stokes equations. It was shown that the aortic segment loses stability by divergence with deformation of the cross section at a critical flow velocity for a given static pressure, exhibiting a strong subcritical behavior with partial or total collapse of the inner wall.

6. Non-linear vibrations of functionally graded shells

In this section, papers that have dealt with geometrically non linear vibrations of functionally graded (FG) shells and panels are reviewed. FGMs are inhomogeneous composites fabricated from a mixture of metal and ceramic with smooth and continuous variation of material properties through the thickness. Compared to isotropic and laminated shells they have reduced thermal stresses and stress concentrations and have the capability of withstanding high temperature gradient environments without losing structural integrity. These advantages, together with their high strength and light weight, have made functionally graded materials a suitable replacement for conventional materials in many industrial applications as thermal barriers. Therefore, studying the vibration characteristics of FG structures under large amplitude thermo mechanical loads has become of paramount importance in the last decade.

6.1. Closed FGM shells

Darabi et al. [274] studied the dynamic stability of FG shells at constant temperature subjected to periodic axial loads. Donnell's non linear shallow shell theory was used and a combination of the Galerkin and the Bolotin methods was employed to solve the equations of motion. Axisymmetric modes were neglected in the displacement expansions, and therefore only hardening type behavior was obtained.

Mahmoudkhani et al. [275] used multi mode Galerkin approximation and the method of multiple scales to study the primary resonance response of simply supported cylindrical shells. Donnell's non linear shallow shell theory was used and the displacement expansion included driven, companion and axisymmetric modes. The effect of FG volume fraction exponent, length to radius and thickness to radius ratios on the trend of non linearity was discussed; however, temperature effects were neglected.

Non linear free vibrations of FG shells surrounded by Pasternak foundation were investigated by Shen [276]. Higher order shear deformation theory was used, but in a version that makes use of Airy's stress function to condensate in plane displacements, so it retains transverse and rotary inertia but neglects in plane inertia. The effective material properties of the FG shell were obtained using the rule of mixtures (Voigt model) and the Mori Tanaka micromechanics scheme, which is usually applicable to regions of the graded microstructure with well defined continuous matrix and discontinuous particulate phase. It was assumed that temperature varies only in the thickness direction and the temperature distribution was obtained by solving a one dimensional steady state heat transfer equation. A two step perturbation approach was utilized to obtain the non linear frequencies of the shell, neglecting the effect of axisymmetric modes. Hardening type non linearity was predicted for thick shells and it was found that, in most cases, Voigt and Mori Tanaka models have the same levels of accuracy in predicting the non linear response of FGM shells. Shen and Xiang [277] also used the same theory and methodology as described in [276] to study non linear vibrations of nano tube FG reinforced shells.

Bich and Xuan Nguyen [278] studied non linear free and forced vibrations of FG shells under pre load compression at constant temperature. Based on Donnell's shell theory and neglecting in plane inertia, they employed a single mode Galerkin approximation to discretize the transverse displacement. An averaging method was used to obtain the backbone curves whereas a Runge Kutta scheme was utilized to obtain the frequency response curves. The effects of axisymmetric (i.e. axisymmetric shell contraction during vibrations) and companion (i.e. 1:1 internal resonance) modes were neglected, and it is well known that this gives wrong results. Results are presented for a mode with 3 circumferential waves, when it has been established that in plane inertia should be retained at least for modes up to 3 circumferential waves (the condition $1/n^2 \ll 1$ must be satisfied in order to neglect in plane inertia, where n is the number of circumferential waves).

Non linear dynamics and bifurcations of FG shells subjected to large amplitude harmonic radial loads were investigated by Hao et al. [279,280] and Zhang et al. [281]. First order shear deformation theory was used and the Galerkin approach was employed to obtain a two degree of freedom non linear system that was solved by a fourth order Runge Kutta method in order to obtain bifurcation diagrams, Poincaré sections and time responses of the shell. In all three papers, the effect of axisymmetric and companion modes was neglected, again with erroneous results.

Sheng and Wang [282] used the first order shear deformation theory and the Galerkin multi mode approach together with the method of multiple scales to study the effects of temperature change, thickness ratio and volume fraction exponent on the non linear vibration response of the shell. Companion modes were neglected but the transverse displacement of the shell was expanded in the form of Ref. [64] and included axisymmetric and driven modes.

Rafiee et al. [283,284] studied non linear vibrations of FG shells with piezoelectric layers in thermal environments and subjected to supersonic airflow. The method of multiple scales was used and the analysis was based on a single degree of freedom approximation that showed hardening type behavior. Single mode Galerkin approximation was also performed by Najafov and Sofiyev [285] to examine the non linear vibrations of FG truncated conical shells at constant temperature. Dynamic buckling of FG conical shells subjected to time dependent compressive axial loads was investigated by Deniz and Sofiyev [286] using the Galerkin and the Runge Kutta methods.

Recently, Strozzi and Pellicano [287] used the Sanders Koiter non linear shell theory and a Lagrangian approach to investigate the forced non linear vibrations of FG circular cylindrical shells subjected to radial harmonic excitation, neglecting the thermal effects. Simply supported, clamped and free boundary conditions were considered and the shell displacements were expanded by using Chebyshev polynomials. By considering different multi dimensional models with sufficient asymmetric and axisymmetric modes, a convergence study was carried out and it was found that very short and thick, and sufficiently long and thin FG shells exhibit hardening type non linearity.

6.2. FGM panels

Liew et al. [288] investigated the non linear free vibrations of a three layer coating FGM substrate cylindrical panel with different boundary conditions and subjected to a temperature gradient across the thickness due to steady heat conduction. First order shear deformation theory was used and the numerical solution was obtained by using the differential quadrature method. The effects of geometry, volume fraction exponent, temperature and

boundary conditions on the non linear fundamental frequency of cylindrical panels were discussed.

Pradyumna et al. [289] used higher order finite element formulation to examine the non linear transient vibrations of FG doubly curved panels with rectangular base. An eight noded C^0 element with nine degrees of freedom at each node was used and the transient response of the system was calculated by the Newmark method. Pradyumna and Nanda [290] also extended their work by including initial geometric imperfections and thermal effects.

Chorfi and Houmat [291] investigated the non linear fundamental frequency of doubly curved shallow shells with clamped boundary conditions using the p version finite element method along with the harmonic balance method. They showed that FG thick panels present a hardening behavior and discussed the effect of volume fraction exponent on that behavior. In their analysis, temperature effects on system response were neglected.

A comprehensive study on the non linear vibrations of thin simply supported FG doubly curved panels at constant temperature was undertaken by Alijani et al. [292]. By deriving the equations of motion in terms of transverse deflection and Airy stress function as the only dependent variables, and in order to overcome the shortcomings of low order Galerkin discretization, the equations of motion were discretized by the full basis Galerkin approach using the complete set of eigen functions of the associated linear system to obtain a system of infinite non linear ordinary differential equations with quadratic and cubic nonlinearities. Then, using the method of multiple scales, the effects of curvature and FGM volume fraction exponent index on the non linear behavior were investigated when (i) the excitation frequency is close to the fundamental frequency (primary resonance case) and (ii) twice that frequency (sub harmonic resonance case). Subsequently, using a numerical scheme, internal resonance and chaotic vibrations were studied. Later, Alijani et al. [293] extended their work by including the thermal effects and discussed the non linear vibration characteristics of doubly curved panels with conventional non linear higher order shear deformation and Amabili Reddy non linear higher order shear deformation theories [24]. The analysis was based on multi mode Lagrangian approach and the pseudo arc length continuation scheme was used to perform bifurcation analysis. It was shown that, for thin FG spherical panels, the response is softening, turning to hardening behavior for large vibration amplitudes. However, thick spherical panels exhibit hardening non linearity. Moreover, it was revealed that the temperature plays different roles depending on the thickness and shallowness of the panel. Alijani and Amabili [294] also studied the chaotic vibrations and internal resonances of doubly curved FG panels. To classify the complex dynamics, the maximum Lyapunov exponent and the full spectrum of Lyapunov exponents were calculated for several multi dimensional models.

Bich et al. [295,296] and Duc [297] investigated the non linear vibrations and dynamic buckling of eccentrically stiffened FG panels, neglecting the temperature effects. In the three papers, the equations of motion were obtained using Donnell's shell theory. In plane inertia was neglected and the problem was reduced to a single degree of freedom non linear equation that was solved using a Runge Kutta technique. In particular, in Ref. [295] only cylindrical panels were studied, in Ref. [296] the work was extended to doubly curved panels and in Ref. [297] the panel was assumed to be on elastic foundation.

7. Non-linear vibrations of hyperelastic shells

Studies dealing with the non linear dynamic behavior of shells made of rubber like materials were primarily discussed in the

1960s and continue to the present. In fact, rubber like materials are capable of withstanding large deformations, and therefore, in addition to geometrical nonlinearities (non linear strain displacement relations), physical or material nonlinearities based on hyperelastic models (non linear stress strain relations) should also be retained in their numerical models [298]. In particular, strain invariant constitutive models such as neo Hookean, Mooney Rivlin, Ogden and Fung models should be used to capture the non linear behavior of hyperelastic materials.

Owing to the complicated nature of these models, reported studies on the dynamic problems of hyperelastic shells are very scarce. Early investigations on the large amplitude radial oscillations of circular cylindrical shells made of incompressible hyperelastic materials were performed by Knowles [299,300]. In these works the governing equation of motion was an autonomous single degree of freedom equation and the free vibrations caused by suddenly applied and subsequently maintained uniform pressures were studied. Later, Shahinpoor and Nowinski [301] and Shahinpoor and Balakrishnan [302] studied the large amplitude vibrations of circular cylindrical shells made of incompressible materials and Mooney Rivlin type rubber. In particular, an analytic solution was presented in Ref. [301] and a numerical study based on a fourth order Runge Kutta technique was employed in Ref. [302] for the single degree of freedom model previously presented by Knowles [299]. Free vibrations of neo Hookean circular cylindrical shells were studied by Wang and Ertepinar [303] and Ertepinar and Akay [304]. The finite amplitude vibrations of non linearly elastic incompressible spherical and cylindrical shells were examined mathematically by Calderer [305]. Akyuz and Ertepinar [306] investigated the stability and asymmetric vibrations of pressurized compressible hyperelastic cylindrical shells. The hyperelastic model that they used was based on Levinson Burgess polynomial compressible material, which is coincident with the neo Hookean law for incompressible materials. In order to solve the problem, they first found the static equilibrium state of the solution for finite deformations, and then used the small perturbations theory to investigate the free vibrations of the shell around the equilibrium state. A shooting method was also utilized to solve the governing equations of motion and the numerical solutions were found to be in good agreement with the finite element results.

Since 2003, new researches performed on hyperelastic shells have mostly been focused on the stability and buckling of elastic shells subjected to external/internal pressure. For instance, based on the non linear theory of elasticity, Zhu et al. [307] discussed the asymmetric bifurcations of thick shells made of isotropic neo Hookean materials and under combined axial loading and external pressure. Zhu et al. [308] extended their previous work [307] by studying the non linear axisymmetric deformations of incompressible hyperelastic shells subjected to zero displacements on their ends and pressure on their external lateral surfaces. Recently, the same authors [309] investigated the three dimensional non linear buckling of pressurized circular cylindrical shells using a finite element algorithm and highlighted the post buckling deformations.

Concerning non linear vibrations of hyperelastic shells, the only works available since 2003 are those of Sansour et al. [176], Yuan et al. [310], Yong et al. [151], He et al. [152] and Ren [311,312]. In particular, Sansour et al. [176] constructed an energy momentum method to study the non linear dynamics of shells in the framework of the finite element method and discussed the large amplitude response of shells considering non linear constitutive laws. Yuan et al. [310] studied the dynamic inflation of infinitely long cylindrical tubes subjected to periodic step pressures on the inner surface. The incompressible Ogden material model was used and the strain energy density function was reduced to neo Hookean type. Sensitivity of the dynamics to the material parameters, geometry and the applied pressures was discussed. In Refs. [151,152] the dynamics of dielectric incompressible neo Hookean spherical and cylindrical

shells was studied, respectively, and it was found that the system is more stable when the thickness of the shell increases. Ren [311] determined the critical applied pressure of an incompressible neo Hookean circular cylindrical shell subjected to periodic or suddenly applied constant load on the inner surface. He found that when the pressure is lower than the critical value, the fixed point is a center surrounded by a homoclinic orbit, and the shell undergoes non linear periodic oscillations. On the other hand, when the pressure is larger than the critical value, the fixed point becomes an unstable saddle node, and the shell's displacements will ultimately diverge. The same author [312] also discussed the dynamics of neo Hookean spherical shells, and by plotting Poincaré maps and phase plane plots showed that the shell may undergo quasi periodic oscillations once it is subjected to periodic internal pressure.

8. Experiments on large-amplitude vibrations of shells

Although a vast literature is available on geometrically non linear vibrations of shells and panels, the experimental investigations are quite scarce. A complete list of experimental works on non linear vibrations and fluid structure interactions of shell structures before 2003 can be found in the review study of Amabili and Paidoussis [5].

Comprehensive experimental analysis on large amplitude vibrations of empty and fluid filled circular cylindrical shells subjected to radial harmonic excitation was carried out by Amabili [56]. Pellicano and Avramov [94] and Pellicano [95] performed experiments on cylindrical shells connected to a rigid disk and under base excitation to obtain damping ratios for numerical simulations. Pellicano [96] extended his previous experimental tests by following a stepped sine approach and obtained the experimental stability diagrams of a shell connected to top mass. Experiments on the stability of shells subjected to fluid flow were carried out by Karagiozis et al. [121,122,124,125]. Shells having clamped ends and subjected to external airflow and internal water flow were tested. Lakiza [313,314] performed experiments on non linear vibrations of compound (truncated cone with spherical shell) and circular cylindrical shells with a gas liquid medium, respectively.

Experiments on non linear vibrations of cylindrical panels have been conducted by Amabili et al. [157] and Amabili [160]. Geometrically non linear dynamic behavior of cylindrical panels with complex base subjected to transverse excitation has been investigated by Kurpa et al. [170]. Experimental investigation of chaos in cylindrical and doubly curved panels has been carried out by Nagai et al. [180,184] and Maruyama et al. [183]. Thomas et al. [192] performed experimental tests on a free edge spherical cap in order to validate the occurrence of a rare modal interaction coupling.

Kubenko and Koval'chuk [222] studied the dynamic stability of a three layer composite shell experimentally. Mallon [223] and Mallon et al. [224] performed experiments on an orthotropic circular cylindrical shell subjected to base (seismic like) excitation. The only available experimental work on geometrically non linear vibrations of composite cylindrical panels has been conducted by Amabili et al. [252].

9. Conclusions

The purpose of this review is (i) to help researchers working in the field of non linear vibrations of shell structures with the required references for their research and (ii) to give ideas of what researches can be undertaken and what researches could be of paramount importance. In writing this review the authors have tried to be as complete as possible. However, it is plausible that some studies are missed out. The authors apologize to the

authors of papers that have accidentally been left out of this review.

It must be observed that the research performed on large amplitude vibrations of shell structures, specifically for those made of advanced materials, is still far from being considered well established. Regarding isotropic materials, the research on geometrically non linear vibrations of conical shells and panels is still in the primary stage and is hoped to be enhanced in the near future. In case of composite shells and panels (i) experimental investigations and (ii) non linear fluid structure interactions can be rarely found in the literature, and therefore need particular attention. For FG shells, most of the studies that are published neglect the fundamental and preliminary features of shell structures, such as the effect of axisymmetric modes in closed shells and in plane modes in curved panels. Thus, in order to clarify the non linear dynamic behavior of this type of shells, a more careful investigation is required. Finally, in case of hyperelastic shells, no comprehensive research has yet been conducted on the dynamic instability and non linear vibrations of highly pressurized shells taking into account shear deformation and thickness variation effects.

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