A mixed model for estimating the probabilistic worst case execution time

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firstname.lastname@inria.fr, AOSTE team, INRIA Paris-Rocquencourt

1 Introduction

Probabilistic analysis are considered nowadays an interesting solution for real-time systems as the probability of appearance of worst-case values is small (10^{-45} per hour of functioning) compared to the accepted probability of failure (10^{-9} per hour of functioning for the highest safety level in avionics). In order to take into account this information, Burns and Edgar [1] have introduced the notion of probabilistic worst case execution time (pWCET). The pWCET of a program is bounding the probability that the execution time of that program exceeds a given value. A possible method to estimate the pWCET is based on measurements and the associated analysis is called measurement-based probabilistic timing analysis (MBPTA). Such method has been proposed by Cucu-Grosjean et al. [2] and the obtained estimate is sensitive to the observed execution times. To our best knowledge this dependence of MBPTA on the observations is an open problem. Within this paper we propose a first solution based on a mixed model using genetic algorithms.

2 Measurement based probabilistic time analysis

MBPTA provides from perfect input data the tightest pWCET estimate of a program on a platform. The perfect input data is a finite data set yet representative with respect to all possible factors that may influence the execution time of the program on that platform. Please note that the user may be happy to have a pWCET estimate that upper bounds the tightest pWCET. In Figure 1 we illustrate the relation between input data that may be the execution times of a program on a platform, a MBPTA and the pWCET of that program on that platform.

2.1 Extreme Value Theory

Extreme Value Theory (EVT) estimates the probability of occurrence of extreme values which are known to be rare events, see Embrecht et al. [4]. More precisely, EVT predicts the distribution function for the maximal values of a set of n observations, which are modeled with random variables. The theoretical basis of the block maxima approach of the EVT is built on the theorem which says that the asymptotic behavior of the maximum of a sufficiently large sample is a Generalized Extreme Value (GEV) distribution. In the same way as for the Central Limit Theorem (which states that the sample's mean converges in distribution to a Gaussian variable, whenever the sample variance is finite) a stability property is the key element to understand and to find this distributional convergence of maxima. Main hypotheses must be verified: that the n random variables are independent and identically distributed and the existence of the sequence of real numbers $(a_n; b_n)$. The GEV distribution is defined by three parameters: shape (ξ), scale (σ) and location (μ). By determining these three parameters, we prove the existence of the sequence of real numbers $(a_n; b_n)$. The values used as input for EVT are grouped into blocks of equal length m by applying the block maxima method. Once we found the block size fitting the best our data we can compute the pWCET.

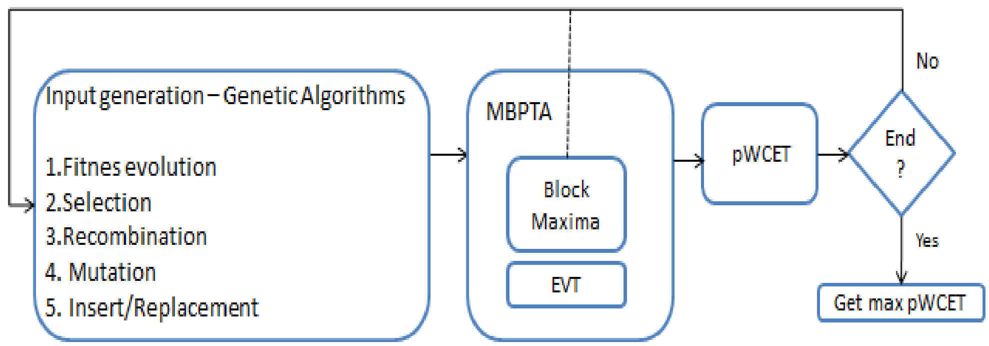

Figure 1: The relation between the input data, genetic algorithms, MBPTA and the obtained pWCET

2.2 Genetic Algorithm

Evolutionary algorithms iteratively generate populations, i.e., sets of individuals, by means of reproduction features using stochastic operators such as selection, recombination and mutation. One technique that follows this pattern is the genetic algorithms [5], where individuals are represented as strings of numbers.
In the first rectangle of Figure 1 there can be seen the main components of a genetic algorithm. We think it is possible to use this technique to generate inputs for a known program [3] and to apply MBPTA on the obtained execution times. Our purpose is to combine the block maxima method of MBPTA with the selection component of the genetic algorithm. Therefore, by selecting only the inputs that yield the maximum observe times, and obtaining new inputs from the recombination of these ones, we get a new population for the next round of experiments. After applying MBPTA on the each set of execution times obtained at each simulation we manage to get a list of execution times with a required probability. Out of this list we will choose the maximum value and compare it with the WCET obtained statically.

3 Conclusions

At this point there are series of question rising. How many numbers of simulations do we need in order to get a pWCET close to the WCET? Is the model managing to produce a total number of inputs that manage to cover large spectrum of possible inputs? Is there a need for a filtering method, like model checking, that can exclude the inputs that produce small execution times? Is it possible to obtain an input that yields a large execution time from the recombination of two inputs that generated small execution times?

4 Bibliography

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