A Single-Phase Two-Winding Transformer Dynamic Model for Circuit Simulators
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Transformers are magnetic components widely used in switched-mode power electronics systems. The non-linear hysteresis behavior of the magnetic material and the high frequency effects in both core and windings have significant effects on system's efficiency, reliability and power losses. This behavior can be modeled using simple fast models or complex accurate models in order to predict and improve the transformer behavior before realization. This paper is summarized by proposing a non-linear dynamic model of transformers for use in circuit simulators. This model allows winding and core modeling including the material’s accurate nonlinear dynamic hysteresis behavior. The magnetic component model is implemented in the circuit simulation software “Simplorer” using VHDL-AMS modeling language. It is validated for a medium-frequency nanocrystalline core transformer. Effects of frequency and waveform on computed efficiencies are discussed and validated thanks to experimental measurements.

Index Terms—Magnetic hysteresis, transformers, energy efficiency, Circuit simulation,…

I. INTRODUCTION

VIRTUAL prototyping is essential in the development of new power electronics devices like power converters and power transformers [1], [2]. Circuit simulators allow the optimization of complex circuit prototypes by associating models of passive and active components [3]. Indeed modeling and simulation allow the prior knowledge of the system’s behavior. Magnetic components constitute a major part of electronic devices. Therefore accurate modeling of magnetic materials is mandatory to predict their behavior under realistic operating conditions. Several efforts have contributed to the modeling of magnetic components such as single and three phase transformers. Transformer models included: Analytical models [4], equivalent circuit models [5], and finite elements models [6]. Unlike most of state-of-the-art models which are limited either to, static conditions [7], sinusoidal waveforms [8], or non-conducting media [9], the model presented in this paper overcomes some of these limitations. The study in this paper is based on the magnetic modeling approach presented in [10], where the authors have presented a non-linear dynamic inductor model for circuit simulators. As the model is generic and modular this paper presents its extension to a transformer model. The effects of transformer’s loading on losses and thus on efficiency are taken into account by the proposed model. High frequencies up to 100 kHz and waveform influence on losses and efficiency are also studied and discussed in this paper.

II. TRANSFORMER MODELING

A transformer is a static device used in the generation, transmission and distribution of electric power. They generally consist of two or more insulated windings, and a magnetic core to insure good coupling between windings [11]. The model must be able to take into account the magnetomotive force (MMF) induced in the secondary winding and thus increase the primary current based on the value of secondary current (transformer’s loading). The model consists of different blocks: windings blocks, a static core block and a dynamic core block (last two blocks represent the magnetic core) as shown in Fig 1. VHDL-AMS language [12] is chosen to describe each block behavior due to its multi-domain modeling feature and since it is supported by several circuit simulators.

A. Winding block

The winding block represents a coupling element between electric and magnetic circuits. It converts electrical energy into magnetic energy and vice versa (bidirectional energy flow). The winding block has two electrical pins, and two magnetic pins. It has a global parameter, the number of turns (N) for coupling quantities between magnetic and electric domains. These domains are related by Ampere’s law linking the magnetic field H to the magnetizing electric current i₀ [1] and by Faraday’s law linking induced voltage e to magnetic induction B [2]. Aₑ and ℓ are the cross section and the mean path length of the core respectively. Consequently, the transformer model has two winding blocks representing the...
primary and secondary windings having respectively $R_{w1}$ and $R_{w2}$ winding resistances.

$$H \cdot \ell = N \cdot i_0$$  

$$e(t) = -NAc \cdot \frac{dB}{dt}$$  

**B. Dynamic block**

The dynamic block is the center of the transformer model representing the overall behavior of the magnetic material core. This block contains a dynamic behavioral model which relates the applied magnetic field $H$ to the flux density $B$. The magnetic core model is based on the principle of separation of losses into static and dynamic contributions, as well as Bertotti’s theory [13]. The applied magnetic field is the sum of static field $H_s$ and dynamic fields due to eddy currents $H_{ec}$ and wall motion effects $H_{wm}$  and 4. As $\gamma$ is related to the electrical resistivity $\rho$ and on the ribbon thickness $d$, it can be identified analytically [13]. The remaining parameter $\alpha$ is identified thanks to dynamic measurements.

$$H(t) = H_s(B) + H_{ec} + H_{wm}$$  

$$H = H_s(B) + \alpha \frac{dB}{dt} + \text{sign}(\frac{dB}{dt}) \cdot \alpha \cdot \left| \frac{dB}{dt} \right|^{1/2}$$  

**C. Static Block**

The dynamic magnetic model requires a static model to calculate $H_s(B)$. This is the role of the static block. It may be composed by one of the existing physical or mathematical static magnetic models such as Polynomial model [14], Jiles-Atherton model [9], or Preisach-Néel model [15]. In this work, the polynomial model, detailed in [14], is used to describe the static behavior of the materials. This mathematical model [5] is based on a piecewise polynomial function of order $n$. It is reversible and limited to materials with very low coercive field.

$$B = \begin{cases} \mu_0(H - H_b) + P(H_b) & \text{if } H \geq H_b \\ P(H) & \text{if } |H| < H_b \\ \mu_0(H + H_b) - P(H_b) & \text{if } H \leq -H_b \end{cases}$$  

where $H_b$ is the field needed to reach saturation and $P$ is a polynomial function such that $P(X) = \sum_{i=0}^{n} p_i X^i$.

**III. MAGNETIC MATERIAL CHARACTERIZATION**

Among available materials for medium frequency transformers, the nanocrystalline Nanophy® cores N14E1, provided by Mecagis®, are chosen due to their interesting properties including high saturation flux density ($B_s$), low coercive field ($H_c$) and high relative permeability ($\mu_r$). Material characterization is mandatory to identify the parameters of the static and dynamic models. The Table I resumes some of the magnetic material physical properties and geometrical dimensions.

**TABLE I**

<table>
<thead>
<tr>
<th>Physical properties and geometrical dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s$ (T)</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>1.25</td>
</tr>
<tr>
<td>$d$ (mm)</td>
</tr>
<tr>
<td>20</td>
</tr>
</tbody>
</table>

**A. Static measurements and modeling**

A simple $B(H)$ major loop measurement (see Fig. 2) and basic mathematical fitting is necessary to identify the needed static parameters that are resumed in the table II. For this material a polynomial model of order 5 is sufficient to represent its static behavior, where the even coefficients are almost zero due to the odd parity of the $B(H)$ curve.

**B. Dynamic measurements and modeling**

Several dynamic measurements are performed on the magnetic core to identify the remaining dynamic parameter $\alpha$ ($\gamma$ is directly calculated). Identified values of $\alpha$ and $\gamma$ are reported in Table II. Fig. 3 shows measured $B(H)$ loops under a 280 mT sinusoidal applied flux density for 3 different frequencies (60, 80 and 100 kHz) and an example of a simulated loop.

**TABLE II**

<table>
<thead>
<tr>
<th>Static and dynamic parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
</tr>
<tr>
<td>0.041</td>
</tr>
</tbody>
</table>

**C. Loss measurements**

Fig. 4 shows the core losses versus frequency for different sinusoidal applied flux densities. These measurements will be useful later to analyze the behavior of the studied transformer.

**IV. SIMULATIONS AND MODEL VALIDATION**

**A. Transformer losses and efficiency determination**

In order to validate the developed model, a 10 W-100 kHz transformer is realized. The circuit design allows variable
voltage, frequency and load. The same magnetic material characterized is used as the transformer’s core with five turns for primary and secondary windings. To measure precisely primary and secondary currents, a shunt resistor (500 mΩ) is added to the primary and secondary sides of the transformer. Set of resistive loads between 5 Ω and 50 Ω are available to vary the load percentage (from 15% to 100%). The transformer is operated so as to have a desired flux density peak value in the core (example $B = 280$ mT) while measuring the primary and secondary voltages and currents for variable loads. From the primary and secondary currents the no-load current $i_o$ related to the applied excitation field $H$ (1) is calculated. From $v_2$ and $i_2$, the induced voltage $e$ then $B$ (2) inside the magnetic circuit can be retrieved. With $H$ and $B$, the core losses $p_c$ can be evaluated precisely for each transformer test conditions. The winding losses $p_w$ are AC only and are calculated using (4). $R_{AC}$ is the AC resistance due to skin and proximity effects and $I_{AC}$ is the RMS current in a single winding. The AC resistance has been determined using an Agilent 4294A impedance analyzer [16]. Choosing a copper wire of 0.5 mm diameter, $R_{AC}$ is about 20 mΩ in the 10 kHz-100 kHz frequency range (no skin effects up to 100 kHz).

Finally, the efficiency $\eta$ can be calculated using (7) with $P_2$ the secondary power.

$$p_w = \frac{R_1 I_{1AC}^2 + R_2 I_{2AC}^2}{P_2}$$

(6)

$$\eta = \frac{P_2}{P_2 + p_c + p_w}$$

(7)

B. Circuit simulation: Results and discussion

1) Results

A transformer circuit identical to the one used for measurements was simulated using Simplorer® software. The static and dynamic parameters were inserted in the static and dynamic model respectively in addition to the core dimensions. Winding resistances and number of turns were inserted to the primary and secondary winding blocks. Simulations were carried out for the same input voltage and frequency and for load variation between 15% and 100%. Primary and secondary voltages and currents were available from simulations. Core and winding losses were then calculated and the transformer’s efficiency was deduced. Simulation results were compared to measured ones. The efficiency for three different frequencies up to 100 kHz at constant flux density level ($B = 280$ mT) are shown in Fig. 5.

From Fig. 5, taking the efficiency curves at a constant frequency (60 kHz for example), we can notice that the efficiency increases with load. As the flux density is almost constant, core losses are not affected by load variation. However, as copper losses are proportional to the square of the RMS current, they become more and more significant in the calculation of (7). On the other hand, as the frequency varies at a constant flux density, the core losses increase with frequency (see Fig. 4). As a result the efficiency decreases with increasing frequency. At any given load, the efficiency difference with measured ones. The efficiency for three different frequencies up to 100 kHz at constant flux density level ($B = 280$ mT) are shown in Fig. 5.

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(7)
simulated efficiencies. In many power electronics applications, magnetic materials are excited with non-sinusoidal waveforms. This induces different losses compared to those provided by manufacturers based on sinusoidal flux density \[17\]. To test the model under non-sinusoidal excitation, a square voltage (thus a triangular flux density) at a frequency of 100 kHz is applied to the transformer. The voltage level is adapted in order to have the same applied flux density peak value as in the sinusoidal case. Resulting measured and simulated efficiencies are compared in Fig. 6.

Fig. 6. Measured (solid lines) and simulated (dashed lines) efficiencies versus relative load for different voltage waveforms at $B = 280$ mT and $f = 100$ kHz

In this case core losses under square voltage are lower than those under sinusoidal voltage (under the condition of 50% duty cycle). This results in a higher efficiency value for square voltage.

2) discussion

In view of all previous results, a complete magnetic model has many advantages. Voltages, currents, magnetic fields, flux densities, losses, hysteresis loops and efficiencies are available from simulation and are in good agreement with measured ones. Besides its precision and its adaptability to different operating conditions, the real benefits of the modeling approach herein presented are its real-time aspect (the power balance is respected at any time of the simulation), ease of implantation and rapidity. However, since the model reproduces core losses accurately, we believe that the major part of the remaining error between simulated and measured efficiency (mostly visible for high relative loads) is related to flux leakages which are not considered in the current model.

V. CONCLUSION

In this paper the development of a middle frequency transformer made with a nanocrystalline core model for use in circuit simulation is presented. Model’s parameters are extracted from magnetic material characterization under various conditions. Simulations of a 10 W-100 kHz transformer including the proposed model are compared to measurements performed at variable voltages, frequencies and loads. The developed model allows precise prediction of core and winding losses. As a consequence the transformer’s efficiency is directly available at the end of the simulation (no post correction calculations needed). Experiments confirm model’s loss prediction accuracy with a mean square error of about $4e^{-4}$ between simulated and measured efficiencies. Adding leakage elements (electric inducances or magnetic reluctances) to the model is of our future interests in the aim of even more accurate virtual prototyping. Simulation of other circuits or materials, including other or adapted static and dynamic laws is also of great interest for future work to help engineers to design innovative and effective new products.

REFERENCES