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Approximate Capacity of the Gaussian Interference Channel with Noisy Channel-Output Feedback

Víctor Quintero, Samir M. Perlaza, Íñaki Esnaola and Jean-Marie Gorce

Abstract—In this paper, an achievability region and a converse region for the two-user Gaussian interference channel with noisy channel-output feedback (G-IC-NOF) are presented. The achievability region is obtained using a random coding argument and three well-known techniques: rate splitting, superposition coding and backward decoding. The converse region is obtained using some of the existing perfect-output feedback outer-bounds as well as a set of new outer-bounds that are obtained by using genie-aided models of the original G-IC-NOF. Finally, it is shown that the achievability region and the converse region approximate the capacity region of the G-IC-NOF to within a constant gap in bits per channel use.

Index Terms—Capacity, Interference Channel, Noisy Channel-Output Feedback.

I. NOTATION

Throughout this paper, $(\cdot)^+$ denotes the positive part operator, i.e., $(\cdot)^+ = \max\{\cdot, 0\}$ and $\mathbb{E}[\cdot]$ denotes the expectation with respect to the distribution of the random variable $\mathcal{X}$. The logarithm function log is assumed to be base 2.

II. SYSTEM MODEL

Consider the two-user G-IC-NOF in Figure 1. Transmitter $i$, with $i \in \{1, 2\}$, communicates with receiver $i$ subject to the interference produced by transmitter $j$, with $j \in \{1, 2\} \setminus \{i\}$. There are two independent and uniformly distributed messages, $W_i \in \mathcal{W}_i$, with $\mathcal{W}_i = \{1, 2, \ldots, 2^{N_i}\}$, where $N_i$ denotes the block-length in channel uses and $R_i$ is the transmission rate in bits per channel use. At each block, transmitter $i$ sends the codeword $X_i = (X_{i,1}, X_{i,2}, \ldots, X_{i,N})^T \in \mathcal{X}_i^N$, where $\mathcal{X}_i$ and $\mathcal{X}_i^N$ are respectively the channel-input alphabet and the codebook of transmitter $i$.

The channel coefficient from transmitter $j$ to receiver $i$ is denoted by $h_{ij}$; the channel coefficient from transmitter $i$ to receiver $i$ is denoted by $h_{ii}$; and the channel coefficient from channel-output $i$ to transmitter $i$ is denoted by $\bar{h}_{ii}$. All channel coefficients are assumed to be non-negative real numbers. At each channel use $n \in \{1, 2, \ldots, N\}$, the channel output at receiver $i$ is denoted by $Y_{i,n}$. During channel use $n$, the input-output relation of the channel model is given by

$$
Y_{i,n} = \bar{h}_{ii}X_{i,n} + h_{ij}X_{j,n} + Z_{i,n}, \quad (1)
$$

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![Figure 1. Gaussian interference channel with noisy channel-output feedback at channel use $n$.](image-url)
Assume that during a given communication, \( T \) blocks are transmitted. Hence, the decoder of receiver \( i \) is defined by a deterministic function \( \psi_i : \mathbb{R}^{NT} \rightarrow \mathbb{W}^T_i \). At the end of the communication, receiver \( i \) uses the vector \((\vec{Y}_{i,1}^{(t)}, \vec{Y}_{i,2}^{(t)}, \ldots, \vec{Y}_{i,NT}^{(t)})\) to obtain an estimate of the message indices
\[
(\hat{W}_{i}^{(1)}, \hat{W}_{i}^{(2)}, \ldots, \hat{W}_{i}^{(T)}) = \psi_i (\vec{Y}_{i,1}^{(t)}, \vec{Y}_{i,2}^{(t)}, \ldots, \vec{Y}_{i,NT}^{(t)}),
\]
where \( \hat{W}_{i}^{(t)} \) is an estimate of the message index sent during block \( t \in \{1, 2, \ldots, T\} \). The decoding error probability in the two-user G-IC-NOF during block \( t \) of a codebook of blocklength \( N \), denoted by \( P_e^{(t)}(N) \), is given by
\[
P_e^{(t)}(N) = \max \left( \Pr \left[ \hat{W}_{i}^{(t)} \neq W_{i}^{(t)} \right], \Pr \left[ \hat{W}_{j}^{(t)} \neq W_{j}^{(t)} \right] \right).
\]
The definition of an achievable rate pair \((R_1, R_2) \in \mathbb{R}^2_+\) is given below.

Definition 1 (Achievable Rate Pairs): A rate pair \((R_1, R_2) \in \mathbb{R}^2_+\) is achievable if there exists at least one pair of codebooks \( \mathcal{X}_1^N \) and \( \mathcal{X}_2^N \) with codewords of length \( N \), and the corresponding encoding functions \( f_1^{(N)} \), \( f_2^{(N)} \) such that the decoding error probability \( P_e^{(t)}(N) \) can be made arbitrarily small by letting the blocklength \( N \) grow to infinity, for all blocks \( t \in \{1, 2, \ldots, T\} \).

The two-user G-IC-NOF in Figure 1 can be fully described by six parameters: \( \text{SNR}_{i1}, \text{SNR}_{i2} \), and \( \text{INR}_{ij} \), with \( i \in \{1, 2\} \) and \( j \in \{1, 2\} \backslash \{i\} \). The functions are defined as follows:
\[
\begin{align*}
\text{SNR}_{i1} &= h_{i1}^2, \\
\text{SNR}_{i2} &= h_{i2}^2, \\
\frac{\text{SNR}_{ij}}{\text{INR}_{ij}} &= h_{ij}^2 (\gamma_{i1}^2 + 2h_{i1}h_{ij} + h_{ij}^2 + 1).
\end{align*}
\]

\section{III. MAIN RESULTS}

This section introduces an achievable region (Theorem 1) and a converse region (Theorem 2), denoted by \( \mathcal{C}_{G-IC-NOF} \) and \( \mathcal{C}_{G-IC-NOF}^c \), respectively, for the two-user G-IC-NOF with fixed parameters \( \text{SNR}_{i1}, \text{SNR}_{i2}, \text{INR}_{i12}, \text{INR}_{i21}, \text{SNR}_{i1}, \) and \( \text{SNR}_{i2} \). In general, the capacity region of a given multi-user channel is said to be approximated to within a constant gap according to the following definition.

Definition 2 (Approximation to within \( \xi \) units): A closed and convex set \( \mathcal{T} \subseteq \mathbb{R}^m \) is approximated to within \( \xi \) units by the sets \( \mathcal{T} \) and \( \tilde{T} \) if \( \mathcal{T} \subseteq \tilde{T} \subseteq \mathcal{T} \) and for all \( t = (t_1, \ldots, t_m) \in \mathcal{T} \) then \((t_1 - \xi, \ldots, t_m - \xi) \in \tilde{T}\).
Denote by \( \mathcal{C}_{G-IC-NOF} \) the capacity region of the 2-user G-IC-NOF. The achievable region \( \mathcal{C}_{G-IC-NOF} \) and the converse region \( \mathcal{C}_{G-IC-NOF}^c \) approximate the capacity region \( \mathcal{C}_{G-IC-NOF} \) to within 4.4 bits per channel use (Theorem 3).

\section{A. An Achievable Region for the Two-User G-IC-NOF}

The description of the achievable region \( \mathcal{C}_{G-IC-NOF} \) is presented using the constants \( a_{1,i}, a_{2,i} : [0, 1] \rightarrow \mathbb{R}_+ \), \( a_{l,i} : [0, 1]^2 \rightarrow \mathbb{R}_+ \), with \( l \in \{3, \ldots, 6\} \); and \( a_{7,i} : [0, 1]^3 \rightarrow \mathbb{R}_+ \), which are defined as follows, for all \( i \in \{1, 2\} \), with \( j \in \{1, 2\} \backslash \{i\} \):
\[
\begin{align*}
a_{1,i} &= \frac{1}{2} \log \left( 2 + \frac{\text{SNR}_{i1}}{\text{INR}_{ij}} \right) - \frac{1}{2}, \\
\text{SNR}_{i1} &= h_{i1}^2, \\
\frac{\text{SNR}_{ij}}{\text{INR}_{ij}} &= h_{ij}^2 (\gamma_{i1}^2 + 2h_{i1}h_{ij} + h_{ij}^2 + 1). \quad (9)
\end{align*}
\]

\section{B. Comments on the Achievability}

The achievable region is obtained using a random coding argument and combining three classical tools: rate splitting, superposition coding, and backward decoding. This coding scheme is described in [1] and is specially designed for the two-user IC-NOF. Consequently, only the strictly needed number of superposition code-layers is used. Other achievable schemes, as reported in [2], can also be obtained as special cases of the more general scheme presented in [3]. However, in this more general case, the resulting code for the IC-NOF contains a handful of unnecessary superposing code-layers, which complicates the error probability analysis.

\section{C. A Converse Region for the Two-User G-IC-NOF}

The description of the converse region \( \mathcal{C}_{G-IC-NOF}^c \) is determined by the ratios \( \frac{\text{INR}_{i1}}{\text{SNR}_{i1}} \) and \( \frac{\text{INR}_{i2}}{\text{SNR}_{i2}} \), for all \( i \in \{1, 2\} \), with \( j \in \{1, 2\} \backslash \{i\} \). All relevant scenarios regarding these ratios.
Theorem 1: The capacity region $\mathcal{C}_{\text{GIC–NOF}}$ contains the region $\mathcal{C}_{\text{G–IC–NOF}}$ given by the closure of the set of all possible non-negative achievable rate pairs $(R_1, R_2)$ that satisfy

\begin{align}
R_1 &\leq \min \left( a_{2,1}(\rho), a_{6,1}(\rho, \mu_1) + a_{3,2}(\rho, \mu_1), a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{4,2}(\rho, \mu_1) \right), \\
R_2 &\leq \min \left( a_{2,2}(\rho), a_{5,1}(\rho, \mu_2) + a_{6,2}(\rho, \mu_2), a_{3,1}(\rho, \mu_2) + a_{4,1}(\rho, \mu_2) + a_{1,2} \right), \\
R_1 + R_2 &\leq \min \left( a_{2,1}(\rho) + a_{1,2}, a_{1,1} + a_{2,2}(\rho), a_{3,1}(\rho, \mu_2) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{7,2}(\rho, \mu_1, \mu_2), \\
as_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{3,2}(\rho, \mu_1), a_{3,1}(\rho, \mu_2) + a_{7,1}(\rho, \mu_1, \mu_2) + a_{3,2}(\rho, \mu_1) + a_{1,2} \right), \\
2R_1 + R_2 &\leq \min \left( a_{2,1}(\rho) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{7,2}(\rho, \mu_1, \mu_2), \\
as_{3,1}(\rho, \mu_2) + a_{1,1} + a_{7,1}(\rho, \mu_1, \mu_2) + 2a_{3,2}(\rho, \mu_1) + a_{5,2}(\rho, \mu_1), a_{2,1}(\rho) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{5,2}(\rho, \mu_1) \right), \\
R_1 + 2R_2 &\leq \min \left( a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{2,2}(\rho) + a_{1,2}, a_{3,1}(\rho, \mu_2) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{2,2}(\rho) + a_{1,2}, \\
2a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{3,2}(\rho, \mu_1) + a_{1,2} + a_{7,2}(\rho, \mu_1, \mu_2) \right),
\end{align}

with $(\rho, \mu_1, \mu_2) \in \left[0, 1 - \max \left(\frac{1}{\text{INR}_1}, \frac{1}{\text{INR}_2}\right)\right] \times [0, 1] \times [0, 1].$

are described by two events denoted by $S_{i,1}$ and $S_{i,2}$, where $(l_1, l_2) \in \{1, \ldots, 5\}^2$. The events are defined as follows:

\begin{align}
S_{1,i} : \text{INR}_j &\leq \min \left( \text{SNR}_j, \text{INR}_{ji} \right), \\
S_{2,i} : \text{INR}_{ji} &\leq \text{SNR}_j < \text{INR}_{ji}, \\
S_{3,i} : \text{INR}_{ji} &\leq \text{SNR}_j < \text{INR}_{ji}, \\
S_{4,i} : \max \left( \text{INR}_{ji}, \text{INR}_{ji} \right) &\leq \text{SNR}_j < \text{INR}_{ji}, \text{INR}_{ji}, \\
S_{5,i} : \text{SNR}_j &\geq \max \left( \text{INR}_{ji}, \text{INR}_{ji}, \text{INR}_{ji}, \text{INR}_{ji} \right).
\end{align}

Note that for all $i \in \{1, 2\}$, the events $S_{i,1}$, $S_{i,2}$, $S_{i,3}$, $S_{i,4}$, and $S_{i,5}$ are mutually exclusive. This observation shows that given any 4-tuple $(\text{SNR}_1, \text{SNR}_2, \text{INR}_{12}, \text{INR}_{21})$, there always exists one and only one pair of events $(S_{i,1}, S_{i,2})$, with $(l_1, l_2) \in \{1, \ldots, 5\}^2$, that identifies a unique scenario. Note also that the pairs of events $(S_{2,1}, S_{2,2})$ and $(S_{3,1}, S_{3,2})$ are not feasible. In view of this, twenty-three different scenarios can be identified using the events in (13). Once the exact scenario is identified, the converse region is described using the functions $\kappa_{1,i} : [0, 1] \to \mathbb{R}_+$, with $(l, i) \in \{1, \ldots, 3\} \times \{1, 2\}$; $\kappa_2 : [0, 1] \to \mathbb{R}_+$, with $l \in \{4, 5\}; \kappa_{6,1} : [0, 1] \to \mathbb{R}_+$, with $l \in \{1, \ldots, 4\}$; and $\kappa_{7,1,i} : [0, 1] \to \mathbb{R}_+$, with $(i,l) \in \{1,2\}^2$. These functions are defined as follows for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

\begin{align}
\kappa_{1,i}(\rho) &= \frac{1}{2} \log \left( b_{1,i}(\rho) + 1 \right), \\
\kappa_{2,i}(\rho) &= \frac{1}{2} \log \left( 1 + b_{5,j}(\rho) \right) + \frac{1}{2} \log \left( 1 + \frac{b_{4,i}(\rho)}{1 + b_{5,j}(\rho)} \right), \\
\kappa_{3,i}(\rho) &= \frac{1}{2} \log \left( \frac{\text{SNR}_j}{b_{1,j}(\rho) + b_{5,j}(\rho) + 1} \right) + 1 \\
&+ \frac{1}{2} \log \left( b_{1,i}(\rho) + 1 \right), \\
\kappa_{4}(\rho) &= \frac{1}{2} \log \left( 1 + \frac{b_{4,1}(\rho)}{1 + b_{5,2}(\rho)} \right) + \frac{1}{2} \log \left( b_{1,2}(\rho) + 1 \right),
\end{align}

where

\begin{align}
\kappa_{5}(\rho) &= \frac{1}{2} \log \left( 1 + \frac{b_{4,2}(\rho)}{1 + b_{5,1}(\rho)} \right) + \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right), \\
\kappa_{6,1}(\rho) &= \begin{cases} \frac{1}{2} \log \left( b_{1,1}(\rho) + b_{5,1}(\rho) \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) \\
+ \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho)}{\text{SNR}_2} \right) \end{cases} \\
&+ \frac{1}{2} \log \left( b_{1,2}(\rho) + b_{5,1}(\rho) \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) \\
&+ \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)}{\text{SNR}_1} \right) + \log(2\pi e), \\
\kappa_{6,2}(\rho) &= \frac{1}{2} \log \left( b_{6,2}(\rho) + \frac{b_{5,1}(\rho)}{\text{SNR}_2} \right), \\
&- \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) + \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)}{\text{SNR}_1} \right) \\
&- \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) + \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho)}{\text{SNR}_2} \right) \\
&+ \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)}{\text{SNR}_1} \right) + \log(2\pi e), \\
\kappa_{7,1,i}(\rho) &= \begin{cases} \frac{1}{2} \log \left( b_{1,i}(\rho) + b_{5,1}(\rho) \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) \\
+ \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho)}{\text{SNR}_2} \right) \end{cases} \\
&+ \frac{1}{2} \log \left( b_{1,2}(\rho) + b_{5,1}(\rho) \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) \\
&+ \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)}{\text{SNR}_1} \right) + \log(2\pi e),
\end{align}
\[ \kappa_{6,3}(\rho) = \frac{1}{2} \log \left( b_{6,1}(\rho) + \frac{b_{5,1}(\rho)\text{INR}_{21}}{\text{SNR}_1} \left( \frac{\text{SNR}_1 + b_{3,1}}{\text{SNR}_1} \right) \right) \]
\[ - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) + \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho)\text{SNR}_2}{b_{1,2}(1) + 1} \right) \]
\[ + \frac{1}{2} \log \left( b_{1,2}(\rho) + b_{5,1}(\rho)\text{INR}_{21} - \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) \right) \]
\[ + \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)}{\text{SNR}_1} \left( \text{INR}_{21} + \frac{b_{3,1}\text{SNR}_1}{b_{1,1}(1) + 1} \right) \right) \]
\[ - \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)\text{INR}_{21}}{\text{SNR}_1} \right) + \log(2\pi e), \quad (15c) \]

\[ \kappa_{6,4}(\rho) = \frac{1}{2} \log \left( b_{6,1}(\rho) + \frac{b_{5,1}(\rho)\text{INR}_{21}}{\text{SNR}_1} \left( \frac{\text{SNR}_1 + b_{3,1}}{\text{SNR}_1} \right) \right) \]
\[ - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) \]
\[ + \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho)}{\text{SNR}_2} \left( \text{INR}_{12} + \frac{b_{3,2}\text{SNR}_2}{b_{1,2}(1) + 1} \right) \right) \]
\[ + \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)\text{INR}_{21}}{\text{SNR}_1} \right) \]
\[ - \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)\text{INR}_{21}}{\text{SNR}_1} \right) \]
\[ + \frac{1}{2} \log \left( b_{6,2}(\rho) + \frac{b_{5,1}(\rho)\text{INR}_{21}}{\text{SNR}_2} \left( \frac{\text{SNR}_2 + b_{3,2}}{\text{SNR}_2} \right) \right) \]
\[ + \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho)}{\text{SNR}_1} \left( \text{INR}_{21} + \frac{b_{3,1}\text{SNR}_1}{b_{1,1}(1) + 1} \right) \right) \]
\[ + \log(2\pi e), \quad (15d) \]

and

\[ \kappa_{7,1,2}(\rho) = \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{ij} \right) \]
\[ + \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho)}{b_{1,2}(1) + 1} \right) \]
\[ + \frac{1}{2} \log \left( b_{1,2}(\rho) + b_{5,1}(\rho)\text{INR}_{ij} \right) \]
\[ + \frac{1}{2} \log \left( 1 + b_{4,1}(\rho) + b_{5,1}(\rho) \right) - \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \right) \]
\[ + 2 \log(2\pi e), \quad (16a) \]

where the functions \( b_{l,i} \), with \((l,i) \in \{1,2\}^2 \) are defined in (11); \( b_{3,i} \) are constants; and the functions \( b_{l,i} : [0, 1] \rightarrow \mathbb{R}_+ \), with \((l,i) \in \{4, 5, 6\} \times \{1, 2\} \) are defined as follows, with \( j \in \{1, 2\} \setminus \{i\} \):
\[ b_{3,l}(\rho) = 2\sqrt{\text{SNR}_i \text{INR}_{ij} + \text{INR}_{ji}}, \quad (17a) \]
\[ b_{4,i}(\rho) = \left( 1 - \rho^2 \right) \text{SNR}_i, \quad (17b) \]
\[ b_{5,l}(\rho) = \left( 1 - \rho^2 \right) \text{INR}_{ij}, \quad (17c) \]
\[ b_{6,i}(\rho) = \sqrt{\text{SNR}_i + \text{INR}_{ij} + 2\rho \sqrt{\text{SNR}_j \left( \sqrt{\text{SNR}_i} - \sqrt{\text{SNR}_j} \right)}} \]
\[ + \frac{\text{INR}_{ij} \sqrt{\text{INR}_{ji}} \left( \sqrt{\text{SNR}_j} - 2\sqrt{\text{SNR}_i} \right)}{\text{SNR}_i}. \]
\[ (17d) \]

Note that the functions in (14), (15), (16) and (17) depend on \( \text{SNR}_1, \text{SNR}_2, \text{INR}_{12}, \text{INR}_{21}, \text{SNR}_1, \) and \( \text{SNR}_2 \). However, these parameters are fixed in this analysis, and therefore, this dependence is not emphasized in the definition of these functions. Finally, using this notation, Theorem 2 is presented below.

**Theorem 2:** The capacity region \( C_{GIC-NOF} \) is contained within the region \( \overline{C}_{GIC-NOF} \) given by the closure of the set of non-negative rate pairs \((R_1, R_2)\) that for all \( i \in \{1, 2\} \), with \( j \in \{1, 2\} \setminus \{i\} \) satisfy:

\[ R_i \leq \min \left( \kappa_{1,1}(\rho), \kappa_{2,1}(\rho) \right), \quad (18a) \]
\[ R_i \leq \kappa_{3,1}(\rho), \quad (18b) \]
\[ R_1 + R_2 \leq \min \left( \kappa_{4}(\rho), \kappa_{5}(\rho) \right), \quad (18c) \]
\[ R_1 + R_2 \leq \kappa_{6}(\rho), \quad (18d) \]
\[ 2R_i + R_j \leq \kappa_{7,i}(\rho), \quad (18e) \]
where $\rho \in [0, 1]$. 

Proof: The proof of Theorem 2 is presented in [1].

D. Comments on the Converse Region

The outer bounds (18a) and (18c) correspond to the outer bounds for the case of perfect channel-output feedback [4]. The bounds (18b), (18d) and (18e) correspond to new outer bounds that generalize those presented in [2] for the two-user symmetric G-IC-NOF. These new outer-bounds were obtained using the genie-aided models shown in Figure 2.

E. A Gap Between the Achievable Region and the Converse Region

Theorem 3 describes the gap between the achievable region $\mathbb{C}_{G-IC-NOF}$ and the converse region $\overline{\mathbb{C}}_{G-IC-NOF}$ using the approximation notion described in Definition 2.

Theorem 3: The capacity region of the two-user G-IC-NOF is approximated to within 4.4 bits per channel use by the achievable region $\mathbb{C}_{G-IC-NOF}$ and the converse region $\overline{\mathbb{C}}_{G-IC-NOF}$.

Proof: The proof of Theorem 3 is presented in [1].

The gap, denoted by $\delta$, between the sets $\mathbb{C}_{G-IC-NOF}$ and $\overline{\mathbb{C}}_{G-IC-NOF}$ can be approximated (Definition 2) as follows:

$$\delta \leq \max\left( \delta_{R_1}, \delta_{R_2}, \frac{\delta_{R_3}}{2}, \frac{\delta_{R_4}}{3} \right),$$

where

$$\delta_{R_1} \triangleq \min \left( \kappa_{11}(\rho), \kappa_{21}(\rho), \kappa_{31}(\rho) \right) - \min \left( a_{2,1}(\rho), \kappa_{11}(\rho), \kappa_{21}(\rho), \kappa_{31}(\rho) \right),$$

$$\delta_{R_2} \triangleq \min \left( \kappa_{12}(\rho), \kappa_{22}(\rho), \kappa_{32}(\rho) \right) - \min \left( a_{2,2}(\rho), \kappa_{12}(\rho), \kappa_{22}(\rho), \kappa_{32}(\rho) \right),$$

$$\delta_{R_3} \triangleq \min \left( \kappa_{13}(\rho), \kappa_{23}(\rho), \kappa_{33}(\rho) \right) - \min \left( a_{3,1}(\rho), \kappa_{13}(\rho), \kappa_{23}(\rho), \kappa_{33}(\rho) \right),$$

$$\delta_{R_4} \triangleq \min \left( \kappa_{14}(\rho), \kappa_{24}(\rho), \kappa_{34}(\rho) \right) - \min \left( a_{3,2}(\rho), \kappa_{14}(\rho), \kappa_{24}(\rho), \kappa_{34}(\rho) \right),$$

$$\delta_{R_5} \triangleq \min \left( \kappa_{15}(\rho), \kappa_{25}(\rho), \kappa_{35}(\rho) \right) - \min \left( a_{3,3}(\rho), \kappa_{15}(\rho), \kappa_{25}(\rho), \kappa_{35}(\rho) \right),$$

$$\delta_{R_6} \triangleq \min \left( \kappa_{16}(\rho), \kappa_{26}(\rho), \kappa_{36}(\rho) \right) - \min \left( a_{3,4}(\rho), \kappa_{16}(\rho), \kappa_{26}(\rho), \kappa_{36}(\rho) \right),$$

$$\delta_{R_7} \triangleq \min \left( \kappa_{17}(\rho), \kappa_{27}(\rho), \kappa_{37}(\rho) \right) - \min \left( a_{3,5}(\rho), \kappa_{17}(\rho), \kappa_{27}(\rho), \kappa_{37}(\rho) \right),$$

$$\delta_{R_8} \triangleq \min \left( \kappa_{18}(\rho), \kappa_{28}(\rho), \kappa_{38}(\rho) \right) - \min \left( a_{3,6}(\rho), \kappa_{18}(\rho), \kappa_{28}(\rho), \kappa_{38}(\rho) \right).$$

Note that $\delta_{R_1}$ and $\delta_{R_2}$ represent the gap between the active achievable single-rate bound and the active converse single-rate bound; $\delta_{R_3}$ represents the gap between the active achievable sum-rate bound and the active converse sum-rate bound; and, $\delta_{R_4}$ and $\delta_{R_5}$ represent the gap between the active achievable weighted sum-rate bound and the active converse weighted sum-rate bound.

Finally, it is important to highlight that, as suggested in [2], [4], and [5], the gap between $\mathbb{C}_{G-IC-NOF}$ and $\overline{\mathbb{C}}_{G-IC-NOF}$ can be calculated more precisely. However, the choice in (19) eases the calculations at the expense of less precision. Figure 3 presents the exact gap existing between the achievable region $\mathbb{C}_{G-IC-NOF}$ and the converse region $\overline{\mathbb{C}}_{G-IC-NOF}$ for the case in which $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$, $\text{INR}_{12} = \text{INR}_{21} = \text{INR}$, and $\overline{\text{SNR}}_1 = \overline{\text{SNR}}_2 = \text{SNR}$ as a function of $\alpha = \log_{\text{SNR}} \text{INR}$ and $\beta = \log_{\text{SNR}} \text{SNR}$.

Note that in this case, the maximum gap is 1.1 bits per channel use and occurs when $\alpha = 1.05$ and $\beta = 1.2$.

IV. CONCLUSIONS

An achievable region and a converse region for the two-user G-IC-NOF have been introduced. It has been shown that these regions approximate the capacity region of the two-user G-IC-NOF to within 4.4 bits per channel use.

REFERENCES


