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Approximate Capacity of the Gaussian Interference Channel with Noisy Channel-Output Feedback

Victor Quintero, Samir M. Perlaza, Iñaki Esnaola and Jean-Marie Gorce

Abstract—In this paper, an achievability region and a converse region for the two-user Gaussian interference channel with noisy channel-output feedback (G-IC-NOF) are presented. The achievability region is obtained using a random coding argument and three well-known techniques: rate splitting, superposition coding and backward decoding. The converse region is obtained using some of the existing perfect-output feedback outer-bounds as well as a set of new outer-bounds that are obtained by using genie-aided models of the original G-IC-NOF. Finally, it is shown that the achievable region and the converse region approximate the capacity region of the G-IC-NOF to within a constant gap in bits per channel use.

Index Terms—Capacity, Interference Channel, Noisy Channel-Output Feedback.

I. NOTATION

Throughout this paper, $(\cdot)^+$ denotes the positive part operator, i.e., $(\cdot)^+ = \max(\cdot, 0)$ and $E_X[\cdot]$ denotes the expectation with respect to the distribution of the random variable $X$. The logarithm function log is assumed to be base 2.

II. SYSTEM MODEL

Consider the two-user G-IC-NOF in Figure 1. Transmitter $i$, with $i \in \{1, 2\}$, communicates with receiver $i$ subject to the interference produced by transmitter $j$, with $j \in \{1, 2\} \setminus \{i\}$. There are two independent and uniformly distributed messages, $W_i \in W_i$, with $W_i = \{1, 2, \ldots, 2^{NR_i}\}$, where $N$ denotes the block-length in channel uses and $R_i$ is the transmission rate in bits per channel use. At each block, transmitter $i$ sends the codeword $X_i = (X_{i,1}, X_{i,2}, \ldots, X_{i,N})^T \in \mathbb{X}_1^N$, where $\mathbb{X}_1$ and $\mathbb{X}_2$ are respectively the channel-input alphabet and the codebook of transmitter $i$.

The channel coefficient from transmitter $j$ to receiver $i$ is denoted by $h_{ij}$; the channel coefficient from transmitter $i$ to receiver $i$ is denoted by $h_{ii}$; and the channel coefficient from channel-output $i$ to transmitter $i$ is denoted by $\hat{h}_{ii}$. All channel coefficients are assumed to be non-negative real numbers. At a given channel use $n \in \{1, 2, \ldots, N\}$, the channel output at receiver $i$ is denoted by $Y_{i,n}$. During channel use $n$, the input-output relation of the channel model is given by

$$Y_{i,n} = h_{ii}X_{i,n} + h_{ij}X_{j,n} + Z_{i,n},$$

where $Z_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise at the input of receiver $i$. Let $d > 0$ be the finite feedback delay measured in channel uses. At the end of channel use $n$, transmitter $i$ observes $Y_{i,n}$, which consists of a scaled and noisy version of $Y_{i,n-d}$. More specifically,

$$Y_{i,n} = \begin{cases} Z_{i,n} & \text{for } n \in \{1, 2, \ldots, d\} \\ \frac{1}{d} Y_{i,n-d} + Z_{i,n} & \text{for } n \in \{d+1, d+2, \ldots, N\}, \end{cases}$$

where $Z_{i,n}$ is a real Gaussian random variable with zero mean and unit variance that represents the noise in the feedback link of transmitter-receiver pair $i$. The random variables $Z_{i,n}$ and $Z_{i,n}$ are independent and identically distributed. In the following, without loss of generality, the feedback delay is assumed to be one channel use, i.e., $d = 1$. The encoder of transmitter $i$ is defined by a set of deterministic functions $f_i^{(1)}, \ldots, f_i^{(N)}$, with $f_i^{(1)} : W_i \rightarrow \mathbb{X}_i$ and for all $n \in \{2, \ldots, N\}$, $f_i^{(n)} : W_i \times \mathbb{R}^{n-1} \rightarrow \mathbb{X}_i$, such that

$$X_{i,1} = f_i^{(1)}(W_i), \quad \text{and} \quad X_{i,n} = f_i^{(n)}(W_i, Y_{i,1}, \ldots, Y_{i,n-1}).$$

The components of the input vector $X_i$ are real numbers subject to an average power constraint:

$$\frac{1}{N} \sum_{n=1}^{N} E(X_i,n^2) \leq 1,$$

where the expectation is taken over the joint distribution of the message indexes $W_1, W_2$, and the noise terms, i.e., $Z_1, Z_2, Z_1$, and $Z_2$. The dependence of $X_{i,n}$ on $W_1, W_2$, and the previously observed noise realizations is due to the effect of feedback as shown in (2) and (3).

Fig. 1. Gaussian interference channel with noisy channel-output feedback at channel use $n$. 

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Assume that during a given communication, $T$ blocks are transmitted. Hence, the decoder of receiver $i$ is defined by a deterministic function $\psi_i : \mathbb{R}_i^{NT} \rightarrow \mathcal{W}_i^T$. At the end of the communication, receiver $i$ uses the vector $(\overrightarrow{Y}_{i,1}, \overrightarrow{Y}_{i,2}, \ldots, \overrightarrow{Y}_{i,NT})$ to obtain an estimate of the message indices

$$
(\overrightarrow{W}_i^{(1)}, \overrightarrow{W}_i^{(2)}, \ldots, \overrightarrow{W}_i^{(T)}) = \psi_i (\overrightarrow{Y}_{i,1}, \overrightarrow{Y}_{i,2}, \ldots, \overrightarrow{Y}_{i,NT}),
$$

where $\overrightarrow{W}_i^{(t)}$ is an estimate of the message index sent during block $t \in \{1, 2, \ldots, T\}$. The decoding error probability in the two-user G-IC-NOF during block $t$ of a codebook of blocklength $N$, denoted by $P_e(t)(N)$, is given by

$$
P_e(t)(N) = \max \left( \Pr [\overrightarrow{W}_1^{(t)} \neq \overrightarrow{W}_1^{(t)}], \Pr [\overrightarrow{W}_2^{(t)} \neq \overrightarrow{W}_2^{(t)}] \right).
$$

The definition of an achievable rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is given below.

**Definition 1 (Achievable Rate Pairs):** A rate pair $(R_1, R_2) \in \mathbb{R}_+^2$ is achievable if there exists at least one pair of codebooks $X_1^N$ and $X_2^N$ with codewords of length $N$, and the corresponding encoding functions $f_1^{(N)}, \ldots, f_1^{(N)}$ and $f_2^{(N)}, \ldots, f_2^{(N)}$ such that the decoding error probability $P_e(t)(N)$ can be made arbitrarily small by letting the blocklength $N$ grow to infinity, for all blocks $t \in \{1, 2, \ldots, T\}$.

The two-user G-IC-NOF in Figure 1 can be fully described by six parameters: $\text{SNR}_i, \tilde{\text{SNR}}_i$, and $\text{INR}_{ij}$, with $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus \{i\}$, which are defined as follows:

$$
\text{SNR}_i = \frac{\rho_i}{h_i^2},
$$

$$
\text{INR}_{ij} = h_{ij}^2 \quad \text{and}
$$

$$
\tilde{\text{SNR}}_i = \frac{h_i^2}{\tilde{\rho}_i} \left( \tilde{\rho}_i^2 + 2 \tilde{\rho}_i h_{ij} + h_{ij}^2 + 1 \right).
$$

### III. MAIN RESULTS

This section introduces an achievable region (Theorem 1) and a converse region (Theorem 2), denoted by $\mathcal{C}_{G-\text{IC-NOF}}$ and $\mathcal{C}_{G-\text{IC-NOF}}$ respectively, for the two-user G-IC-NOF with fixed parameters $\text{SNR}_1, \text{SNR}_2, \text{INR}_{12}, \text{INR}_{21}, \tilde{\text{SNR}}_1$, and $\tilde{\text{SNR}}_2$. In general, the capacity region of a given multi-user channel is said to be approximated to within a constant gap according to the following definition.

**Definition 2 (Approximation to within $\xi$ units):** A closed and convex set $\mathcal{C} \subseteq \mathbb{R}_+^m$ is approximated to within $\xi$ units by the sets $\mathcal{C} \subseteq \mathcal{C} \subseteq \mathcal{T}$ and for all $t = (t_1, \ldots, t_m) \in T$ then $(t_1 - \xi, \ldots, t_m - \xi) \in \mathcal{T}$.

Denote by $\mathcal{C}_{G-\text{IC-NOF}}$ the capacity region of the two-user G-IC-NOF. The achievable region $\mathcal{C}_{G-\text{IC-NOF}}$ and the converse region $\mathcal{C}_{G-\text{IC-NOF}}$ approximate the capacity region $\mathcal{C}_{G-\text{IC-NOF}}$ to within $4.4$ bits per channel use (Theorem 3).

#### A. An Achievable Region for the Two-User G-IC-NOF

The description of the achievable region $\mathcal{C}_{G-\text{IC-NOF}}$ is presented using the constants $a_{1,i}, \text{ the functions } a_{2,i} : [0, 1] \rightarrow \mathbb{R}_+, a_{3,i} : [0, 1]^2 \rightarrow \mathbb{R}_+$, with $l \in \{3, \ldots, 6\}$; and $a_{7,i} : [0, 1]^3 \rightarrow \mathbb{R}_+$, which are defined as follows, for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$:

$$
a_{1,i} = \frac{1}{2} \log \left( 2 + \frac{\text{SNR}_i}{\text{INR}_{ij}} \right) - \frac{1}{2},
$$

$$
a_{2,i}(\rho) = \frac{1}{2} \log \left( b_{1,i}(\rho) + 1 \right) - \frac{1}{2},
$$

$$
a_{3,i}(\rho, \mu) = \frac{1}{2} \log \left( \frac{\text{SNR}_{i} b_{2,i}(\rho) + 1]{1, 2} \} \setminus \{i\}$. All relevant scenarios regarding these ratios

Note that the functions $b_{1,i} : [0, 1] \rightarrow \mathbb{R}_+$, with $(l, i) \in \{1, 2\}^2$ are defined as follows:

$$
b_{1,i}(\rho) = \text{SNR}_i^2 + 2 \rho \text{SNR}_i \text{INR}_{ij} + \text{INR}_{ij}$
$$

$$
b_{2,i}(\rho) = \left( 1 - \rho \right) \text{INR}_{ij} - 1,
$$

with $j \in \{1, 2\} \setminus \{i\}$. However, as these parameters are fixed in this analysis, this dependence is not emphasized in the definition of these functions. Finally, using this notation, Theorem 1 is presented on the next page.

**Proof:** The proof of Theorem 1 is presented in [1].

#### B. Comments on the Achievability

The achievable region is obtained using a random coding argument and combining three classical tools: rate splitting, superposition coding, and backward decoding. This coding scheme is described in [1] and it is specially designed for the two-user IC-NOF. Consequently, only the strictly needed number of superposition code-layers is used. Other achievable schemes, as reported in [2], can also be obtained as special cases of the more general scheme presented in [3]. However, in this more general case, the resulting code for the IC-NOF contains a handful of unnecessary superposing code-layers, which complicates the error probability analysis.

#### C. A Converse Region for the Two-User G-IC-NOF

The description of the converse region $\mathcal{C}_{G-\text{IC-NOF}}$ is determined by the ratios $\frac{\text{INR}_{ij}}{\text{SNR}_i}$, and $\frac{\text{INR}_{ij}}{\text{SNR}_i}$, for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$. All relevant scenarios regarding these ratios
Theorem 1: The capacity region $\mathcal{C}_{\text{GIC-NOF}}$ contains the region $\mathcal{C}_{\text{G-IC-NOF}}$ given by the closure of the set of all possible non-negative achievable rate pairs $(R_1, R_2)$ that satisfy

\begin{align*}
R_1 &\leq \min \left( a_{2,1}(\rho), a_{6,1}(\rho, \mu_1) + a_{3,2}(\rho, \mu_1), a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{4,2}(\rho, \mu_1) \right), \\
R_2 &\leq \min \left( a_{2,2}(\rho), a_{3,1}(\rho, \mu_2) + a_{6,2}(\rho, \mu_2), a_{3,1}(\rho, \mu_2) + a_{4,1}(\rho, \mu_2) + a_{1,2} \right), \\
R_1 + R_2 &\leq \min \left( a_{2,1}(\rho), a_{1,2} + a_{1,1} + a_{2,2}(\rho), a_{3,1}(\rho, \mu_2) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{7,2}(\rho, \mu_1, \mu_2), \\
&\quad a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{3,2}(\rho, \mu_1), a_{3,1}(\rho, \mu_2) + a_{7,1}(\rho, \mu_1, \mu_2) + a_{3,2}(\rho, \mu_1) + a_{1,2} \right), \\
2R_1 + R_2 &\leq \min \left( a_{2,1}(\rho) + a_{1,1} + a_{2,2}(\rho, \mu_1) + a_{7,2}(\rho, \mu_1, \mu_2), \\
&\quad a_{3,1}(\rho, \mu_2) + a_{1,1} + a_{3,2}(\rho, \mu_1, \mu_2) + 2a_{3,2}(\rho, \mu_1) + a_{5,2}(\rho, \mu_1), a_{2,1}(\rho) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{5,2}(\rho, \mu_1) \right), \\
R_1 + 2R_2 &\leq \min \left( a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{2,2}(\rho) + a_{1,2}, a_{3,1}(\rho, \mu_2) + a_{7,1}(\rho, \mu_1, \mu_2) + a_{2,2}(\rho) + a_{1,2}, \\
&\quad 2a_{3,1}(\rho, \mu_2) + a_{5,1}(\rho, \mu_2) + a_{3,2}(\rho, \mu_1) + a_{1,2} + a_{7,2}(\rho, \mu_1, \mu_2) \right),
\end{align*}

with $(\rho, \mu_1, \mu_2) \in [0, 1 - \max \left( \frac{1}{\text{INR}_2}, \frac{1}{\text{INR}_1} \right)] \times [0, 1] \times [0, 1].$

are described by two events denoted by $S_{1,i}$ and $S_{2,i}$, where $(l_1, l_2) \in \{1, \ldots, 5\}^2.$ The events are defined as follows:

\begin{align*}
S_{1,i} &: \text{SNR}^*_j < \min \{ \text{INR}_{i_j}, \text{INR}_{j_i} \}, \\
S_{2,i} &: \text{INR}_{i_j} \leq \text{SNR}^*_j < \text{INR}_{i_j}, \\
S_{3,i} &: \text{INR}_{i_j} \leq \text{SNR}^*_j < \text{INR}_{j_i}, \\
S_{4,i} &: \max \{ \text{INR}_{i_j}, \text{INR}_{j_i} \} \leq \text{SNR}^*_j < \text{INR}_{i_j} \text{INR}_{j_i}, \\
S_{5,i} &: \text{SNR}^*_j \geq \max \{ \text{INR}_{i_j}, \text{INR}_{j_i} \text{INR}_{j_i} \}.
\end{align*}

Note that for all $i \in \{1, 2\}$, the events $S_{1,i}$, $S_{2,i}$, $S_{3,i}$, $S_{4,i}$, and $S_{5,i}$ are mutually exclusive. This observation shows that given any 4-tuple ($\text{SNR}_1, \text{SNR}_2, \text{INR}_{12}, \text{INR}_{21}$), there always exists one and only one pair of events $(S_{1,i}, S_{2,i})$, with $(l_1, l_2) \in \{1, \ldots, 5\}^2$, that identifies a unique scenario. Note also that the pairs of events $(S_{2,1}, S_{2,2})$ and $(S_{3,1}, S_{3,2})$ are not feasible. In view of this, twenty-three different scenarios can be identified using the events in (13). Once the exact scenario is identified, the converse region is described using the functions $\kappa_{6,1}, \kappa_{6,2}, \kappa_{6,3}, \kappa_{6,4}, \kappa_{7,1,1}, \kappa_{7,1,2}$ with

\begin{align*}
\kappa_{6,1}(\rho) &= \frac{1}{2} \log \left( 1 + \frac{b_{4,2}(\rho)}{1 + b_5(\rho)} \right) + \frac{1}{2} \log \left( b_1(\rho) + 1 \right), \\
\kappa_{6,2}(\rho) &= \frac{1}{2} \log \left( 1 + \frac{b_5(\rho)}{1 + b_5(\rho)} \right) + \frac{1}{2} \log \left( b_1(\rho) + 1 \right), \\
\kappa_{6,3}(\rho) &= \frac{1}{2} \log \left( b_{4,2}(\rho) + b_{5,1}(\rho) + 1 \right) + \frac{1}{2} \log \left( b_1(\rho) + 1 \right), \\
\kappa_{6,4}(\rho) &= \frac{1}{2} \log \left( \frac{\text{SNR}_2}{b_1(\rho) + 1} \right) + \log(2\pi e),
\end{align*}

where

\begin{align*}
\kappa_{7,1,1}(\rho) &= \frac{1}{2} \log \left( b_6(\rho) + b_{5,1}(\rho) \text{INR}_{21} \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) + \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) + \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) + \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) - 1 \log(2\pi e), \\
\kappa_{7,1,2}(\rho) &= \frac{1}{2} \log \left( b_6(\rho) + b_{5,1}(\rho) \text{INR}_{21} \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) + \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) + \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) - 1 \log(2\pi e),
\end{align*}

and
\[ \kappa_{6,3}(\rho) = \frac{1}{2} \log \left( b_{6,1}(\rho) + \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_1} \left( \sqrt{\text{SNR}_1} + b_{3,1} \right) \right) \]
\[ - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) + \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho) \sqrt{\text{SNR}_2}}{b_{1,2}(1) + 1} \right) \]
\[ + \frac{1}{2} \log \left( \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_1} \right) \left( \sqrt{\text{SNR}_1} + b_{3,1} \right) \left( \sqrt{\text{SNR}_1} + b_{1,1} (1) + 1 \right) \]
\[ - \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_1} \right) + \log(2\pi e), \quad (15c) \]

\[ \kappa_{6,4}(\rho) = \frac{1}{2} \log \left( b_{6,1}(\rho) + \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_1} \left( \sqrt{\text{SNR}_1} + b_{3,1} \right) \right) \]
\[ - \frac{1}{2} \log \left( 1 + \text{INR}_{12} \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{21} \right) \]
\[ + \frac{1}{2} \log \left( 1 + \frac{b_{5,2}(\rho) \sqrt{\text{SNR}_2}}{b_{1,2}(1) + 1} \right) \]
\[ - \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_1} \right) \]
\[ - \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_1} \right) \]
\[ + \frac{1}{2} \log \left( \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_2} \right) \left( \sqrt{\text{SNR}_2} + b_{3,2} \right) \]
\[ + \frac{1}{2} \log \left( 1 + \frac{b_{5,1}(\rho) \text{INR}_{21}}{\text{SNR}_1} \right) \left( \sqrt{\text{SNR}_1} + b_{3,1} \right) \left( \sqrt{\text{SNR}_1} + b_{1,1} (1) + 1 \right) \]
\[ + \log(2\pi e), \quad (15d) \]

and

\[ \kappa_{7,1,2}(\rho) = \frac{1}{2} \log \left( b_{1,1}(\rho) + 1 \right) - \frac{1}{2} \log \left( 1 + \text{INR}_{ij} \right) \]
\[ - \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \right) + \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \right) \]
\[ + \frac{1}{2} \log \left( 1 + \text{INR}_{ij} \right) \]
\[ + \frac{1}{2} \log \left( 1 + b_{4,1}(\rho) + b_{5,1}(\rho) \right) - \frac{1}{2} \log \left( 1 + b_{5,1}(\rho) \right) \]
\[ + 2 \log(2\pi e), \quad (16a) \]

where the functions $b_{l,i}$, with $(l, i) \in \{1, 2\}^2$ are defined in (11); $b_{3,i}$ are constants; and the functions $b_{l,i} : [0, 1] \rightarrow \mathbb{R}_+$, with $(l, i) \in \{3, 5, 6\} \times \{1, 2\}$ are defined as follows, with $j \in \{1, 2\} \setminus \{i\}$:

\[ b_{3,3} = \sqrt{\text{SNR}_i} + 2\sqrt{\text{SNR}_i \text{INR}_{ij}} + \text{INR}_{ij}, \quad (17a) \]
\[ b_{4,1}(\rho) = (1 - \rho^2) \sqrt{\text{SNR}_i}, \quad (17b) \]
\[ b_{5,1}(\rho) = (1 - \rho^2) \text{INR}_{ij}, \quad (17c) \]
\[ b_{6,1}(\rho) = \sqrt{\text{SNR}_i + \text{INR}_{ij} + 2\rho \sqrt{\text{INR}_{ij}} \left( \sqrt{\text{SNR}_i} - \sqrt{\text{SNR}_j} \right)} \]
\[ + \frac{\text{INR}_{ij} \sqrt{\text{INR}_{ij}}}{\sqrt{\text{SNR}_i}} \left( \sqrt{\text{SNR}_j} - 2\sqrt{\text{SNR}_j} \right). \quad (17d) \]

Note that the functions in (14), (15), (16) and (17) depend on $\text{SNR}_1$, $\text{SNR}_2$, $\text{INR}_{12}$, $\text{INR}_{21}$, $\sqrt{\text{SNR}_1}$, and $\sqrt{\text{SNR}_2}$. However, these parameters are fixed in this analysis, and therefore, this dependence is not emphasized in the definition of these functions. Finally, using this notation, Theorem 2 is presented below.

**Theorem 2:** The capacity region $C_{\text{GIC-NOF}}$ is contained within the region $\mathcal{C}_{\text{GIC-NOF}}$ given by the closure of the set of non-negative rate pairs $(R_1, R_2)$ that for all $i \in \{1, 2\}$, with $j \in \{1, 2\} \setminus \{i\}$ satisfy:

\[ R_i \leq \min \left( \kappa_{1,1}(\rho), \kappa_{2,1}(\rho) \right), \quad (18a) \]
\[ R_i \leq \kappa_{3,1}(\rho), \quad (18b) \]
\[ R_1 + R_2 \leq \min \left( \kappa_{4}(\rho), \kappa_{5}(\rho) \right), \quad (18c) \]
\[ R_1 + R_2 \leq \kappa_{6}(\rho), \quad (18d) \]
\[ 2R_i + R_j \leq \kappa_{\gamma,1}(\rho), \quad (18e) \]
with $\rho \in [0, 1]$.

Proof: The proof of Theorem 2 is presented in [1].

D. Comments on the Converse Region

The outer bounds (18a) and (18c) correspond to the outer bounds for the case of perfect channel-output feedback [4]. The bounds (18b), (18d) and (18e) correspond to new outer bounds that generalize those presented in [2] for the two-user symmetric G-IC-NOF. These new outer-bounds were obtained using the genie-aided models shown in Figure 2.

E. A Gap Between the Achievable Region and the Converse Region

Theorem 3 describes the gap between the achievable region $C_{G \rightarrow IC-NOF}$ and the converse region $C_{G \rightarrow IC-NOF}$ using the approximation notion described in Definition 2.

Theorem 3: The capacity region of the two-user G-IC-NOF is approximated to within 4.4 bits per channel use by the achievable region $C_{G \rightarrow IC-NOF}$ and the converse region $C_{G \rightarrow IC-NOF}$.

Proof: The proof of Theorem 3 is presented in [1].

The gap, denoted by $\delta$, between the sets $C_{G \rightarrow IC-NOF}$ and $C_{G \rightarrow IC-NOF}$ can be approximated (Definition 2) as follows:

$$\delta \leq \max \left( \delta_{R_1}, \delta_{R_2}, \frac{\delta_{R_3}}{3}, \frac{\delta_{R_4}}{3} \right),$$

where

$$\delta_{R_1} = \min \left( \kappa_{1,1}(\rho), \kappa_{2,1}(\rho), \kappa_{3,1}(\rho) \right) - \min \left( a_{2,1}(\rho), a_{6,1}(\rho, \mu_1) + a_{3,2}(\rho, \mu_1), a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{4,2}(\rho, \mu_1) \right) \tag{20a},$$

$$\delta_{R_2} = \min \left( \kappa_{1,2}(\rho), \kappa_{2,2}(\rho), \kappa_{3,2}(\rho) \right) - \min \left( a_{2,2}(\rho), a_{3,1}(\rho, \mu_2) + a_{6,2}(\rho, \mu_2), a_{3,1}(\rho, \mu_2) + a_{4,1}(\rho, \mu_2) + a_{1,2} \right) \tag{20b},$$

$$\delta_{R_3} = \min \left( \kappa_{1,4}(\rho), \kappa_{5}(\rho), \kappa_{6}(\rho) \right) - \min \left( a_{2,1}(\rho) + a_{1,2}, a_{3,1}(\rho, \mu_2) + a_{1,1} + a_{3,2}(\rho, \mu_1) + a_{7,2}(\rho, \mu_1, \mu_2), a_{3,1}(\rho, \mu_2) + a_{7,1}(\rho, \mu_1, \mu_2) + a_{3,2}(\rho, \mu_1) + a_{1,2} \right) \tag{20c}.$$