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Viscosity scaling in hydrodynamic instabilities in porous media

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Abstract. The importance of viscosity scaling in the context of viscous fingering in a finite slice with viscosity dependent diffusivity is investigated theoretically. Choosing the characteristic viscosity classically as either the displacing or displaced fluid viscosity for both more and less viscous slice leads to inappropriate theoretical predictions, which do not support the physics. With an appropriate choice of the characteristic viscosity, we show that the onset of instability and the initial dynamics of the finger patterns are the same for both more and less viscous slices. Our analysis will be helpful in the theoretical understanding of buoyancy-driven convection in a variable viscosity layer in vertical porous media or VF with non-monotonic viscosity profiles.

1 Introduction

Viscous fingering (VF) in various fluid flow problems of industrial and environmental interest, such as oil recovery [1], pollution remediation [2], CO₂ sequestration [3], chromatography separation [4–7], etc. have drawn attention of many researchers over many decades. This hydrodynamic instability is featured when a less viscous fluid displaces a more viscous and hence less mobile one in porous media [1]. Based on the geometry of the interface featuring the instability in the form of finger-like structure, VF problems can be broadly classified into two categories. The first one being the classical Saffman-Taylor instability in two semi-infinite fluids separated by a flat interface [1], while the other is the displacement of a finite sample, typically in chromatography column [4]. In the latter case, the sample is confined in a rectangular region that can feature VF either at the frontal or rear interface depending on the viscosity contrast in the downstream direction. The influence of positive and negative log-mobility ratio was investigated in the context of solute adsorption on porous matrix [8]. In the absence of adsorption and for constant diffusivity of the solute concentration, Mishra *et al.* showed that the onset of instability and the subsequent dynamics of the fingers are identical for more and less viscous slices [9]. Pramanik and Mishra [12] showed that in the presence of the Korteweg stress [10, 11] the fingering dynamics of a miscible slice of more viscosity are identical to that of a less viscosity. This was achieved with an appropriate scaling. In this paper, we are interested to explore whether such dynamics are also possible with concentration dependent diffusivity in absence of the Korteweg stress effect. In this context,

we propose a modified viscosity scaling that helps to answer this question when the diffusivity is inversely proportional to the viscosity of the fluid. We structure this paper as follows. Mathematical formulation and numerical solution of the present problem is discussed in Sect. 2, followed by result discussion and conclusion in Sects. 3 and 4, respectively.

2 Mathematical formulation and numerical solution of the problem

Consider a rectilinear displacement of a finite sample of viscosity μ_2 by another fluid of viscosity μ_1 at a uniform velocity Ue_x , where e_x is the unit vector in the x -increasing direction. The displacing fluid consists of the same sample solvent having no solute concentration in it, i.e. $c = 0$, while the finite sample has solute concentration $c = c_2$. We assume the variation of the diffusion coefficient with concentration is governed by the generalized Stokes-Einstein relation [13, 14],

$$D(c) \cdot \mu(c) = \text{constant}, \quad (1)$$

so that $D(c) = D_0/\mu(c)$, where D_0 denotes the diffusion coefficient of an infinitesimally small amount of the displaced fluid in the displacing fluid. Fluids are assumed to be incompressible, miscible and neutrally buoyant. For nondimensionalization we choose D_0/U , D_0/U^2 , U , $\mu_{\text{ch}}D_0/\kappa$, μ_l , c_2 as the characteristic length, time, velocity, pressure, viscosity, and concentration, respectively [12]. Here $\mu_l = \min\{\mu_1, \mu_2\}$, κ is the permeability of the porous media, and D_0 represents a reference diffusivity. The related dimensionless equations in a frame of reference moving with the velocity Ue_x can be written as [1],

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla p = -\mu(c)(\mathbf{u} + e_x), \quad \frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \nabla \cdot (D(c)\nabla c), \quad (2)$$

where $D(c)$ represents concentration dependent dispersion. The viscosity of the fluids depends exponentially on the solute concentration, i.e., $\mu(c) = e^{Rf(c)}$, where $R = \ln(\mu_m/\mu_l)$ is the log-mobility ratio and $f(c)$ is c (more viscous slice) or $1 - c$ (less viscous slice), and $\mu_m = \max\{\mu_1, \mu_2\}$. Therefore, the dimensionless form of the concentration dependent viscosity becomes $D(c) = 1/\mu(c)$. For two dimensional incompressible flow we define the stream-function $\psi(x, y, t)$, such that $u = \psi_y, v = -\psi_x$ are the longitudinal and transverse component of the Darcy velocity $\mathbf{u}$. Therefore, we can write (2) as,

$$\frac{\partial c}{\partial t} + J(x, y, t) = 0, \quad \nabla^2 \psi = -Rf'(c)N(x, y, t). \quad (3)$$

The nonlinear terms $J(x, y, t)$ and $N(x, y, t)$ in (3) are defined as,

$$J(x, y, t) = \psi_y c_x - \psi_x c_y - e^{-Rf(c)} (\nabla^2 c - Rf'(c)|\nabla c|^2), \quad (4)$$

$$N(x, y, t) = (\nabla \psi + e_y) \cdot \nabla c, \quad (5)$$

where e_y is the unit vector in y -direction.

Equations (3) are solved using a Fourier pseudo-spectral method [15, 4] to analyze the influence of variable diffusivity on VF of more and less viscous miscible slices. We apply Discrete Fourier transform (DFT) to $c(x, y, t)$, $\psi(x, y, t)$, $J(x, y, t)$ and $N(x, y, t)$ that converts (3) into differential algebraic equations,

$$\frac{d\hat{c}_{m,n}(t)}{dt} + \hat{J}_{m,n}(t) = 0, \quad \hat{\psi}_{m,n}(t) = \hat{N}_{m,n}(t)/(k_m^2 + k_n^2), \quad (6)$$

Fig. 1. Spatial structure of the solute concentration at $t = 800$ in the moving frame of reference for $R = 2$, $l = 256$: (a) variable diffusivity and (b) constant diffusivity, where $k_m = 2\pi m/L_x$, $k_n = 2\pi n/L_y$, $m, n = 0, 1, 2, \dots$, L_x and L_y being the dimensionless length and width of the computational domain, respectively. Equation (6) is solved using Adams-Bashforth predictor and trapezoidal corrector method in the Fourier space to obtain the Fourier modes at next time step $t + \Delta t$. Inverse DFT has been performed for $\hat{c}_{m,n}$ and $\hat{\psi}_{m,n}$ to update the concentration and stream function at time $t + \Delta t$, $c(x, y, t + \Delta t)$, $\psi(x, y, t + \Delta t)$. Details of the numerical method used can be found in the paper of Tan and Homsy [15].

3 Results and discussion

First we investigate the influence of variable diffusivity on miscible VF in a finite slice. The spatial structure of a more viscous slice of dimensionless width $l = 256$ is presented for variable diffusivity (Fig. 1a) and constant diffusivity (Fig. 1b) with $R = 2$. Figure 1 depicts that the fingering dynamics become more complex for variable diffusivity as compared to constant diffusivity case. In the former case the wavelength of the fingers becomes shorter and the fingers propagate faster, which is readily evident from an early interaction of the fingers with the frontal interface.

Figure 2 shows the spatio-temporal evolution of the solute concentration of more and less viscous slice for $R = 2$, $l = 256$ with concentration dependent diffusivity. This figure depicts that, before interacting with the stable interface the fingering dynamics in more and less viscous slices are identical. We also perform simulations for a less viscous slice by considering $\mu_{ch} = \mu_1$ [9], such that $\mu = e^{-2c}$. It is observed that the finger patterns differ significantly in this case (for brevity not shown) as compared to that shown in Fig. 2b. In particular, the onset of instability delays when choosing $\mu = e^{-2c}$, which is attributed to the fast diffusion of the solute concentration as compared to $\mu = e^{2(1-c)}$. Thus we conclude, in order to compare between two similar flows one should be careful about the choice of the dimensionless values in such a way that they represent the same dimensional values. With the prescribed scaling of the viscosity of the fluids this has been achieved in the case of variable diffusivity VF in finite slice.

The temporal evolution of degree of mixing $\chi(t)$ [16] has been investigated for both more and less viscous finite sample of width $l = 256$. We mentioned in §2 that the displacement of a more or less viscous finite sample is represented by $\mu(c) = e^{Rc}$ or $\mu(c) = e^{R(1-c)}$, respectively, where the log-mobility ratio $R > 0$.

Fig. 3. Temporal evolution of the degree of mixing for $l = 256$ with $\mu(c) = e^{2c}$ (dash-dotted line), $\mu(c) = e^{2(1-c)}$ (continuous line) and $\mu(c) = e^{-2c}$ (dashed line). Inset images show the concentration distribution for the last two cases at $t = 500$ and 1500 .

Figure 3 depicts that the temporal evolution of the degree of mixing, $\chi(t)$, for these two cases are almost identical, even after the interaction of the fingers with the respective stable interface. The displacement of a less viscous finite sample can alternatively be represented by $\mu(c) = e^{Rc}$, $R < 0$ [9]. It is shown that at early times, say at $t = 500$ in Fig. 3, mixing due to diffusion for $R < 0$ is more than the mixing due to VF for $R > 0$. However, at later times, for example at $t = 1500$, VF enhances the fluid-fluid interface and hence the degree of mixing becomes larger for $R > 0$ than its $R < 0$ counterpart. This is due to the fact that the dimensionless diffusion coefficient, which depends on the local solute concentration, c . For $R = 2$, the dimensionless diffusion coefficient is $e^{-2(1-c)}$, while for $R = -2$, the dimensionless diffusion coefficient is e^{2c} . Therefore, when the displacement of a less viscous finite sample is represented by $\mu(c) = e^{-2c}$, the finite slice diffuses faster than that when the displacement is represented by $\mu(c) = e^{R(1-c)}$. This leads to a faster diffusive mixing for $\mu(c) = e^{-2c}$ than that for $\mu(c) = e^{2c}$.

4 Conclusion

The influence of viscosity scaling on miscible VF in a finite slice with viscosity dependent diffusivity is investigated through numerical simulations. Choosing an appropriate characteristic viscosity we show that the fingering dynamics of a more viscous slice is identical to that of a less viscous one. We believe, the present scaling analysis shall be important to investigate the stability of a variable viscosity buoyantly unstable layer in vertical porous media [17] or miscible VF with non-monotonic viscosity profiles [18], and pave new way to understand fluid mixing in porous media.

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5 Scientific validation

This paper has been unanimously validated in a collaborative review mode with the following reviewers:

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