An application of the inverse power index problem to the design of the PSL electoral college

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Abstract

In this paper we shortly introduce an application of a method based on power indices and coalitional games to design a weighted majority voting system in practice. More specifically, we solve an inverse Banzhaf index problem in order to decide the weight of "great electors" within the electoral college for the election of the members of the Administration Board and the Academic Senate of the *Paris Sciences & Lettres* University. The method used and the relevant parameters of our analysis are presented and discussed.

Keywords: apportionment, coalitional games, Banzhaf index, inverse power index problem.

1 Introduction

Paris Sciences & Lettres (PSL) is a federal university that brings together 25 education and research institutions in Paris (to be hereafter denominated the *PSL Institution Members* or, simply, the *Institutions*). Founded in 2010, the organisation of PSL is based on two main bodies[2]: the PSL Foundation for Scientific Cooperation (*Fondation de Coopération scientifique*), which is mainly responsible for the management of key actions of the PSL project (e.g., the recruitment of chairs of excellence, the development of strategic international partnerships, the implementation of innovative programs in research and training, etc.) and the Community

[1]I am very grateful to Jean-Claude Petit for his valuable comments about the PSL University's organization, functions and duties and his useful suggestions to improve this paper.

[2]For a more detailed description of the governance bodies of PSL, see the PSL official website: https://www.univ-psl.fr/fr

of Universities and Institutions (*Communauté d'Universités et Etablissements*, also denominated *ComUE*) which is responsible of the decisions concerning the training and graduation policy of PSL, and of other decisions over the common actions related with the educational and research community (e.g., the joint coordination of research policies and international projects for knowledge dissemination, the activation of digital actions, the implementation of joint strategies concerning students' life, etc.).

The *ComUE* is governed by an Administration Board (AB) (*Conseil d'Administration*) of 30 members, assisted by an Academic Senate (AS) (*Conseil Académique*) of 120 members with a consultative role. The members of the AB and the AS of the ComUE are representatives of the different Institutions but, for statutory reasons, only 16 Institutions (see Table 1) out of the 25 of PSL participate to the electoral process for their designation. Notice that the PSL Foundation and the *ComUE* itself are both represented as independent establishment within the AB and the AS of the *ComUE*.

The members of the AB and the AS are indirectly elected, among the candidates of the different Institutions, by a college of "great electors" designated by the Institutions according to their own statutes (usually, via general elections within their respective Institutions). Moreover, the members of the AB and the AS must be appointed in the respect of their professional categories (e.g., teachers, researchers, administrative and technical staff, etc.) according to the proportion specified in the Internal Regulations [13], addressing the general recommendations of the relevant national legislation. The same internal rules also specify that the "great electors" must have different amounts of say (weights) and should appoint candidates members for the AB and the AS using a simple majority mechanism.

The *ComUE* is also provided with a Steering Committee (SC) (*Comité des Membres*) formed by the PSL institutions heads, the president and the vice-president of PSL, and the deans of the main departments. The objective of the SC is to ensure the proper functioning of PSL and to address the implementation of the guidelines provided by the Internal Regulations [13].

In this paper, we shortly introduce an approach based on coalitional games and on power indices to establish a "fair" distribution of the weights among the "great electors" within the PSL electoral college for the appointment of the AB and the AS members. In the game theoretic literature, several power indices (e.g. the well known Banzhaf index [4] and the Shapley-Shubik [15] index), have been proposed to evaluate the (*a priori*) distribution of power on committee decisions (see for instance [6] or [9] for a complete introduction to the literature). For instance, the Banzhaf index, that will play a central role in this paper, measures the ability of a voter to cast the decisive vote in winning coalitions when all coalitions have the same probability to form (see Section 2 for a more detailed discussion). Applications of power indices to existing electoral systems include the analysis of the United Nations Security Council [15, 14], the International Monetary Fund [10], the European Union Council of Ministers [1], the Electoral College of the United States [12] and many others.

Another very interesting and related issue is the *inverse power index problem*: if a vector of desired individual powers is given (for instance, as the outcome of a negotiation process), can we determine a voting method where a certain power index yields a good approximation of the desired vector? For instance, assuming that the Banzhaf index is used to evaluate the power of n voters, the inverse power index problem can be formulated as follows: given a vector $P = (p_1, \ldots, p_n)$ of n real numbers and an appropriate "metric" to evaluate the distance between real-valued vectors, find a voting system with n voters such that the distance (or *error*) between the Banzhaf index computed on such a voting system and the vector P is smaller than a predefined value (see [3, 2] for further details, and [7] for a recent review of the existing methods to solve this problem).

According to the recommendation of the SC [16], an "ideal" voting system should take into account the following three criteria: 1) all the Institutions should participate with a non-negligible power to the process of taking decisions in the AB and the AS of the *ComUE*; 2) larger Institutions (in terms of number of staff employed) should play a more relevant role; 3) Institutions with a major academic offer should be fairly represented. Because of an important disproportion of students over the different Institutions (many Institutions have no student at all), the SC of the *Co-*

mUE considers this last principle less relevant than the first two, in order not to destabilise the “global economy” of the decision-making process within PSL.

Consequently, a triple majority system (with thresholds over the total number of Institutions, staff and students) has been adopted as an “ideal” voting system (see Section 3 for more details on the three respective quotas). Our goal, is then to solve an *inverse Banzaf index problem* [2] with the objective to compute the weights of the “great electors” of PSL such that the Banzhaf index of the electoral college (based on a simple majority mechanism) is as close as possible to the Banzhaf index of the “ideal” triple majority system recommended by the SC [16].

We start with some preliminary notations and definitions in the next section. Section 3 is devoted to the discussion of the method used to solve the inverse Banzhaf index problem in our setting. Section 4 concludes.

2 Preliminaries and notations

A *coalitional game*, also known as *game in characteristic function form* or *Transferable Utility (TU) game*, is a pair (N, v), where N denotes a finite set of *players* and v is the *characteristic function*, assigning to each $S \subseteq N$, a real number $v(S) \in \mathbb{R}$, with $v(\emptyset) = 0$ by convention. If the set N of players is fixed, we identify a coalitional game (N, v) with the corresponding characteristic function v. A group of players $S \subseteq N$ is called a *coalition* and $v(S)$ is called the *worth* of this coalition.

A coalitional game (N, v) such that $v(S) \in \{0, 1\}$ (i.e., the worth of every coalition is either 0 or 1) for each $S \in 2^N \setminus \{\emptyset\}$ and $v(N) = 1$ is said a *simple game*. The standard interpretation for these games is to consider coalitions as “winning” ($v(S) = 1$) or “losing” ($v(S) = 0$).

A particular class of simple games is the one of *weighted majority games*, where the players in N are associated to a vector of $n = |N|$ *weights* $(w_1, \dots, w_n)$ and a *majority quota* q is given. A weighted majority game (N, v^w) on the weight w and the quota q is such that for each $S \in 2^N \setminus \{\emptyset\}$:

$$v^w(S) = \begin{cases} 1 & \text{if } \sum_{i \in S} w_i > q, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

	Institution	Short name
1	*École nationale supérieure de chimie de Paris*	Chimie ParisTech
2	*Centre national de la recherche scientifique*	CNRS
3	*Conservatoire national supérieur d'art dramatique*	CNSAD
4	*Conservatoire national supérieur de musique et de danse de Paris*	CNSMDP
5	*Communauté d'universités et établissements PSL*	ComUE PSL
6	*École normale supérieure*	ENS
7	*École nationale supérieure des arts décoratifs*	ENSAD
8	*École nationale supérieure des beaux-arts*	ENSBA
9	*École supérieure de physique et de chimie industrielles de la ville de Paris*	ESPCI
10	*Fondation de coopération scientifique PSL*	FCS PSL
11	*Institut national de recherche en informatique et en automatique*	INRIA
12	*Institut Curie*	Institut Curie
13	*Fondation européenne des métiers de l'image et du son*	La Fémis
14	*École Nationale Supérieure des Mines de Paris*	Mines ParisTech
15	*Observatoire de Paris*	Observatoire
16	*Université Paris-Dauphine*	Paris-Dauphine

Table 1: The 16 Institutions taking part to the electoral college for the appointment of the AB and AS members.

This paper mainly deals with a very famous *index* for coalitional games: the Banzhaf index [4]. The Banzhaf index of a game v is denoted by $\mathcal{B}(v)$ and can be obtained directly from the following relation:

$$\mathcal{B}_i(v) = \sum_{S \in 2^{N \setminus \{i\}}} \frac{1}{2^{n-1}} m_i(S), \tag{2}$$

where $m_i(S) = v(S \cup \{i\}) - v(S)$ is the *marginal contribution* of i entering in S, for each $i \in N$ and $S \in 2^{N \setminus \{i\}}$. So, $\mathcal{B}_i(v)$ is the expected marginal contribution of player $i \in N$ in game v over all possible coalitions (not containing i) in which i can enter, and assuming that all coalitions not containing i have the same probability to form (i.e., probability equal to $\frac{1}{2^{n-1}}$). The Banzhaf index [4] of coalitional games arising from voting situations (e.g., the simple majority games) has been widely used in the literature as a *power index*: the vector of values assigned by the Banzhaf index to the players of a (monotonic) simple game is often interpreted as the probability to cast a decisive vote or, more in general, as the ability of players to influence a voting mechanism.

A simple game (N, v) is said *monotonic* if $v(S) = 1$ implies that $v(T) = 1$ for each $S, T \in 2^N$ with $S \subseteq T$. For instance, every weighted majority game is a monotonic simple game. For a monotonic simple game (N, v) and a player $i \in N$, a coalition $S \in 2^N \setminus \{\emptyset\}$ with $i \in S$ and such that $v(S) = 1$ and $v(S \setminus \{i\}) = 0$ is said a *swing* for i and we denote by $s_i(v)$ the number of swings for player i in game v. It is easy to check that the *normalized Banzhaf index* on v can be defined for monotonic simple games as follows:

$$\beta_i(v) = \frac{\mathcal{B}_i(v)}{\sum_{i \in N} \mathcal{B}_i(v)} = \frac{s_i(v)}{\sum_{i \in N} s_i(v)}, \tag{3}$$

for each $i \in N$.

Example 1. *Consider a parliament with three political parties* 1*,* 2 *and* 3 *with weights (i.e., seats)* 47*,* 33 *and* 20*, respectively, and suppose that to pass any motion, a qualified majority of two-third of the seats is needed. This situation can be described as a simple majority game* (N, v^w) *with* $N = \{1, 2, 3\}$*,* $w = (47, 33, 20)$ *and* $q = \frac{200}{3}$*. As a consequence,* $v^w(\{1, 2, 3\}) = v^w(\{1, 2\}) = v^w(\{1, 3\}) = 1$ *(*$\{1, 2, 3\}$*,*$\{1, 2\}$ *and* $\{1, 3\}$ *are winning coalitions) and* $v^w(\{1, 2, 3\}) = 0$ *for all the remaining coalitions. Notice that the number of swings for player* 1 *is* $s_1(v^w) = 3$ *(*$\{1, 2, 3\}$*,*$\{1, 2\}$ *and* $\{1, 3\}$ *are all swing for player* 1*), whereas for* 2 *and* 3 *we have* $s_2(v^w) = s_3(v^w) = 1$ *(*$\{1, 2\}$ *is the only swing for* 2 *and* $\{1, 3\}$ *is the only swing for* 3*). So the normalized Banzhaf index is* $\beta(v^w) = (\frac{3}{5}, \frac{1}{5}, \frac{1}{5})$*. The reader can immediately verify that the weight* w_i *does not represent at all the effective power of players* $i \in N$*.*

3 The proposed apportionment

In this section, we introduce the inverse Banzhaf index problem used for the apportionment of the weights of the "great electors" within the electoral college for the appointment of the AB members (the analysis for the AS is very similar and therefore omitted from these notes). To be more specific, we consider the inverse Banzhaf index problem as introduced in the next definition.

Definition 1 (Inverse Banzhaf index problem)**.** *Let $N = \{1, \ldots, n\}$ be a finite set of players, let $\lambda \in [0, 1]$ be a predefined quota (expressed as a fraction of the total weight) and let $\epsilon \in [0, 1]$ be the maximum tolerated error.*

Given another vector $P = (p_1, ..., p_n) \in [0, 1]^N$ of n real numbers on the interval $[0, 1]$ and such that $\sum_{i \in N} p_i = 1$ (i.e., P is normalized), find a vector of non-negative integer numbers $w = (w_1, \ldots, w_n) \in \mathbb{Z}^N$ with $w_i \geq 0$ for each $i \in N$ and such that

$$\sum_{i \in N} |\beta_i(v^w) - p_i| < \epsilon, \tag{4}$$

where v^w is the weighted majority game on N defined according to relation (1) with weights w and a quota $q = \lambda \sum_{i \in N} w_i$, and $|\beta_i(v^w) - p_i|$ is the absolute value of the difference between the normalized Banzhaf index $\beta_i(v^w)$ of player $i \in N$ and in game v^w and the "ideal" value p_i.

Notice that in the above definition, it is asked to find a vector of integer weights and the quota is provided *a priori* (for different formulations of the inverse Banzhaf index problem see [3, 2]).

According to the aforementioned "ideal" triple-majority rule recommended by the SC [16] a subset of the 16 Institutions of the *ComUE* shown in Table 1 is a winning coalition if it satisfies the following three criteria:

i) it is formed by a simple majority ($> 50\%$) of the Institutions of the *ComUE*, all in favour;

ii) it represents a qualified majority ($> 66\%$) of the total number of employees working in the Institutions of the *ComUE*;

iii) it represents at least one-fourth (> 25%) of the entire population of students enrolled in the Institutions of the *ComUE*;

The data concerning the number of employees and students of each Institution of the *ComUE* are shown in Table 2.

Institution	e_i	e_i%	s_i	s_i%
Chimie ParisTech	262	3.7%	410	2.4%
CNRS	128	1.8%	0	0.0%
CNSAD	68	1.0%	97	0.6%
CNSMDP	556	7.8%	1203	7.0%
ComUE PSL	87	1.2%	230	1.3%
ENS	1591	22.3%	1706	10.0%
ENSAD	284	4.0%	654	3.8%
ENSBA	293	4.1%	500	2.9%
ESPCI	354	5.0%	368	2.2%
FCS PSL	30	0.4%	0	0.0%
INRIA	42	0.6%	0	0.0%
Institut Curie	703	9.9%	0	0.0%
La Fémis	262	3.7%	195	1.1%
Mines ParisTech	609	8.6%	831	4.9%
Observatoire de Paris	543	7.6%	103	0.6%
Université Paris-Dauphine	1309	18.4%	10778	63.1%
Total	*7121*	*100.0%*	*17075*	*100.0%*

Table 2: Relevant number of employees (e_i) and students (s_i) of each Institution i of the ComUE in the electoral college for the appointment of the AB members.

A simple game (N, v^t), with $N = \{1, \ldots, 16\}$ as the set of players representing the 16 Institutions Members of PSL, was defined according to the above triple majority mechanism. Precisely, let e_i and s_i, be, respectively, the number of employees and of students of each Institution $i \in N$, and take a coalition of Institutions $S \subseteq N$,

$S \neq \{\emptyset\}$, then we have that:

$$v^t(S) = \begin{cases} 1 & \text{if } |S| > 8 \text{ and } \sum_{i\in S} e_i > \frac{2}{3}\rho \text{ and } \sum_{i\in S} s_i > \frac{1}{4}\sigma, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where $\rho = \sum_{i\in N} e_i$ is the total size of personnel affiliated to the 16 Institutions of the *ComUE* and $\sigma = \sum_{i\in N} s_i$ is the total number of enrolled students. The normalized Banzhaf index $\beta(v^t)$ of game v^t has been computed according to relation (3) and by means of the computer program introduced in [8] and the *Mathematica* [17] functions introduced in [5] (see the Appendix for the *Mathematica* instructions). The vector $\beta(v^t)$ is shown in the first column of Table 3.

A weighted majority game (N, v^w) is also defined according to relation (1), where the weights are computed solving the inverse Banzhaf index problem defined in Definition 1 with $P = \beta(v^t)$ and $\lambda = \frac{1}{2}$ (i.e., according to the guidelines of the Internal Regulations [13] we focus on a simple-majority systems with quota $q = \sum_{i\in N} \frac{w_i}{2}$). The total error introduced by the approximation of the Banzhaf index computed on the "ideal" game v^t, is defined as follows:

$$\delta(\beta(v^w), \beta(v^t)) = \sum_{i\in N} |\beta_i(v^w) - \beta_i(v^t)|,$$

which is bounded by the the vector ϵ in the problem of Definition 1 (for our purposes, we define ϵ equal to 5%).

In order to find a solution of the previously introduced instance of the inverse Banzhaf index problem (specifically, the one introduced in Definition 1 with $P := \beta(v^t)$, $\lambda = \frac{1}{2}$ and $\epsilon = 0.05$), we apply a trial-and-error procedure. At the initial step of the procedure, we define an initial vector $w = (w_1, \ldots, w_n)$ of n integer weights ordered according to $\beta_i(v^t)$, i.e. such that $w_i \geq w_j :\Leftrightarrow \beta_i(v^t) \geq \beta_j(v^t)$, for each $i, j \in N$. Using this vector of initial integer weights w, we compute the associated weighted majority game v^w and the corresponding Banzhaf index $\beta_i(v^w)$ together with the initial total error $\delta(\beta(v^w), \beta(v^t))$. If this initial total error $\delta(\beta(v^w), \beta(v^t))$ is smaller than ϵ, then the procedure stops and the vector w is a solution of the inverse Banzhaf problem; otherwise, the vector of integer weights is adjusted as follows: each w_i such that player i has the largest absolute difference $|\beta_i(v^w) - \beta_i(v^t)|$ is decreased by 1, if $\beta_i(v^w) - \beta_i(v^t) > 0$, and increased by 1, otherwise. A new weighted majority game v^w associated to the adjusted weight vector is computed, together with the corresponding Banzhaf index $\beta_i(v^w)$ and the new total error $\delta(\beta(v^w), \beta(v^t))$. Again, if the new total error $\delta(\beta(v^w), \beta(v^t))$ is smaller than ϵ, then the procedure stops and the new vector w is a solution of the inverse Banzhaf problem; otherwise, a new adjustment of the weights occurs, and the process described above is repeated till finding a vector of integer such that the corresponding total error $\delta(\beta(v^w), \beta(v^t)) \leq \epsilon$. Of course, using this iterative procedure, in general we cannot guarantee to obtain a solution of the inverse Banzhaf index problem as specified in Definition 1 (see [3] for a more detailed discussion of the problem). In our case, however, the procedure yielded the vector of integer weights shown in the last column of Table 3 together with the Banzhaf value $\beta_i(v^w)$ of the corresponding weighted majority game (see also Figure 3 for a comparison between $\beta_i(v^w)$ and $\beta_i(v^t)$ of PSL members ordered according to their Banzhaf index in the AB).

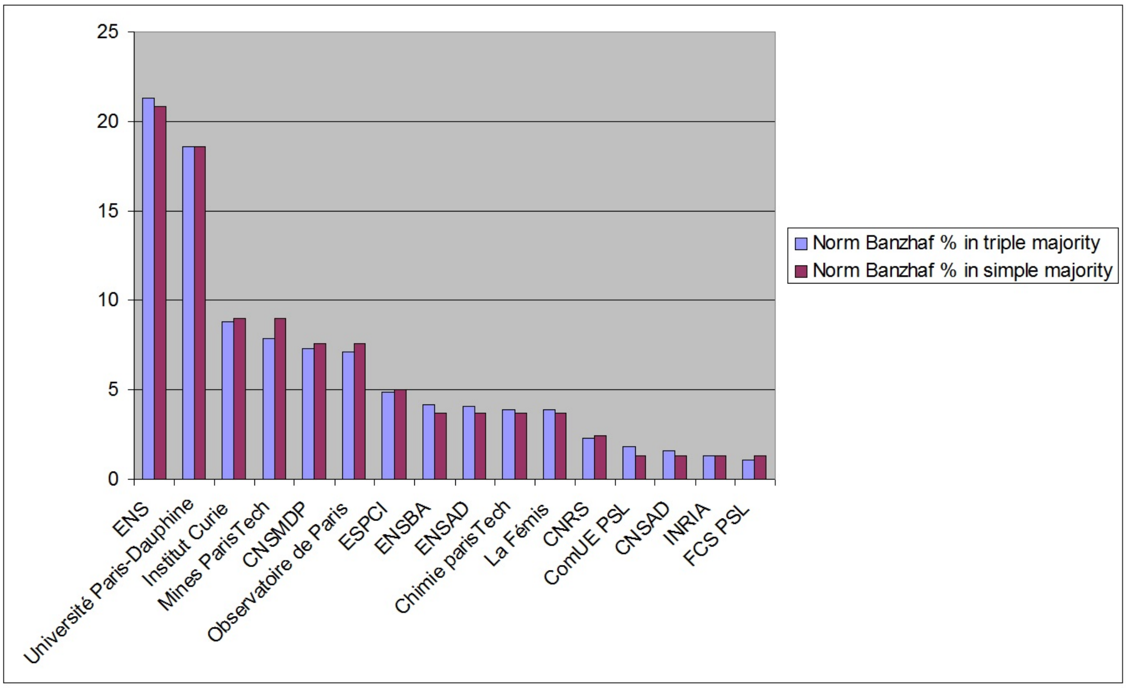

Figure 1: Comparison of the normalized Banzhaf index of the "ideal" triple-majority game v^t (left column) and the Banzhaf index of the weighted majority game v^w (right column) for each Institution of the *ComUE*.

A more sophisticated algorithm to solve the inverse Banzhaf index problem has

Institution	$\beta(v^t)$	$\beta(v^w)$	$\lvert\beta(v^w)-\beta(v^t)\rvert$	*Weight*
Chimie ParisTech	0.039	0.037	0.002	3
CNRS	0.023	0.024	0.001	2
CNSAD	0.016	0.013	0.003	1
CNSMDP	0.073	0.076	0.003	6
ComUE PSL	0.018	0.013	0.005	1
ENS	0.213	0.208	0.005	15
ENSAD	0.041	0.037	0.004	3
ENSBA	0.042	0.037	0.005	3
ESPCI	0.049	0.050	0.001	4
FCS PSL	0.011	0.013	0.002	1
INRIA	0.013	0.013	0	1
Institut Curie	0.088	0.090	0.002	7
La Fémis	0.039	0.037	0.002	3
Mines ParisTech	0.079	0.090	0.011	7
Observatoire de Paris	0.071	0.076	0.005	6
Université Paris-Dauphine	0.186	0.186	0	14
Total	*1*	*1*	*0.049*	*77*

Table 3: Normalized Banzhaf index of the "ideal" triple-majority game v^t and of the weighted majority game v^w. The absolute value of the differences between the two Banzhaf indices $|\beta(v^w) - \beta(v^t)|$ is shown in the third column (notice that the total error is less than the maximum tolerated one $\epsilon = 0.05$). The vector of weights is shown in the last column (the quota is then fixed at $q = \frac{77}{2}$).

been introduced in [3]. Using the implementation of Algorithm 1 in [3] to solve our instance of the inverse Banzhaf index problem, we obtain the vector of weights shown in Table 10 in the Appendix (see the Appendix also for the *Mathematica* code adapted from [3] to our problem). The vector of weights found with this algorithm is very close to the one we obtain by our procedure, and the total error with respect to the Banzhaf index of the "ideal" game is the same ($\delta(\beta(v^w), \beta(v^t)) = 0.049$).

4 Concluding remarks

In this report we shortly introduce an application of the inverse Banzhaf index problem to the apportionment of the weights of "great electors" in the electoral college for the appointment of the AB and AS members of the ComUE PSL [13]. Game theoretic power indices [6, 9] are often used in literature to describe the *a priori* power of players in existing voting systems [15, 14, 1, 12]. The objective of this report is to show that the inverse power index problem [7] can also provide the basis to "prescribe" a good electoral system. We are aware that many aspects of our analysis can be improved and embedded in a more general framework. For instance, the choice of the appropriate power index to evaluate the influence of the players in the "ideal" game is a delicate issue that deserves further discussion. Another interesting issue that should be further investigated is related to the algorithmic aspects of the inverse power index problem (for more sophisticated algorithms see, for instance, [11, 3, 2]). With respect to the inverse Banzhaf index problem, we would like to conclude with some open questions raised by [3] that to the best of our knowledge are still without a definitive answer:
[3]: "...*The questions we are interested in exploring are: is there a way of directly transforming a multiple majority weighted game into a weighted voting game with the same voting powers. Is there any loss of information in the transformation? Is it possible to identify and remove redundant weighted games from the multiple majority weighted voting game*?"

In our opinion, the method we propose for the design of the electoral college of the *ComUE* should be seen as an attempt to provide an objective basis for the negotiation of the weights among the PSL Institutions. This is at least, we believe, how it has been perceived by the members of the SC of the *ComUE* [16], where our proposition has been debated and finally approved under few modifications during the PSL-SC meeting held in Paris on April 21st, 2015.

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Appendix: *Mathematica* instructions

For the computation of the Banzhaf index of games v^t and v^w we used the *Mathematica* [17] functions introduced in [5]. In this appendix we show the *Mathematica* instructions to compute $\beta(v^t)$ and $\beta(v^w)$.

```
institutions16PSL = {"ENS", "Université Paris-Dauphine", "Institut Curie",
"Mines ParisTech", "CNSMDP", "Observatoire de Paris", "ESPCI",
"ENSBA", "ENSAD", "Chimie ParisTech", "La Fémis", "CNRS",
"ComUE PSL", "CNSAD", "INRIA", "FCS PSL"};
staffpermil = {223, 184, 99, 86, 78, 76, 50, 41, 40, 37, 37, 18, 12, 10, 6, 4};
studpermil = {100, 631, 0, 49, 70, 6, 22, 29, 38, 24, 11, 0, 13, 6, 0, 0};
members16 = Table[1, {16}];
swingsnumb = banzhaf3swings[staffpermil, studpermil, members16, 667, 250, 8]
ban3aPSL16 = SetPrecision[swingsnumb/Plus @@ swingsnumb, 3]
TableForm[ Transpose[{staffpermil, studpermil, members16, swingsnumb,
ban3aPSL16}],
TableHeadings − > {institutions16PSL, {"Staff", "Students", "Membership",
"Swing", "Banzhaf Normalized"}}]
```

Table 4: Computation of the Banzhaf index $\beta(v^t)$ of the “ideal” game v^t (code adapted from [5]).

```
weight16s = 15, 14, 7, 7, 6, 6, 4, 3, 3, 3, 3, 2, 1, 1, 1, 1
swingssimple = banzhafPower[weight16s, 39]
ban3aPSL17simple = SetPrecision[swingssimple/Plus @@ swingssimple, 3]
TableForm[Transpose[{weight16s, swingssimple, ban3aPSL17simple}],
TableHeadings − > {institutions16PSL, {"Weight", "Swing",
"Banzhaf Normalized" }}]
```

Table 5: Computation of the Banzhaf index $\beta(v^w)$ of the “ideal” game v^w (code adapted from [5]).

```
Off[InterpolatingFunction::"dmval", General::"spell1", NumberForm::"sigz"]
Clear[P, RW, VD];
g[v_] := Apply[Times, Map[(1 + x^#) &, v]];
s[r_] := Normal[Series[1 / (1 - x), {x, 0, r - 1}]];
h[r_, v_] := Expand[g[v] / (1 + x^r)];
p[r_, v_] := Coefficient[s[r] h[r, v], x^Round[Total[v] / 2 - 1]];
(*strict majority rule*)
Ind[v_] := Map[p[#, v] &, v];
(*raw index*)NBI[v_] := (temp = Ind[v];
  temp / N[Total[temp]]);
(*normalized index*)FirstEqual[u_, v_] := u[[1]] == v[[1]];

F[r_, RW_] := Total[(FractionalPart[RW r + 1 / 2] - 1 / 2) ^2];

T = {0.213, 0.186, 0.0876, 0.0791, 0.0732, 0.0714, 0.049, 0.042, 0.0413, 0.0388, 0.0388, 0.0233, 0.0176, 0.0156, 0.0128, 0.0105};

P[n_] := P[n] = NBI[V[n]] 1.0002; (*correcting for target excess*)DT[n_] := P[n] - T;
Err[n_] := Total[DT[n] ^2];
Errabs[n_] := Total[Abs[DT[n]]];
RW[n_] := RW[nD] = Map[Interpolation[Union[Transpose[{P[n - 1], V[n - 1]}], SameTest → FirstEqual], InterpolationOrder → 2], T];

Go[n_] := (Print["Next real weights RW[", n, "] = ", NumberForm[RW[n], 4]];
   r = Minimize[F[s, RW[n]], 0.8 < s < 1.2, s][[2, 1, 2]];
   Print["Good multiplier =", r];
   V[n] = Round[RW[n] r];
   Print["Next integer weights RW[", n, "] = ", V[n]];
   Print["Error l2=" NumberForm[Err[n], 4]];
   Print["Error l1=" NumberForm[Errabs[n], 4]];);

V[0] = Round[77 Map[InverseErf, T] / Total[Map[InverseErf, T]]];
(*initial approximation*)

Print["Initial integer weights V[0]=", V[0]];
Print["Initial error l2=", NumberForm[Err[0], 4]];
Print["Initial error l1=", NumberForm[Errabs[0], 4]];
Go[1]
```

Table 6: Weights computation using the algorithm introduced in [3] (code adapted from [3]

	Staff	Students	Membership	Swing	Banzhaf Normalized
ENS	223	100	1	10771	0.213
Université Paris-Dauphine	184	631	1	9415	0.186
Institut Curie	99	0	1	4433	0.0876
Mines ParisTech	86	49	1	3999	0.0791
CNSMDP	78	70	1	3705	0.0732
Observatoire de Paris	76	6	1	3613	0.0714
ESPCI	50	22	1	2477	0.0490
ENSBA	41	29	1	2123	0.0420
ENSAD	40	38	1	2089	0.0413
Chimie ParisTech	37	24	1	1961	0.0388
La Fémis	37	11	1	1961	0.0388
CNRS	18	0	1	1181	0.0233
ComUE PSL	12	13	1	891	0.0176
CNSAD	10	6	1	789	0.0156
INRIA	6	0	1	647	0.0128
FCS PSL	4	0	1	531	0.0105

Table 7: The output of the instructions in Table 4.

	Weight	Swing	Banzhaf Normalized
ENS	15	$\frac{4091}{19695}$	0.208
Université Paris-Dauphine	14	$\frac{94}{505}$	0.186
Institut Curie	7	$\frac{118}{1313}$	0.0899
Mines ParisTech	7	$\frac{118}{1313}$	0.0899
CNSMDP	6	$\frac{499}{6565}$	0.0760
Observatoire de Paris	6	$\frac{499}{6565}$	0.0760
ESPCI	4	$\frac{991}{19695}$	0.0503
ENSBA	3	$\frac{736}{19695}$	0.0374
ENSAD	3	$\frac{736}{19695}$	0.0374
Chimie ParisTech	3	$\frac{736}{19695}$	0.0374
La Fémis	3	$\frac{736}{19695}$	0.0374
CNRS	2	$\frac{37}{1515}$	0.0244
ComUE PSL	1	$\frac{19}{1515}$	0.0125
CNSAD	1	$\frac{19}{1515}$	0.0125
INRIA	1	$\frac{19}{1515}$	0.0125
FCS PSL	1	$\frac{19}{1515}$	0.0125

Table 8: The output of the instructions in Table 5.

```
Initial integer weights V[0]={17, 14, 7, 6, 6, 5, 4, 3, 3, 3, 3, 2, 1, 1, 1, 1}
Initial error l2=0.001177
Initial error l1=0.08273
Next real weights RW[1] = {16.01, 14.7, 6.759, 6.168, 5.724, 5.585, 3.895, 3.348, 3.294, 3.098, 3.098, 1.876, 1.418, 1.257, 1.03, 0.8429}
Good multiplier =0.937229
Next integer weights RW[1] = {15, 14, 6, 6, 5, 5, 4, 3, 3, 3, 3, 2, 1, 1, 1, 1}
Error l2= 0.0001912
Error l1= 0.04912
```

Table 9: The output of the instructions in Table 6.

Institution	Weight
Chimie ParisTech	3
CNRS	2
CNSAD	1
CNSMDP	5
ComUE PSL	1
ENS	15
ENSAD	3
ENSBA	3
ESPCI	4
FCS PSL	1
INRIA	1
Institut Curie	6
La Fémis	3
Mines ParisTech	6
Observatoire de Paris	5
Université Paris-Dauphine	14

Table 10: The weight of vector obtained using Algorithm 1 in [3] and using the code in Table 6.