Contradictions and shifts in teaching with a new curriculum: The role of mathematics

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HAL Id: hal-01289878
https://hal.science/hal-01289878

Submitted on 17 Mar 2016

Contradictions and shifts in teaching with a new curriculum: The role of mathematics

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In the framework of Activity Theory (AT), contradictions are sources of change and development. Borrowing concepts from AT, we attempt an interpretation of identified contradictions in a collaborative context where teachers plan and evaluate their teaching in the process of enacting a new curriculum. We examine the connection between contradictions and shifts in teaching activity, with a special focus on the mathematical character of these contradictions. We claim that dialectical oppositions lying in the background of these contradictions promote teachers to broaden their teaching activity by embedding into it new mathematical and pedagogical possibilities.

Keywords: Activity theory, contradictions, curriculum, teacher choices.

INTRODUCTION

A new, reform oriented curriculum was introduced and piloted in a small number of schools in Greece. During the year 2012–13 we collaborated with the teachers of three of these schools supporting them to enact the new curriculum in their classrooms. This collaboration was taking place in group meetings at the school where the teachers worked. During the meetings they planned and evaluated their teaching while the first author, who was also a member of the team that developed the curriculum, supported them by providing explanations about the rationale of the curriculum as well as teaching resources. Our main research goal is to understand the teachers' decisions and choices in relation to the curriculum documents and resources and the factors that framed them. Analysing the data we found that a variety of contradictions appeared to trigger discussions in most cases. Drawing on AT we explored further the possible role

of these contradictions for teachers' choices and the development of the teaching activity. In general, the development of mathematics teaching has been studied in the context of planned interventions aiming to teachers' professional development. Few studies though investigate teachers' professional learning and the shift of their teaching activity in contexts where learning is not promoted by an expert. Some of them refer to teachers' learning through reflection on their own teaching (Chapman & Heater, 2010) or to joint reflection in collaborative contexts (Potari, Sakonidis, Chatzigoula, & Manaridis, 2010). In these contexts professional learning is a complex and long process framed by many different factors and conditions. Our study attempts to contribute in understanding of this complexity by using contradictions as a tool for our analysis.

In this paper, we refer to the emerged contradictions "as sources of change and development" (Engeström, 2001, p. 137). Especially, we focus on contradictions that seem to challenge teachers' choices and on possible shifts in their teaching activity.

THEORETICAL FRAMEWORK

Current research in mathematics education recognizes that in the context of reform teachers are not expected to implement a predefined set of methods in their classroom, because there are not such methods. On the contrary, teachers are required to play a substantial role as a link between the curricular and other reform priorities and their classroom (Skott, 2004). This requirement, which Skott calls "forced autonomy", brings the teacher at the center of the curriculum enactment but also creates new challenges and conflicts. Within this perspective, teachers are not considered as mere transmitters of a curriculum

formulated by some experts outside their classroom, but as active agents and designers. Teachers' instructional actions are influenced by curricular materials but also shaped by their interactions with the students in the classroom (Remillard, 2005).

Analyzing data through a grounded theory approach we found ourselves trying to understand and interpret the emerged contradictions and we turned to AT for this purpose. AT is trying to capture the complexity of teaching, integrating dialectically the individual and the social – collective and focusing on the activity of the subject. The activity is driven by subject's motivation and directed towards an object (Leont'ev, 1978). The unit of analysis in this context is the activity system (AS) that incorporates social factors that frame the relations between the subject and the object with the mediation of tools. These factors are related to the communities in which the subject acts, the rules of these communities and the division of labor (Engeström, 2001).

In the present study we consider the activity of the participating teachers to be the teaching of mathematics in the context of introducing a new set of curricular materials. As the subject of this activity we see teachers as a group and as individuals. A main object and motive of the teachers' activity is their students' mathematical learning, in combination with other professional obligations such as implementing the mandates of the educational authorities. Their activity is mediated by tools such as curricular documents, school textbooks and other teaching-learning materials, instructional strategies, and lesson plans. They constantly balance school community, students' and parents' communities, mathematics teachers' community and other communities that influence the teaching activity. Teachers' activity is framed by rules such as institutional commitments (e.g., examinations, time constraints, timetables) or traits of mathematics as discipline and as a school subject. The division of labor refers to the teacher's role in the classroom or in the school and to the distribution of classrooms among the mathematics teachers in their school.

One of the fundamental characteristics of every AS is contradictions. They emerge when an AS adopts new elements from the outside, such as a new tool or a new rule, causing a conflict with the old elements. Contradictions are neither everyday solvable problems nor temporary conflicts that may easily be overcome. Moreover, the term "contradiction" in AT has not at all the meaning of a logical contradiction. "Contradictions are historically accumulating structural tensions within and between activity systems" (Engeström, 2001, p. 137). Roth and Radford (2011) refer to a special type of contradictions as "inner contradictions" to describe the often mutually exclusive aspects of the same phenomenon that coexist dialectically and "cannot be removed". Contradictions create learning opportunities for the subject and may broaden its activity to a wider horizon of possibilities (Engeström, 2001; Potari, 2013).

There is an increasing amount of research literature about contradictions in mathematics education. Some of them use the concepts of AT to identify, describe and interpret contradictions in teaching (for example, Barab, Barnett, Yamagata-Lynch, Squire, & Keating, 2002; Jaworski & Potari, 2009) and in teachers' professional development (Potari, 2013). In these studies contradictions refer mainly to pedagogical or professional issues, paying less attention to mathematical and epistemological ones. Another dimension of this research is related to the use of contradictions to stimulate expansive learning in developmental interventions with groups of teachers (Engeström, 1994; Jaworski & Goodchild, 2006). Our initial research goal although developmental, was not based on stimulating contradictions as the ground of expansive learning. So, our view in this paper related to expansive learning is restricted in a snapshot of what is called expansive cycle. However, we adopt Engeström's position that professional learning often is "something that is not stable, not even defined or understood ahead of time" and "there is no competent teacher" who knows what must be learned (Engeström, 2001, p. 137–138).

In this study we understand contradictions in two ways. First, as conflicting elements of the teaching-learning activity. Such is the contradiction between the tools and the rules of the activity (e.g., between the choice of computer based instructional tools and time restrictions). Second, we identify contradictions as conflicting opinions, practices or choices between two teachers or a teacher and some external agent (e.g., the curriculum, the students, etc.). An example of the latter is the contradiction between the use of tasks that require conceptual understanding of divisibility (promoted by the new curriculum) and, the use of tasks that can be solved using key-

words such as "less than" or "at least" that indicate LCM or GCF.

Some of the emerging contradictions are characterized by a dialectical opposition. Here, as dialectical opposition we consider some opposing aspects of a mathematical concept or of how it is transformed in teaching. Often, these opposing aspects are complementary, they can't be separated, and both constitute the concept. For example, the distributive property encompasses two opposing but complementary uses: it can be used to transform a product to a sum or a sum to a product. In our analysis such oppositions appeared in some cases underlying a contradiction. Dialectical oppositions of this kind allow us to consider more deeply in our analysis the mathematical dimensions (e.g., content, processes) of teaching.

METHODOLOGY

The research was conducted during the pilot implementation of the new curriculum in three junior high schools (grades 7–9). The new curriculum emphasizes students' mathematical activity that promotes mathematical reasoning and argumentation, connections within and outside mathematics, communication through the use of tools and metacognitive awareness. It also attributes a central responsibility for the teacher in the process of designing teaching. In this study, the mathematics teachers at each participating school (school A, B and C) worked together to enact the new curriculum with the support of the first author. The main tasks undertaken in the groups were planning lessons and reflecting on their experiences with teaching some modules of the designed curriculum in the classroom.

For this paper data consist of transcriptions of audiotaped conversations and written documents (worksheets, lesson plans) from 8 meetings with the 5 mathematics teachers of school A. This school is an experimental and model school where participating teachers have long teaching experience and are familiar with educational innovations. In general, the school culture is characterized by an innovative spirit. In this paper, we refer to two teachers, Marina and Linda, with over 25 years of teaching experience, with extra qualifications (both have masters degree, Marina in mathematics and Linda in mathematics education) and both with experience of innovative teaching approaches in their classrooms. In the past,

both had participated in teacher collaborative groups developing classroom materials or writing papers for mathematics teacher journals and conferences. In general, Marina was more informed than Linda about the research activities of the mathematics education community in Greece. Both had a critical stance to innovations in general, adopting some of them and rejecting others and had strong views about their teaching choices. Concerning the new curriculum, in an interview at the beginning of this research study they had said that it came as a legitimizing umbrella over their practice.

The transcribed conversations were analyzed with grounded theory methods (Charmaz, 2006). The written documents were used to exemplify the conversations. The initial open coding resulted in the identification of discussion themes for each meeting, forming thematic units. In each unit teachers' choices, their rationale and emerging contradictions were identified. As indicators of a contradiction were disagreements among the participants or between the participants and an external source. Each identified contradiction, was formulated as a dichotomy (e.g., the choice of tasks aimed at conceptual understanding or at procedural fluency). For every identified contradiction we used descriptive codes related to its content, the agents (e.g., a contradiction between participants and the curriculum) and teachers' awareness (whether or not they recognise the contradiction). Then, contradictions were categorized and traced through data for possible effects on teaching.

RESULTS

The content of the identified contradictions concerned issues such as: teaching planning and strategies, students' activity and difficulties, institutional constraints, teacher collaboration, classroom management and epistemological issues. Data analysis showed that in some cases contradictions led teachers to question their own teaching and start transforming it. Below we elaborate two of these cases as exemplars of shifts in teaching activity. The first exemplar underlies a dialectical opposition in the teaching of a mathematical property, while the second shows an epistemological opposition concerning the validation of school mathematical knowledge.

Contradicting goals and dialectical oppositions in teaching the distributive law

In the second meeting at school A (A2, turn 138) Marina described her teaching plans for algebraic transformations in grade 9 (operations with polynomials, identities and factorization). Starting from her observations on students' mathematical activity she said that in the expression $3 \cdot (a + b + c)$ "children see completely different things than us … they see addition and multiplication [while] we see … a product with two factors". Seeking to obtain "a common language" with the children about the structure of algebraic expressions Marina decided to emphasize this issue in her teaching. She designed to teach multiplication of polynomials in parallel to the factorization of a polynomial. In the meeting Marina presented the worksheet she used in the classroom and described that she divided the blackboard in two parts, with the expression $3 \cdot (a + b + c) =$ on the left and the $3a + 3b + 3c =$ on the right. With this approach she hoped to make clearer to the students that the use of distributive property depends on the structure of the expression we have and the structure we want to get.

At the next meeting (A3, turns 105–114) Marina said that she used this approach in the teaching of algebraic identities and she expressed her satisfaction commenting:

> I think … they have understood better that this way or the other is in fact about polynomials' operations. Until now identities have been presented as something to be learned by heart, and it was a special thing, completely away from the other operations, as well as the factorization was … say these things were not connected at all. And I think that their connection helped students to understand them and to use them in a flexible way. (A3, 113)

In this episode a contradiction comes to the fore: the teacher has the goal for the students to identify the structure of an algebraic expression as sum or product, while the students recognize only the operations that the expression calls them to do, a well-known problem in learning algebra (Sfard, 1991). In the background of this contradiction lays a dialectical opposition concerning the use of the mathematical object: distributive property may be used to transform a product to a sum or to factorize a sum to a product. Recognition of this contradiction by Marina was based

on her past experience and observation of students' mathematical activity. However, it was triggered by the new curriculum, which called to emphasize the structure of algebraic expressions, and by the discussions in the group related to the new curriculum. Marina also recognized the dialectical opposition in the distributive property and based on this her teaching attempts to overcome the contradiction between her goals to emphasize the algebraic structure and her students' tendency to see the tasks operationally.

A "traditional" teaching approach leads to two separated readings of distributive property, attempting to plan teaching based once on the first (operations to get a sum) and then on the other (factorization to get a product). This is the structure of the school textbook, as there was not a new textbook in the philosophy of the new curriculum. This was also the approach of the other teachers in the group, as with different rationale everyone adopted the well known teaching sequence. Especially Linda explained her choice saying that she believed that students needed time to consolidate their knowledge in operations with polynomials. However, she also valued the recognition of the algebraic structure: "I also ask the students: is this a sum or a product?" (A3, 251)

So, Marina's instructional choice on this topic can be regarded as a change to what she was doing before and to what usually her colleagues and most mathematics teachers in Greece used to do. The new perspective that Marina adopted considers the two usages of distributive property as two dialectically opposite ways that need to become explicit to the students and, consequently, to highlight the structure of algebraic expression. This perspective, stemming from the recognition of the initial contradiction, can be considered as an indication of broadening the horizon of Marina's teaching activity, encompassing new possibilities to it. On the other hand, Linda recognized the same contradiction but she did not choose to change her teaching, following the mainstream approach.

Contradicting tools and dialectical oppositions in using geometrical transformations in teaching congruence

Although some elements of reflectional and rotational symmetry existed in the previous curriculum and in the textbook, the geometrical transformations, namely translation, reflection and rotation were introduced as a distinct topic in the new curriculum mainly in

the 8th grade. The rationale of this introduction was connected with the development of students' spatial sense and with the value of transformations in tackling congruence and similarity. The topic emphasized the transformation of a figure as a whole supporting more intuitive and dynamic approaches to the geometric shapes and their properties. The focus was on the relationship between the two figures (original and image), highlighting the relation of congruence or similarity and attributing to the transformations the character of a proving tool (for further discussion on this relationship see Battista, 2007). Therefore, geometrical transformations constitute an alternative approach to the Euclidean perspective in school geometry indicating a different epistemology: the use of the moving figure as a proving tool is not compatible with the rigorous deductive rationale of Euclidean geometry.

In the discussion in the fourth meeting (A4), Marina referred to her introductory lesson on the congruence of triangles (grade 9) and she was pleased that in her question "how could we ascertain that these two triangles are congruent?" some students answered "if the triangles match after translation or reflection or rotation". She refers to Freudenthal's claim that Euclidean geometry is abused in school and she says that she is thinking to use tasks with geometrical transformations in teaching the congruence of triangles (A4, 132). However, she was questioning how this could be introduced in her teaching: "but there is a need of investigation and inquiry before doing so" (A4, 126); "I want them [the students] to understand that when we compare angles or segments or generally elements of polygons, we have two tools. One is the transformation and the other the criteria of triangle congruence" (A4, 136).

Linda listened to what Marina said and asked for clarifications. Finally, she commented that Marinas' thoughts were interesting but "every topic has its purpose". She did not criticize Marina's choices, but she claimed that "there is a purpose to learn to write, to observe the shape, to distinguish the given from the required, to make conclusions, and to prove … [Congruence] has its meaning" (A4, 137).

In the next meeting (A5) Marina described the way that students of grade 9 worked with the congruence of triangles in parallel to geometrical transformations to prove the congruence of segments or angles.

She argued that there are tasks that can show to the students when one approach is more appropriate than the other. For example, the task "the two triangles formed by three pairs of diametrically opposing points, are congruent" can be easily tackled by a 180° rotation, while the use of the criteria of triangle congruence is very complex (A5, turns 7, 9). On the basis of these special tasks, epistemological issues were also discussed in the meeting, about the rigor and the intuition of different approaches (A5, turns 11–15). Marina's descriptions show that her students used transformations as an alternative way to triangle congruence. This happened regularly in the classroom she had also taught in the previous year, but with more difficulty in a classroom she has been teaching only this year (A5, turn 7). Linda follows the discussion expressing positive opinions on Marina's strategies (turns 8, 10, 16, 20).

In the sixth meeting (A6) Marina said that in a test she asked her students to prove the congruence of two segments with two ways and many of them referred to rotation. Reflecting on her attempt to use transformations as an alternative to triangle congruence, Marina admitted:

> the introduction of transformations in the 8th grade gives you the opportunity to change the framework [of proving] in the 9th grade … [for the students] to see that you can cope with the proof of geometrical properties with two strategies … using transformations and the triangle congruence … And it was done easily … it came from the students. … And I think it is very nice that for the first time it is given the possibility to get away from Euclidean geometry… (A6, turns 324–334)

In the 8th meeting (A8) Marina mentioned that some students used transformations in other topics, such as trigonometry, indicating that they used them as an operational tool to visualize and prove congruence. Reflecting on her favour for transformations, she mentioned a seminar on transformations she had attended three years ago and her experimental teaching in another school. Linda expressed her disagreement to such intertwining of different topics. She said: "I like transformations per se. I don't like overusing them later in congruence … I don't find the reason to [do so]" (A8, turns 123,125)

In our interpretation, the discussions on transformations and triangle congruence reveal a contradiction between these two concepts as two different tools that a student can use to prove geometrical properties. In the background of this contradiction lies the epistemological difference between rigorous, deductive foundation of knowledge in Euclidean geometry and more intuitive, visual, dynamic aspects of geometrical transformations. We see this epistemological difference as a dialectical opposition, because the opposing aspects can be synthesized in a way that benefits students in grasping the concepts and properties of congruence. This is the intention of the new curriculum. Marina recognizes the contradiction between the two tools and their epistemological differences (as a dialectical opposition) although she does not use this terminology. This allows her to attempt a shift in her teaching by using the two tools in parallel and synthesizing them in the students' mathematical activity. This shift is a change in comparison with Marina's previous teaching and with the other teachers' teaching, for example, Linda's.

Any attempt to interpret Marina's shift, must incorporate social and cultural factors. Here we discuss some of them, trying to operationalize some concepts from AT. First of all is the obligation of implementing the mandated curriculum (rules) to which Marina's perspective was in accordance. The idea of using transformations in parallel to triangle congruence was triggered by students (community) on the basis of the curriculum philosophy (tools) and Marina's content knowledge and professional experiences (tools). The distribution of classrooms among the teachers (division of labor) led Marina to teach students she had taught transformations in the previous year. Marina's conversations in the group of teachers and with other mathematics teachers (community) helped her to clarify and identify her approach. The specific approach was consistent with the norms of classroom and the active role of students (division of labor). The aforementioned factors embed historical evolution, both in the teachers' biography and in the formation of the tools, rules and communities, but the space limits hinder any further reference on this.

Linda seemed to like the introduction of geometrical transformations by the new curriculum, and recognized them as a proving tool. However, she chose not to synthesize the two tools, pursuing the benefits of emphasizing the deductive approach of congruence in Euclidean geometry. Linda and Marina share similar perspectives about the new curriculum and similar experiences on teaching geometrical transformations. Both work in the same school with innovative culture and participate in the same collaborative group for planning and reflecting on teaching. The apparent differences can be possibly explained by the different tools they use (e.g., content knowledge on the topic of transformations) or the different communities they had participated). But what is making the difference in their activity are the different learning and teaching goals the two teachers set for their students concerning geometrical transformations.

DISCUSSION

Activity theory views contradictions as a prerequisite for the transformation of activity through an expansive cycle (Engeström, 2001). Here, we cannot follow the expansive cycle of the teaching activity but we observe some snapshots, some instances of "creative externalization ... in the form of discrete individual violations and innovations" (Cole & Engeström, 1993, p. 40). This is the way we see the shifts in Marina's teaching. In the two presented examples, dialectical oppositions of epistemological character underlie the identified contradictions. In the first example, the contradiction is between the teacher's goals and students' operational understandings while the dialectical opposition is in the use of distributive property. In the second, the two contradictory proving tools are underpinned by opposing epistemologies that can be dialectically synthesized.

Barab and colleagues (2001, p. 104) argue that "when systemic tensions are brought into a healthy balance they can facilitate a meaningful interplay that enriches and adds dynamism to the learning process". These claims highlight the dialectical dimension of tensions and contradictions that have emerged in our study. It appears from our analysis that recognizing the contradiction and deciding to incorporate both opposite aspects dialectically, has an effect on "broadening the horizon of the activity" (Engestrom, 2001; Potari, 2013) as in the case of Marina. The dialectical oppositions attribute mathematical and epistemological characteristics to the contradictions that can form the basis for a shift to the teaching activity. The above claims are in accordance with Chapman's and Heater's position (2010) that key issues on teacher change are: the experience of authentic tensions based on actual,

personal classroom experiences, the willing to take ownership of the change and the acceptance of a degree of uncertainty. Although from the AT analysis we see commonalities in the social factors that frame both Linda's and Marina's teaching activity, we identified different teaching and learning goals for their students. In other words, their activity is motivated by different objects as "images of thought" (Leont'ev, 1978, p. 86), that is, they hold different anticipations about students' learning.

We don't know if the identified shifts in Marina's teaching will be sustained and if they can be expanded in the collective activity of mathematics teaching in Greek schools. Such an investigation requires long periods of time and different research methods. What we can claim from this study is that contradictions may be overcome in a dialectical way that challenges dichotomies between "effective" and "non effective" teaching towards a more dynamic view of teaching.

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