

Problem solving teaching practices: Observer and teacher's view

Patricio Felmer, Josefa Perdomo-Díaz, Valentina Giaconi, Carmen G. Espinoza

▶ **To cite this version:**

Patricio Felmer, Josefa Perdomo-Díaz, Valentina Giaconi, Carmen G. Espinoza. Problem solving teaching practices: Observer and teacher's view. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.3022-3028. hal-01289734

HAL Id: hal-01289734
https://hal.science/hal-01289734

Submitted on 17 Mar 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Problem solving teaching practices: Observer and teacher's view

Patricio Felmer, Josefa Perdomo-Díaz, Valentina Giaconi and Carmen G. Espinoza

University of Chile, Center for Mathematical Modeling and Center for Advanced Research in Education, Santiago, Chile, pfelmer@dim.uchile.cl

In this article, we report on an exploratory study on teaching practices related to problem solving of a group of 29 novel secondary mathematics teachers. For this purpose, two independent instruments were designed, the first one is based on lesson observations, and the second one is a questionnaire answered by teachers about their teaching practices while working on non-routine problem solving with their students. For each instrument, we perform a statistical analysis to define relevant dimensions regarding problem solving teaching practices and we compare these new dimensions. We find that results from the two instruments are coherent in the case of quantity of problem solving in lessons and quality of teaching practices. These results are encouraging for further studies in this direction.

Keywords: Problem solving, observed teaching practices, self-reported teaching practices.

THE IMPORTANCE AND CHALLENGE OF ANALYZING TEACHING PRACTICES

No doubt that the analysis of teaching practices is crucial for many aspects of education, for teacher, school or program evaluation, for more general purposes of defining and understanding good practices or for theoretical studies aiming to understand the teaching-learning process.

Methods used to collect teaching practices data have an ample variety, for example teachers/students reports, lesson observations and questionnaires, just like the type of analysis done with these data that could be qualitative, quantitative or a vast variety of mixed approaches. Having all of them advantages and disadvantages, depending on the type of collected data and the analysis method used, the research could show in-depth, small-scale results as in the case of qualitative analysis of videotaped lessons, or a more general view of teaching practices as in the case of quantitative analysis of teachers' surveys.

In particular, the analysis of teaching practices through questionnaires is of great value since it allows studies with a high number of teachers. PRIMAS project is an example of this analysis, in which teachers' self-reported practices in inquiry based learning in different countries are described (Engeln, Euler, & Maaß, 2013). On the other hand, teaching practices observations by a research team, without the teacher intervention (external analysis), has been widely used in mathematics education too. These analyses have been conducted with different purposes: to study a specific feature of teaching and compare different types of teaching practices in relation with this feature, for example the mathematical quality of instruction (Hill et al., 2008) or the presence of mathematical elements as definitions, properties or processes (Badillo, Figueras, Font, & Martínez, 2013), to identify teachers' participation patterns in the class (Skott, 2013) or to analyze attributes of teachers that could affect their teaching practices, for example their beliefs about mathematics and teaching (Stipek, Givvin, Salmon, & MacGyvers, 2001).

Nevertheless, how self-reported practices and externally observed practices are related is not an obvious issue. Important differences may come from distinct conceptions of "practices", that is, the researcher has something in mind which differs from what teacher have in mind or they may come from biased answers based in social desirability.

To sum up, there are many important reasons to analyse teaching practices and, at the same time, collecting data to do that is a critical point (Maaß & Artigue, 2013).

Our interest in methods for collecting teaching practices data is in the context of a R&D project related with mathematics teacher's professional development on problem solving practices. In this project, teachers design and implement problem solving lessons with their students, which are later discussed jointly between teachers and researchers. In the context of two previous projects we have designed two instruments to collect data from practices from a group of secondary mathematics teachers: a Lesson Observation Rubric (LOR) and a Questionnaire about Problem Solving (QPS). We are interested in the information that can be obtained from each instrument separately and we want to explore on the information that can be drawn using both instruments at the same time.

The research question of this study is: Which are the dimensions that characterize teaching practices in relation with problem solving when using each of the two instruments and how do they correlate? To answer this question is relevant from a data collection methodological point of view and also for deepening in the understanding of the teaching practices related with problem solving.

TEACHING PRACTICES RELATED WITH PROBLEM SOLVING

The framework for designing research instruments and analysing data to answer our research question includes a conception of problem and of teaching practices that promote students' work in a mathematical environment.

We agree with Schoenfeld (1985) considering that

> Being a problem is not a property inherent in a mathematical task. Rather it is a particular relationship between the individual and the task that makes the task a problem for that person [...] if one has already accessed to a solution schema for a mathematical task, that task is an exercise and not a problem. (p. 74)

Other characteristic that some authors give to a problem is that it needs more time to be solved (e.g., Kloosterman & Stage, 1992).

Our experience with in-service teachers shows that some teachers think that a problem is just a math exercise contextualized in the "reality", and that many teachers use the term "non-routine problem" to refer to what Schoenfeld call "problem". In order to face this common misconception among Chilean teachers, the QPS teachers answered starts with the following definition:

> We are going to consider that a non-routine problem is one where the person that solves it doesn't know a strategy or algorithm to solve it. Some of the characteristics that non-routine problem usually have are: They need more reflection and time than regular problems or exercises; they cannot be solved with a simple rule or just remembering and applying a known fact; usually they can be solved using different strategies; they can have one or more solutions; they are challenging for the person that is solving them and they don't have to be confused with problems with real context, to have or not real context doesn't determine a non-routine problem.

In relation with teaching practices the key pedagogical aspect of problem solving tradition is that students have to think mathematically. To accomplish this it is necessary to have student centred teaching practices and a climate that support for the autonomy. Swan (2006) uses the term "student-centred' to describe practices where

> [T]he teacher takes students' needs into account when deciding what to teach, treats students as individuals rather than a homogeneous body, is selective and flexible about what is covered and allows students to make decisions, compare different approaches and create their own methods. Mathematics is seen as a subject open for discussion. It is not necessary to cover everything on the syllabus and time may be taken to explore and discuss. (p. 63)

METHOD

Participating teachers

This study considers teaching practices from 29 novel secondary mathematics teachers (between 1 and 4 years of teaching) randomly selected from graduates of three of the Chilean reference universities for teacher formation (10 teachers from two of the universities and 9 from the other one).

Each teacher selected a class, in which he/she was teaching, to be observed during three consecutive lessons. The level of each class went from 7th to 12th and the topic treated during the observed lessons was varied, like numbers, algebra or geometry. Some months later (from 6 to 12 months) teachers answered a questionnaire about their teaching in relation with problem solving practices taking into consideration the same class that was observed.

Classroom observation

For each teacher, three consecutive lessons of the selected class were observed and coded (2 live lessons and 1 video recorded lesson), completing 270 minutes of observation. It was chosen to observe only three lessons since, according to methodological research findings; no significant information is obtained by observing more than 3 lessons (Hill et al., 2008). A lesson observation rubric (LOR) was designed to code lessons by sections of 7.5 minutes. The LOR is divided in three main parts described next, with 14 practice indicators in total.

Work dynamics: in this part, the rubric identifies if time is dedicated to mathematics or not and the general way teacher and students interact. It is a first approximation to know if the lesson focus is on the teacher or the students. There are 5 indicators: down time (DT), direct instruction (DI), individual work (IW), group work (GW) and class work (CW). Each 7.5 minutes the indicator which is most observed is marked.

Class management: in this part, more specific characteristics of teaching practices are collected using 7 indicators: teacher does not participate (TNP), teacher makes questions (TMQ), teacher returns responsibility (TRR), teacher does not answer (TNA), teacher gives solutions (TGS), non-routine intervention of a student (NRIS) and teacher promotes discussion (TPD). When one of these indicators was observed during a segment it was marked.

Type of task: in this part, the type of mathematical task was collected using 2 indicators: routine task (RT) and non-routine task (NRT), depending if student were performing routine or non-routine tasks. Just one of them was marked in each time segment except in the case both were present during a very similar amount of time.

For our analysis we use all variables described above as percentages. In the case of DT, DI, IW, GW, CW, RT and NRT they are the percentage of segments were the indicator was marked, for example if for a teacher 9 segments out of 36 were marked non routine task, his score in NRT was 25%. In the case all the other variables, they are percentages of the total number of variables marked in the three lessons, for example if TRR was marked 3 times in the 36 segments and all other variables were marked 15 times, TRR was 20%

Questionnaire (QPS)

The questionnaire answered by teachers (QPS) is designed to measure the quantity and quality of problem solving (PS) activities used by them. The QPS starts with the definition of non-routine problem indicated in the framework (section 2) and it requires teachers to answer considering that definition. The questionnaire was answered through an online form sent by e-mail.

Quantity of PS was measured asking about the frequency of use of PS in their lessons with one item with options from 1 (never) to 5 (almost every lesson). Quality of PS was measured with 19 items (see Appendix A) based in scales of autonomy supportive climate (Leroy, Bressoux, Sarrazin, & Trouilloud, 2007) and scales to measure the level of student-centred approach and teacher-centred approach (Swan, 2006). Also some items were created to complete our view of a good problem solving activity as indicated in the framework. The teachers were asked to answer each question with the following prompt "When I use non-routine problems". Each item in this part has 6 options in a Likert scale ranging from 1 (never) to 6 (always).

Statistical methods

To summarize the variables from each instrument we made a Principal Components Analysis. The purpose of this analysis is to create interpretable and reliable scales that allow us to simplify and organize the data, understanding the limitations of using this method with low number of individuals. We made the analysis over the correlations matrix and, in order to have more interpretable results, we applied a Varimax rotation. The number of factors was chosen with the analysis of the scree plot. We define new scales averaging the items that had the biggest loadings in each factor. If a factor had items with negative and positive loadings we multiplied the items with negative loadings by -1 after calculating averages and reliabilities.

In the case of QPS, the principal components analysis was made with a bigger sample (N=240) of secondary and primary mathematics teachers applying for participating in a Problem Solving Workshop. Nevertheless, when we look for correlations between variables of both research instruments we used the common sample of 29 novel secondary mathematics teachers described at the beginning of the section.

ANALYSIS

Dimensions characterizing teaching practices by external observation

The statistical method applied to data from the LOR retained 4 factors explaining 71.2% of the total variance. The first factor explains 27.9 % of variance and the items with highest loadings were GW, TNP, TMQ(-) and DI(-) (the items with (-) had negative high loadings). With these items we create the new scale or dimension *Classroom Management* with a Cronbach's alpha of 0.681. We interpreted this factor as a dimension that describes the class management because a high positive score in this factor means that the teacher used most group work (GW) without intervention (TNP) and a negative score means that the teacher used mostly direct instruction (DI) and make questions (TMQ).

The second factor explains 16.2% of the variance and the items with highest loadings were TGS(-), NRT, TRR and CW. With these items we create the new scale or dimension *Quality Practices* with a Cronbach's alpha of 0.62. This factor put together variables that we presented in our framework as related with a good problem solving activity. First the frequency of non-routine tasks (NRT) and return the responsibility (TRR) are key variables. To have the frequency of working with whole class (CW) also makes sense, because CW was distinguished from direct instructions (DI), and only was coded if the protagonist were the students and they were working in a mathematical task. Teacher return responsibility was coded when at the moment of been asked teacher answered with a question or guide that allows the student to build, develop or discover a mathematical idea. The frequency of teacher gives solutions (TGS) has a negative loading; this makes sense to us because a teacher that gives the answers kills any attempt to think. So, having a high score in this dimension means that a teacher has a high frequency (compared with the other teachers) of non-routine tasks, returns the responsibility to students, uses whole class discussions and usually does not give the answers or solutions to their students.

The interpretation of the other two factors was not clear enough so we decided not to use them in the subsequent analysis. Studies with larger samples would be necessary to clarify these factors.

Dimensions characterizing problem solving teaching practices by teachers' self-report

The statistical method applied to data from the QPS retained 4 factors explaining 67.9% of the total variance (see Table 1) The first factor is formed with items that indicate students' behaviour denoting student centred practices, for example student's discussions (8,16) and work independently (3). The second factor was formed by items that indicate teacher's behaviour describing student centred practices, for example the teacher promotes that students takes their time (12) and guides with questions (11). The third factor is interpreted as a teacher centred dimension because most of the items situated the teacher as the protagonist. The last factor is about class organization because a high value in the factor means that teachers group their students (1) and a low factor means that teachers set their students working individually (6). Since this dimension has a very low reliability we do not use it for the correlation analysis.

In the next two sections we will combine the information obtained with the analysis of data from both research instruments: LOR and QPS.

Factor	% of Variance	Items with highest loadings	New Dimension	Reliability Cronbach's alpha
1	31.8	8, 13, 16, 14, 19, 3, 4, 9, 7	*Student centred (students)*	0.919
2	16.2	12, 18, 5 11	*Student centred (teacher)*	0.841
3	12.9	17, 10, 2, 15	*Teacher centred*	0.768
4	7	6 (-), 1	*Class Organization*	0.291

Table 1: Description of factors and the new dimensions from the QPS

Relation between observed and self-reported quantity of problem solving

Regarding the quantity of problem solving, the QPS has one item asking explicitly for the frequency of non-routine problems, and the LOR has an indicator of non-routine task. The correlation between the average of observed non-routine task and self-reported quantity of non-routine PS is r=0.447, p=0.015, N=29. This shows that information reported by teachers is not so different from information registered in lessons observations, in relation with the quantity of non-routine tasks or problems used with students of the observed sample. Nevertheless there are some particular cases as teachers reporting they use PS most of the time and where non-routine task was not coded during the observed 3 lessons. Cases like this require a deeper analysis we expect to make in a future study.

Relation between observed and self-reported quality of problem solving

In the analysis of the relation between the observed and self-reported quality of problem solving teaching practices we have to recall that all 19 items of QPS were answered under the prompt "When I use non-routine problems". This lead us to consider with a different weight the answers to items, depending on the reported frequency of use of PS in classes: we multiply the answer of each item by 1, 2, 3, 4 or 5 according to frequency of use of PS reported by the teacher.

In the following table, we present correlations between the new reliable dimensions obtained through principal components analysis starting with data from LOR and from QPS.

Regarding the dimensions from the QPS, we highlight the significant high value between the Student centred (student) and Student centred (teacher), dimensions 1 and 2, which may be interpreted as coherence in the answers of the teachers, those teachers that have a student centred practices have students performing student centred activities. In the same direction the significant positive correlations between 3 with 1 and 3 with 2 suggests a sort of incoherence in teacher's answers. However, having weighted the items answered by teachers with a weight based on a question in the same QPS (quantity of RP) may have increased artificially the correlations between the variables within QPS.

The most important result in Table 2 is the significant correlation between the dimension 5 with the dimensions 1 and 2. In this case, we are correlating dimensions obtained through our statistical analysis for the two instruments, LOR and QPS. These three dimensions express quality in teaching practices related to problem solving measured with two different instruments. These findings are in the same direction as the positive correlation between the frequency of observed non-routine task and self-reported quantity of non-routine PS mentioned above, telling that the two instruments are coherent. Certainly the results of this analysis are not conclusive, but it encourages going deeper in the analysis and tuning of these two instruments with larger samples.

DISCUSSION

We would like to summarize some results found with each LOR and QPS instrument, highlighting those characteristics of teaching practices related with problem solving where both instruments coincide. This is to be done having in mind that this is an exploratory study and that the instruments were not designed for information triangulation purposes. Teaching practices are multidimensional in nature, nevertheless a principal components analysis of both instruments lead us to summarize the information obtained through a big quantity of indicators, (14) and items (19), in two main types of dimensions: those related with classroom management or classroom organization and those linked with teaching practices promoting students development of problem solving skills and autonomy, in the terms described in our

		1	2	3	4	5
QPS	1. Student centred (student)	_				
QPS	2. Student centred (teacher)	0.858**	_			
QPS	3. Teacher centred	0.549**	0.583**	_		
LOR	4. Classroom Management	0.143	-0.018	0.275	_	
LOR	5. Quality Practices	0.369*	0.543**	0.178	-0.075	_

Table 2: Correlations between the new QPS and LOR dimensions $^{**}p < 0.01$ $^{*}p < 0.05$

framework. From our point of view, this is an interesting result although, since both instruments are new, further research is needed to prove the stability of these dimensions.

Correlations indicated some coherence between the information obtained with both instruments. For example, teachers that reported to use more non-routine problem solving activities were the same for whom non-routine task was coded in the external observation of the lessons.

Correlations between dimensions in the case of observed teaching practices point to that there is no relationship between classroom management and quality practices.

Correlations between the three dimensions expressing quality in teaching practices related to problem solving measured with two different instruments were high and significant, showing that the two instruments are coherent in this crucial dimension of our analysis. This is the most important finding in our study that encourages to conduct further studies in this direction.

To sum up, both instruments became useful to identify dimensions related with teaching practices on problem solving and there was coherence between the information obtained using each of them, what it is interesting from a methodological point of view, especially having in mind that classroom observation is very expensive in relation to questionnaire answering.

Moreover, it will be interesting to investigate if obtained dimensions for problem solving teaching practices are stable or not, taking a bigger sample, and go deeply in some cases with a qualitative study that let us to understand better some of the results of this analysis.

ACKNOWLEDGEMENT

Funding from FONIDE F721209, CONICYT-PCHA/Doctorado Nacional/2013 21130684, PIA-CONICYT Basal Funds for Centers of Excellence Project BF0003 and Basal CMM project is gratefully acknowledged.

REFERENCES

Badillo, E., Figueiras, L, Font, V., & Martínez, M. (2013). Visualizing and comparing teachers' mathematical practices. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the 8th Congress of European Research in Mathematics Education (CERME8)* (pp. 2927–2935). Ankara, Turkey: METU and ERME.

Engeln, K., Euler, M., & Maass, K. (2013). Inquiry-based learning in mathematics and science: A comparative baseline study of teachers' beliefs and practices across 12 European countries. *ZDM: The International Journal of Mathematics Education, 45*(6), 823–836.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430–511.

Kloosterman, P., & Stage, F. K. (1992). Measuring beliefs about mathematical problem solving. *School Science and Mathematics, 92*(3), 109–115.

Leroy, N., Bressoux, P., Sarrazin, P., & Trouilloud, D. (2007). Impact of teachers' implicit theories and perceived pressures on the establishment of an autonomy supportive climate. *European Journal of Psychology of Education, 22*(4), 529–545.

Maaß, K., & Artigue, M. (2013). Implementation of inquiry-based learning in day-to-day teaching: a synthesis. *ZDM: The International Journal of Mathematics Education, 45*, 779–795.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York, NY: Academic press.

Skott, J. (2013). Understanding the role of the teacher in emerging classroom practices: searching for patterns of participation. *ZDM: The International Journal of Mathematics Education, 45*, 547–559.

Stipek, D.J., Givvin, K.B., Salmon, J.M., & MacGyvers, V.L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*, 213–226.

Swan, M. (2006). Designing and using research instruments to describe the beliefs and practices of mathematics teachers. *Research in Education, 75*(1), 58–70.

APPENDIX A: QPS

We present an English translation of the items. The original version was applied in Spanish. The items with an (i) are considered to measure practices that we hypothesized are not related to good problem solving activities.

1) I organize my students working in groups

2) If my students take too much time in find the solution of a problem, I solve it in the board. (i)

3) My students solve problems independently

4) My students express their different strategies to solve their problems even though they are wrong

5) I walk around the tables watching my students work

6) I organize my students working individually (i)

7) Usually I get amazed with my students ideas

8) My students are able to discuss with each other different ways to solve non-routine problems

9) I use plenary discussions with all the class

10) My students depend too much on my help to move forward with the problems (i)

11) If a student is too frustrated with a problem, I try to guide him/her only with questions

12) I promote students to take time to solve non-routine problems

13) My students generate different solution strategies

14) My students make interesting questions

15) If a student is too frustrated with a problem, I show him/her how to solve it (i)

16) My students discuss their own mistakes

17) My students progress too slow in the problems resolution (i)

18) I ask questions all the time

19) My students explore new problems arising as a result of the problems we are working on