Teacher empowerment and Socioepistemology: An alternative for the professional development of teachers

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Teacher empowerment is an alternative proposal from Socioepistemology that postulates it as a tool for the professional development of teachers. The concept of empowerment is accompanied by the "problematization of knowledge" in both senses: mathematical knowledge and school mathematical knowledge. We assume that teachers will be better able to transform their educational reality, since they will have taken possession of the teaching knowledge. This new relationship to knowledge is not based more on mnemonics, but on what we consider to be the essence and "raison d'être" of knowledge that will allow the teacher to develop various strategies considering his group of students. In this paper, we will discuss "proportionality" for its high cultural value and its transversality in education.

Keywords: Socioepistemological Theory, teacher empowerment, problematization of knowledge, proportionality.

POSITIONING THEORY

While "the best-selling question" of the 90's was *how* to teach using various teaching strategies in order to make the understanding of certain mathematical knowledge more accessible to students at different educational levels, the Socioepistemological Theory posed a somewhat different question: *What* is it that we are teaching? *What* is it that our students are learning? That is to say, lets study and discuss the *nature of mathematical knowledge*, and thence, "reflect on" the *school mathematical knowledge*. Studying its nature does not implies just making an epistemological study, but getting a systemic perspective of the epistemological, didactical, social and cognitive dimensions of mathematical knowledge, it means, looking at them as a whole.

In terms of teaching practice, while the classical currents analyzed the tasks that teachers use in the classroom, the teacher-student interactions, the competition brought into play to solve math problems, the teacher's knowledge on how students think, know or learn a specific mathematical content, among many others, Socioepistemology wondered what and how is the professor's relationship to knowledge in a specified didactic relationship? It is for this reason that our line of research considers a necessary articulation between two theoretical elements: the functionality of the mathematical knowledge of proportionality (transversal notion in the educational system with high practical value in everyday life) and the theoretical construct of teacher empowerment (Reyes-Gasperini & Cantoral, 2014). On the basis of such an articulation we wove a conceptual framework in order to show that the *teacher empowerment, from a socioepistomological vision*, is a little known alternative to study the professional development of teachers *problematizing* mathematical knowledge.

Socioespistemological theory studies the social construction of mathematical knowledge. The education problem is not that of the constitution of abstract objects, but *their shared significance by its culturally situated use*. It is assumed that since this knowledge is socially constituted in non-school settings, its diffusion to and from the educational system forces it to a number of changes that directly affect their structure and functioning, so that also affects relationships established between students and their teacher. The socioepistemological research promotes a *decentration of the object*, that is, to pay attention to the *practices* from which it emerges and not just on the mathematical object per se. Socioepistemology delimits the role of historical, cultural and institutional setting in human activity, so the problem that motivates the research can be student's difficulties in learning a particular concept; however, studying it seeks to contribute to an alternative vision that includes the associated social practices and, to that extent, provide a social and cultural look of mathematical knowledge (Cantoral, 2013, Cantoral, Reyes-Gasperini, & Montiel, 2014).

SOME LINKS TO TEACHER EMPOWERMENT WITH MATHEMATICAL KNOWLEDGE

Empowerment is a *social phenomenon* typically studied in various disciplines and approaches, from social (Martínez Guzmán Dreyer & Silva, 2007), feminist (Camacho, 2003), from the Psychology Community (Montero, 2006), or from an educational point of view (Howe & Stubbs, 1998, 2003; Stolk, de Jong, Pilot, & Bulte, 2011). While each of the disciplines has a particular focus on the phenomenon, they all concur in their main characteristics that we have synthesized in the following way: empowerment is understood as a process of the individual in collective work (interaction is required in collective work), which parts from the reflection to be consolidated in action, which is produced by the individual without the possibility of being granted (collaborative work will be necessary but not sufficient to promote empowerment) and, above all things, *transforms the reality of the individual and his context.*

In particular the projects that aim to promote teacher empowerment (Howe & Stubbs, 1998, 2003; Stolk, de Jong, Bulte, & Pilot, 2011) provide teachers with tools to design new situations emphasizing contextualization, either by knowledge of new research related to the topic, as well as by the sample of situations that provide a context to what they already know. All with the aim of obtaining an attitude of leadership, confidence and improvement in their practices towards education, emphasizing the fact that they may acquire the power to take the reins of their own growth. While we may coincide with the results that are expected to be achieved, we believe that this type of analysis is reduced to only a pedagogical interpretation.

Our proposal, given the socioepistemological character that is added to this phenomenon, incorporates the notions of *problematization of mathematical knowledge (PMK) and problematization of school mathematical knowledge (PSMK)* keys to boost teacher empowerment. The action of *problematizing* the school mathematical knowledge is done with the knowledge that teachers use in the educational system. Now, why do we differentiate *PMK* from *PSMK*? The *PMK* refers to the fact of "making a problem out of knowledge", an object of training analysis, locating and analyzing its use and its *raison d'être*, namely refers to the study of the nature of said mathematical knowledge, for example related to the proportionality, on the basis of questions like: What problem did the *notion of proportion* come to resolve that could not be resolved without them? Are the problems more difficult when the magnitudes are heterogeneous than when they are homogeneous? Why are problems on the fourth missing value worked on if the comparison is not represented there? Where do the proportions appear in the civilization? What characterizes the relation of proportionality? Among many others. The socioepistemological study based on teaching, epistemological, social and cognitive dimensions of knowledge can make up a *unit of socioepistemic analysis* (*USeA, UASE in spanish*) that causes a singular symbiosis between, and from, the four dimensions, in order to generate a theoretical framework that challenges mathematical knowledge, and subsequently, school mathematical knowledge.

In contrast, when we work with the *PSMK*, we draw on the knowledge that is fundamental to the educational system. Based on the *USeA* an activity guide is designed which confronts the typical educational activities in order to put the teacher in a learning situation and thus generate spaces for the *PSMK* to be performed (Reyes-Gasperini, Cantoral, & Montiel, 2014). We understand the *PSMK* as the action part of the introspection, the gaze of the learner and uses available in their everyday life.

It is necessary to mention at this point that the socioepistemological theory rests on four fundamental principles (Cantoral, 2013): the regulatory principle of *social practice*, the principle of *contextualized rationality*, the principle of *epistemological relativism* and the principle of *progressive resignification* or *appropriation*. These four principles underlie the *PSMK*, well this problematization will allow the teacher to consider that social practices are the foundation of the construction of knowledge (regulation of social practices), and that the context will determine the type of rationality with which an individual or group – as a member of a culture – builds knowledge whilst he/she can express it and put it to use (rationality contextualized). Once this knowledge is put to use, that is to say, it is consolidated as knowledge, its validity will be relative to the individual or the group, as it emerged from their construction and their arguments, which gives that knowledge *epistemological relativism* Thus, because of the evolution of the life of the individual or group and its interaction with various contexts, such enriching knowledge of new meanings will be redefined constructed to this moment (progressive redefinition).

Therefore, the links between empowerment and mathematical knowledge are given by the articulation of the typically social phenomenon with the socioepistemologic character that underlies its main action: the *PSMK*.

PROBLEMATIZATION OF MATHEMATICAL KNOWLEDGE (PMK) AND PROBLEMATIZATION OF SCHOOL MATHEMATICAL KNOWLEDGE (PSMK): THE CASE OF PROPORTIONALITY

In our current research, we take the proportionality, as a mathematical notion, to work the *PMK* and the subsequent *PSMK* with teachers. In order to give some guidelines for what leads to the construction of the USeA, we part from an idea rooted in the educational system and society in general, however, we should make it plain that here there will only be presented some examples of what the USeA is in its entirety. If we asked in a generic way, "What is proportionality?" most would answer that it is a form of variation: "When *more is more* or *less is less*", or, "when using the rule of three". In these responses, we find two aspects to consider, first, the memory of a "recipe" to identify a problem of proportionality; on the other, an algorithm with which to solve said problem. Some typical examples given in this respect are: the price in buying tortillas in kilos, the distance traveled in a certain time, among others; but they all have the same prototype. The use of colloquial language allows the fluidity of a set mathematical thought, but will inevitably redefined subsequently, for example, at a formal level, it should be reflected in written form at a level of symbolic object and consider "the notion of constants proportionality" as the product or the reason for the magnitudes. Piaget's theory (1958, quoted in Noelting, 1980) considers proportionality as the hallmark in the development of formal operations, therefore, we ask, have students (or teachers in our case), developed this type of reasoning? The idea of spending an additive relationship to a multiplicative relationship seems to be the fundamental idea that has been pursued in studies concerning proportionality.

In problems of proportionality, usually, it is asked, "How many hours it take to travel 25 miles?", for us it is important to reflect on the notion of speed, considering it as relation between distance and time, rather than solely the notion of "missing value". Perhaps this is an obstacle that so far may not have been considered as an issue that cuts across the colloquial and has to do with a germinal idea of what the notion of reason is. To do this we ask, what is the nature of proportionality? Is it a continuous or a discrete nature?

Given this situation, we question whether it is possible that the question "how many" can generate a centered answer among students in the quantification rather than the relationship. Well, if we asked what is or what are the relationships that can be established between the magnitudes, perhaps we would be giving the students a relationship beyond the concentration in number and quantity to think about. That is, if from a psychologist point of view the research reports that proportional reasoning is related to the development of formal operations and is complex, if not impossible, to achieve such reasoning, it would seem that we should think that the way that said knowledge is addressed is alien to the reality of the student and teacher in our case.

We conducted a formal analysis of the theory of proportions addressed by Elements of Euclid in his Book V, we work on par with the idea of incommensurability. Hence we affirm: if there is no such thing as a common measure, how can these quantities be measured? The problem of measuring, was replaced by Euclid as the problem of comparing. This is the fundamental question that gave rise to the theory of proportions between magnitudes. So, is the condition caused by the inability to measure what has led to the need to compare? Just as the inability to advance time which has led us to predict (Cantoral, 2013).

In this respect it is stated that this theory emerges to address two specific problems of the time. On the one hand, before the conflict that had suffered the Pythagorean theory regarding the impossibility of assigning a number to the ratio of two quantities, theory of proportion was redesigned in such a way that "you could talk about reasons and proportions, without specifying whether or not they were considered commensurable magnitudes" (Guacaneme, 2012, p. 104), where the greatest merit of the theory described in Book V is the possibility of comparing incommensurable magnitudes (Corry, 1994, quoted in Guacaneme, 2012). On the other hand, the elements are intended to present the mathematical theories under a deductive axiomatic scheme. Now, if the theory of proportionality arises as the possibility of comparing incommensurable magnitudes, it is logical to think that if most of the problems encountered in the literature that are to do with the missing fourth, these kind of problems does not always require a proportionate reasoning (Lamon, 1999), for there will be nothing to compare because the amounts are given and you have to operate on them arithmetically, applying the rule of three most of the time. In addition, they can announce themselves with the structure of the missing fourth, without there being a proportional relationship between the magnitudes, however, the students will resolve it since the characteristic they believe to be enough to apply the simple rule of three is that both magnitudes increase (while the issue is to characterize the type of growth). So far, we conjecture that it is necessary to return to emphasize the relationship between the magnitudes by *comparing* them.

During the process of research on proportionality, we have studied the famous "Cauchy functional" that gave light to analyze the difference between the additive and multiplicative thinking in depth, creating a new look at the nature of the proportionality. There are four functional Cauchy equations (Roa, 2010), in a later study we related each of the four functional equations at school-level with four functions that are of significant importance: <u>Exponential function:</u> $f(x + y) = f(x) \cdot f(y); x, y \geq 0$; <u>Logarithmic function:</u> $f(x \cdot y) = f(x) + f(y); x, y \geq 0$; <u>Power function:</u> $f(x \cdot y) = f(x) \cdot f(y); x, y \geq 0$; <u>Proportional function:</u> $f(x + y) = f(x) + f(y); x, y \geq 0$.

The Cauchy's functional served as sustenance to give evidence, in an analytical and graphical way, for the differences between a proportional linear function and a non-proportional linear function, since that function is only true in the first case. This allows us to analyse the proportional function from a particular property and not just from the classical ownership "double receives double" or from "the rule of three".

In our case, we study in depth the related with the proportional function, which, based on the USeA of Reyes-Gasperini (2011), it was stated with the models of proportional thinking reported in the literature. Let's start now by thought patterns, there is a qualitative thought that is the first to appear in individuals (Inhelder & Piaget, 1972) and is exemplified by the idea of the chorus "more is more; less is less".Godino and Batanero (2002) conducted a study based on (Noelting, 1980) where they report the following types of reasoning used by students to decide between two jugs of juice which is the one with the "stronger" taste. Their arguments are based on the comparison of the number of glasses of water and juice placed. The question posed is: "My mother has prepared two jugs of lemonade. In jug A she has mixed two glasses of water and one glass of lemon juice. In jug B she has mixed three glasses of water and one glass of lemon juice. In which of the two jugs is the lemon flavor is more intense?"

Even if the amounts are in play, the answer is not a quantity, but a relationship between them: which is more intense? As the authors say "the additive strategy would be to compare the difference between the glasses of water and the lemon juice in each jar" (Godino & Batanero, 2002, p. 439), but they ensure that this strategy will not be sufficient to address problems of greater complexity. Regarding the above, Carretero

$$¿f(x) + f(y) = f(x + y)?$$

$$\begin{gathered} f(x) = mx + b, b \neq 0; f(y) = my + b, b \neq 0 \\ (mx + b) + (my + b) = m(x + y) + b \\ mx + my + b + b = m(x + y) + b \\ m(x + y) + 2b \neq m(x + y) + b \\ \text{So, } f(x) + f(y) \neq f(x + y) \end{gathered}$$

$$\begin{gathered} f(x) = mx; f(y) = my \\ mx + my = m(x + y) \\ m(x + y) = m(x + y) \\ \text{So, } f(x) + f(y) = f(x + y) \end{gathered}$$

Figure 1

(1989) distinguishes two types of structures. On the one hand, those having a given relationship between homogeneous magnitudes (also called extensive by other authors) to those called scalar multiplicative models; and on the other hand, those having a relationship between heterogeneous magnitudes (also called intensive), to those called functional multiplicative models. Subsequently Lamon (1993) also makes a distinction as strategies for students to find the missing value in a proportion. He calls them inter models (corresponding to the multiplicative model scalar) and intra models (corresponding to the functional multiplicative model). The work with the different types of multiplicative structures around the acquisition of the notion of proportionality allowed Carretero to conclude that "the division is evidently a more difficult operation than multiplication, despite the underlying multiplicative structure" (Carter, 1989, p. 95). Thus, we conclude that the additive model precedes the scalar multiplicative model, which is less complex than the functional multiplicative model, however, they are all thoughts that underlie the idea of proportionality.

Moreover, G. Vergnaud works on the theory of conceptual fields considering them a set of situations that can be "analyzed as a combination of tasks of which are important to know their own nature and difficulty" (Vergnaud, 1990, p. 140). Regarding proportionality, he compares the conceptual fields of additive structures (those that require an addition, subtraction, or a combination of the two) and the multiplicative structures (those that require multiplication, division or a combination of the two). This allows him to generate a classification and an analysis of cognitive tasks and in procedures that are potentially at stake in each. This allows her to generate a classification and an analysis of cognitive tasks and in procedures that are potentially at stake in each.

He concludes by stating that "it is not superfluous, on the contrary, to emphasize that the analysis of the multiplicative structures is profoundly different from the additive structures." (Vergnaud, 1990, p. 144). This is to say, we can ensure that there will be tasks that demand a multiplicative structure, and others, an additive structure.

Therefore, not all problems deserve to postulate a proportional reasoning in terms of Lamon (1993), but as Vergnaud (1990) says, there are problems that can be solved by additive structures or pre-proportional reasoning, for example: "If one coconut costs 35, how much do 10 coconuts cost?".

This example is tackled by Carraher, Carraher and Schiemann (1991), where they see how a child solves "on the street a sales situation": <u>Client:</u> How much does one coconut cost?; <u>M</u>: Thirty-five; <u>C</u>: I want ten coconuts; How much is it for ten coconuts ?; <u>M</u>: (Pause) Three are 105 plus three is 210. (Pause) we are four short. It is ... (pause) ... it seems to be 350.

An immediate question, at this level of analysis is: Has the child developed proportional thinking? Our answer is yes, because the situation does not require a multiplicative structure, but reaches an additive structure (additive model composed seen above), and behold, our assumptions about the mathematical knowledge of proportionality: proportional reasoning viewed as the relationship between two magnitudes that remain constant, should be assessed whether developed or not, whenever the situation warrants a comparison, that is, an analysis of relationship type between the magnitudes, and not the discovery of a missing value. Hence the need to draw up a learning situation that involves a sequence of activities where different thoughts are progressively and systematically put into play.

Proportionality arises to address the inability to measure incommensurable magnitudes; therefore, as it has already been shown as evident, the current school significance induces us to look at a math problem with a different rationality for offering its epistemological nature. This may explain the academic failure of students to proportionality.

Given the fundamental idea that the inability to measure generates the need to compare, let's see what happens with a purely mathematical problem to which the scales for working have been removed with the idea of the type of relationship between the magnitudes more than in the quantification of the values.

Activity: To the right of the graph of the function f is presented. Does this represent a direct or inverse proportional function? Justify your answer.

Most teachers with whom we have worked on this activity (both secondary school teachers and students) argue that it is inversely proportional because "a plus x, minus y" (qualitative thought). This was the trigger

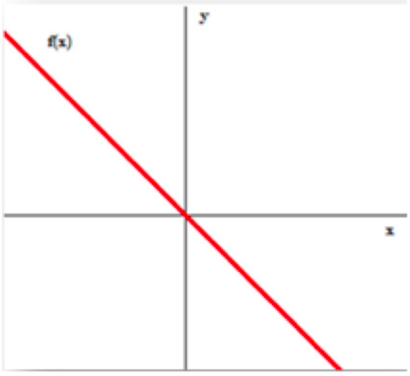

Figure 2

to replant the way we work proportionality in the education field, because it generates germinal mathematical errors supported by "rote simple recipes".

The work with teachers leads us to characterize "what it is proportional" (the proportional, as we use to say in our language of practices) as a relationship between two magnitudes whose ratio remains constant. First, is analyzed the constant rate of change, that characterizes all linear relationship () and then analyzes the constant ratio, that is maintained between its variables, which characterize directly proportional linear relations (). Thus, was worked on the relationship between the magnitudes as from different properties of proportional relationships. So rote recipes make sense and meaning.

Figure 3

To illustrate, we will show an activity we did with teachers, in order to address the idea that not every relationship which have the simultaneous increase or decrease characterizes a proportional relationship:

"Considers that the first figure is the original. Which of them could be considered an extension or reduction of it?"

After the teachers' discussions, where the notion of scale was at stake, we address them to reflect that it is not enough to consider the presence of an increase, we must emphasize the way this increase is done.

DEBATE AND FINAL CONDITIONS

The mathematical treatment of a transversal mathematical subject in all of mathematics education shows that you not only need to work with teachers on pedagogical issues of general teaching process, or only the contents as they are addressed in school. But to this type of study we add the need to problematize mathematical knowledge to then work with teachers posing questions of school mathematical knowledge and thus contribute to the professional development of teachers through the *change of relation to mathematical knowledge*, and not solely based in mnemonic rules or formulas with little meaning, but based on what we call "the reason for this mathematical knowledge and its frameworks that allow their use."

What we propose to be discussed in in the Group of Mathematics Teacher Education and Professional Development is how to generate, within the professional development of the teacher, areas in which the knowledge of the teacher is not classified, but through what the teacher has in its repertoire (background), deepen and challenge the school mathematical knowledge and change accordingly their relationship to knowledge. Thus we assume that teachers will be better able to transform their educational reality, since they will have taken possession of knowledge that teaches. This new way to relationship to knowledge no longer based on mnemonics, but on what we consider the essence of its purpose and allow the teacher to develop various strategies by the group of students he/she may work with. In short, we are studying the process of teacher empowerment, which we postulate as a tool that contributes to teacher development.

The line of research on teacher empowerment provided by socioepistemology brings a fresh, different look on dominant versions in the literature of professional development of the teacher in the field of school mathematics.

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