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How to improve spatial visualization ability of preservice teachers of childhood education: A teaching experiment

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In this paper, we describe a teaching experiment designed and implemented in a classroom of preservice teachers of Childhood Education with the main purpose of measuring the improvement of their spatial visualization ability. Indeed, taking into account some common materials and games, we have designed a four-sessions experiment in mathematical classroom of an Undergraduate Degree of Childhood Education of the University of the Basque Country. After the implementation of the experiment in a classroom with 27 students, the analysed results assert that the spatial visualization ability of the students is better than before, implying that this ability can be achieved and improved even in adults.

Keywords: Spatial visualization ability, teaching experiment, common games, pre-service teachers, childhood education.

INTRODUCTION

Nowadays, we get more and more information through symbols and images that has to be analysed and interpreted in order to extract the meaning.

Understanding symbols is not the only application of the visual-spatial ability that we use in daily life, moreover we are actually surrounded by actions that need it like parking a car, where ideally we need to be able of visualizing the parking spot and calculate if our car fits in. This involves making decisions about how we see objects in a three-dimensional world and how they behave in it. That means we are constantly using our natural ability to position objects, project them on a plane, moving or rotating them mentally. But how can [these abilities] be acquired? And more important, are they susceptible of being taught?

Among the objectives of the book “principles and standards for school mathematics” of the National Council of Teachers of Mathematics (NCTM, 2000) we can find we need to develop the spatial sense and appreciate the geometry as a way to describe and model the physical world. These goals are completely related with the orientation and spatial visualization and, as “Curriculum Focal Points” of geometry points out, they should be studied from childhood Education to 8th course.

In our case, the curriculum of the Childhood Education of the Basque Country has as one of its main goals the “Identification of flat and three-dimensional elements of the environment” (therefore, the visualization skill is required) (EJ / GV, 2010).

These facts imply the visualization ability should be taken into account in the mathematics classroom, developing the necessary activities to improve the skills related with spatial visualization. In this sense, many studies have been done with the purpose to research the factors involved in the teaching-learning of visualization (Bishop, 1983; Gaulin, 1985; Gutiérrez, 1996; Gonzato, 2013; Hershkowitz, Parzysz, & Van Dormolen, 1996; Presmeg, 2006) and one of the open questions is the influence of teacher’s ability in this process.

SPATIAL VISUALIZATION

According to Eurydice and Culture Executive Agency (EACEA, 2011), mathematical competence is one of the key skills for personal development that facilitates social and labour inclusion of citizens satisfactorily. After a thorough analysis of the results of the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) studies at European level,
training and professional development of teachers of mathematics highlights as one of the key issues in education.

About mathematical competence, one of the sections analysed by PISA is the “space and form” section, in which the phenomena and geometric and spatial relationships are examined. Indeed, the development of this competence requires observe similarities and differences, analyse the components, recognize shapes in different representations and dimensions and understand the properties of objects and their relative positions. Summarizing it, we must learn how to visualize objects in space and understand the two-dimensional representation thereof, but what does “visualize” mean?

Spatial visualization can be defined as “the ability, the process, and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper, or with technology tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings” (Sarama & Clements, 2009, p. 183).

Furthermore, Presmeg specifies spatial visualization ability as a collection of processes involved in generating and manipulating mental images, as well as guiding the drawing of figures or diagrams on paper or computer screens (as cited in Sarama & Clements, 2009, p. 184). These processes and skills of subjects have been extensively studied to perform certain tasks that require spatial visualization ability. In fact, these investigations figure out the visualization throughout focusing on different tasks depending on mathematical content: planar representations of 3D-objects (Gutiérrez, 1996; Hershkowitz, Parzysz, & van Dormolen, 1996), planar developments of 3D-objects and 3D-constructions of planar developments (Cohen, 2003; Fischbein, 1993), classification of figures, comprehension of concepts and properties, geometrical transformations (rotations: Battista et al., 1982), compose and decompose 3D-objects in their parts (Bishop, 1983).

About visualization in education, according to NCTM and Basque government, spatial thinking can be learned and should be taught at all educational basic levels. Regarding this, Bishop (1980) proposes questions about teaching that, nowadays, some remain still unanswered: a) Should experimental teaching methods in this area take into account the spatial abilities of the teacher; b) How much responsibility should mathematics teachers take for the training and teaching of spatial abilities? Is this perhaps an area like language, which is every teacher’s responsibility?

Hershkowitz, Parzysz, and van Dormolen (1996) underline that the nature of mathematics is the search of patterns and therefore visualization is a fundamental tool to recognize them, but even though this relevance, the visual education is often neglected in the curriculum.

In the same way, but some years later, Presmeg (2006, p. 227) gives a big perspective of the researches done in mathematics education and, again, she proposes 13 open questions, some of them related with visual ability: 5. What conversion processes are involved in moving flexibly amongst various mathematical registers, including those of visual nature, thus combating the phenomenon of compartmentalization? 9. How may use of imagery and visual inscriptions facilitate or hinder the reification of processes as mathematical objects? 10. How may visualization be harnessed to promote mathematical abstraction and generalization? 13. What is the structure and what are the components of an overarching theory of visualization for mathematics education?

Actually, different studies have been published giving partial answers to these questions, for example, Gaulin (1985) outlines some activities connecting coding and decoding of spatial information through representations. One of the conclusions was that even the teachers had problems to interpret some graphical representations.

Likewise, Battista, Wheatley and Talsma (1982) show that a specific design of a course based on manipulative materials and some concrete models (symmetry of polygons and polyhedral, paper folding, tracing and using a Mira), improves the specific part of the spatial ability of prospective teachers related with rotations (the test used was “Purdue Spatial Visualization”).

In this sense, the work presented here tries to analyse the influence of a specific teaching experiment designed for prospective teachers of Childhood Education, where the methodology is based on com-
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mon materials, games and working as collaborative teams (see, for example, Gutiérrez & Berciano, 2012).

METHODOLOGY

Context of the study and objectives of the research
The research showed here is included in a bigger project where the main objective is to improve the skills of our students with respect to their didactical competency in spatial visualization as teachers of childhood education (this project has the restriction of time: in 60 hours a big curriculum related with childhood mathematics has to be given); but, first of all, we will analyse if their spatial visualization ability can be improved and how. For this purpose, we will focus on the first question, measuring the improvement of the spatial visualization ability of our students after the implementation of a designed teaching experiment.

Participants
The group chosen for the teaching experiment was a Spanish group of the 3rd level of the Undergraduate Degree of Childhood Education of the University of the Basque Country of the topic “Mathematical thinking and its Didactics”, formed by 27 persons, 26 women and 1 man. All of them usually went to classroom and participated actively. This group was divided in small groups of 2 or 3 persons to work in a collaborative way during the entire course.

Instruments to measure the spatial visualization ability
The instruments used to analyse the results have been mainly two: the diaries of the students, where their evolution could be seen, and a test to measure their competency about spatial visualization (used as a pre-test and a post-test).

About this second tool, we have seen that different authors focused their research in a specific task related with spatial visualization ability, but in our case, to measure it, we have taken into account all of them, that is, we have focused on studying the tasks of interpreting perspectives of three dimensional objects (activities requiring recognize and change views (change of perspective)), rotate objects mentally, interpret different planar representations of three-dimensional objects (perspectives, views,...), turn a planar representation into another, build objects from one or more planar performances, .....

For this purpose, we have used the test called “Test of Three-Dimensional Objects” (VOT), designed and validated by Gonzato (Gonzato, Fernández, & Godino, 2011; Gonzato, Godino, & Neto, 2011; Gonzato, 2013). In this test, the mathematical knowledge and the knowledge of teaching Spatial Visualization in elementary education are measured. As the author shows in her PhD thesis, the tasks proposed in the test involve the following actions: change the representation type (planar or three-dimensional object); rotate the object or parts of the object, or equivalently, change mentally the perspective of it; fold a planar development to create a three-dimensional object (physical or represented), or vice versa, expand the object for obtaining one of its developments; compose and decompose in parts; and, given a solid, count the component parts (units of volume, faces, edges, vertices, etc.).

In fact, the test has 5 items (with sub items), each of them dedicated to a previous action. Let’s see the first item and its parts (extracted from (Gonzato, 2013), question 1b’) has been adapted to Childhood Education):

1a) From which positions have been taken the pictures you can see at right side? 1a’) Justify the answer. 1b) Identify the knowledge involve in the resolution of 1a). 1b’) Indicate how should you change 1a) to be able to work it at Childhood Education classroom.

1c) The following figure shows a building drawn from front-right angle. Draw the view from back. 1c’) Justify your answer.

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Figure 1

Figure 2
The way to correct this test is:

1) To measure the skills our students have about mathematical knowledge, we have analysed the results given by them in the first sections of each item. In this case, the correction is right, partially right or wrong (1a, 1c). In the same way, it is possible to do a qualitative analysis of the errors given by the students to obtain more information about the error types.

2) To study the skills that the students have about didactical knowledge, we have examined the answer given in sub item 1b' (not included in this paper).

We have used this test as a pre-test and post-test to, on one hand, study the starting point of our students and, on the other hand, see the variance of the ability of our students after the implementation of the teaching.

Mathematical task
In all the activities of the experiment, the task requested to the students is to determine which mathematical concepts and properties are needed to realize a specific exercise where visualization skills are involved. These are some questions:

1) Which properties of a 3d-object should be used to do its planar development (and the reverse)?

2) Given a 3d-object, how many different projections can be done, and which mathematical properties are involved?

3) Which are the mathematical concepts involved in the movement of a 2d-object in the plane, which properties do they have and how can they be composed?

Design of the experiment, schedule of the materials used and the activities done
Previously to the experiment, the students filled out the pre-test. The experiment has been realized inside the topic “Geometry” in 3 sessions of 2 hours each and another hour and a half of a fourth session has been used to evaluate the results after the implementation using the post-test. The way to design the session has been the same, that is, first the students should become familiar with a selected material (a specific game or common material); second, they should realize different activities to explore the characteristics of the material from mathematical point of view; third, they should formulate a hypothesis about a given question; fourth, they must compare the hypothetical results with the real answer; fifth, they had to give an explanation about the result with mathematical arguments. All of these activities were done in small collaborative teams and the students had to talk to each other at all times.

Next, we describe each session with the given instructions to the students:

Session 1: Visualization and spatial representation (01/04/2014)
The activities done: VOT test and playing with boxes, where the main objective is to learn how are the cardboard boxes constructed from their planar development, and vice versa.

This activity has two different parts, but the way to work with the material is always the same. Each pair of students should complete a diary where the next steps should be done:

1) Observe the given object (3d or 2d).

2) Describe it verbally and graphically.

3) Make a hypothesis about its planar development/ the 3d object that can be done with it (3d and 2d object respectively).

4) Do the planar development/ the 3d object.

5) Describe verbally and graphically the planar development/ the 3d object.

6) Compare the hypothesis with the experimental result.

The materials selected were a commercial box for the first activity and a common planar development for the second activity.

Next, let’s see a small part of the diary realized by a student with respect to the first exercise, the planar development of a cardboard box that contains coffee capsules.
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Session 2: Identification of 3d objects and their component parts (08/04/2014). The activities done: constructions with multilink cubes and constructions of buildings with the game called “Skyscraper”.

The way to give the instructions is the same than in session 1. Two different parts, but with the same line of argue.

With the multilink cubes, the students should construct different structures, describe them verbally to other colleagues of the same collaborative group and the other students should replicate the construction only with the explanations, without see it.

In the second part of the session, students should play with Puzzle Skyscraper. Each puzzle consists of a 4x4 grid with some clues along its sides. The object is to place a skyscraper in each square, with a height between 1 and 4, so that no two skyscrapers in a row or column have the same number of floors. In addition, the number of visible skyscrapers, viewed from the direction of each clue, is equal to the value of the clue. Higher skyscrapers block the view of lower skyscrapers located behind them.

When the construction is done, the students should describe all the projections of the construction verbally and graphically.

Session 3: Geometric transformations (symmetries, turns) (15/04/2014). In last session, the main purpose is to recognize the differences between symmetries, translations and turns. For this end, the material used was: game reflections, game “the prince and Munster” and mirrors. The instructions were given following the next steps:

1) To distinguish between symmetries and translations, the students should find examples at the classroom.
2) Next, taken into account the symmetry, a classification of alphabetic letters was done.

**Table 1**: Theoretical planar development of a given box and part of the diary of a student

<table>
<thead>
<tr>
<th>Take the box</th>
<th>Describe it verbally</th>
<th>Describe it graphically</th>
<th>Make an hypothesis about its planar development</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Image" /></td>
<td>It is a cube-shaped box that has all the edges equal (where two faces meet), which is measuring the same. It also has 6 faces.</td>
<td><img src="image2" alt="Image" /></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 3**: A given square and its construction with all the perspectives done by a student (left and right respectively)
3) Find the differences between a photo and an image in front of a mirror.

4) Finally, the students should anticipate the hypothetical answer of the reflections of the mirrors to create a copy of the asked image (reflection) or the face of the prince (prince and Munster). Again a verbal explanation should be given to their colleagues.

Session 4: Evaluation of the spatial visualization ability (29/04/2014). Again, VOT test was used to evaluate the results after the experiment.

RESULTS

To show the main results, first of all we’ve focused in the analysis of the differences between the results in the pre-test and the post-test considering that the maximum score is 20.

As Figure 3 shows (left graphic), it is clear that in a big percentage, the students have improved their results in the test. Only 4 students have obtained worse result in the post-test than in the pre-test (right graphic), but this can be biased because two of them showed no interest in doing right both the pre and the post-test. Furthermore, the average in the pre-test is 10.04 and 11.56 in the post-test, the median is 10 and 12 respectively and the standard deviations are almost the same (3.5 and 3.6). All this implies that their spatial visualization ability is better, which is the main objective of this teaching experiment.

On other hand, if we count the students who have done successfully each task in pre-test and post-test (Figure 4, left graphic), we can see that, apart from task 1A and 5A, this number has increased, implying again that the students have developed their spatial visualization (see Figure 4, right graphic).

CONCLUSIONS

As we have shown along the paper, we have designed and implemented a new didactical proposal to work visualization with preservice teachers of childhood education. The main part of the design is the use of common materials and games, which normally are known by the students, but with the main purpose of focusing their attention in activities related with visual skills. The activities have the same structure, beginning with an experimental part with verbal and
graphical descriptions; later, they ask for a hypothesis about a question and a verification of the veracity of the hypothesis and finally end with a question about the description of mathematical properties and concepts involved on it. Regarding to results, our students have significantly improved their spatial visualization.

This new perspective allows us to go from a simple activity to abstraction, opening a new research line where Realistic Mathematics Education is the basis of the methodology for designing new teaching experiments, where the goal is to evaluate the impact and the possible improvement of mathematical competencies.

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