Competency level modelling for school leaving examination

Hans-Stefan Siller, Regina Bruder, Tina Hascher, Torsten Linnemann, Jan Steinfeld, Eva Sattlberger

Competency level modelling for school leaving examination

Hans-Stefan Siller[1], Regina Bruder[2], Tina Hascher[3], Torsten Linnemann[4], Jan Steinfeld[5] and Eva Sattlberger[5]

1 Universität Koblenz-Landau, Campus Koblenz, Koblenz, Germany, siller@uni-koblenz.de

2 TU Darmstadt, Darmstadt, Germany

3 Universität Bern, Bern, Switzerland

4 Paedagogische Hochschule FHNW, Solothurn, Switzerland

5 BIFIE, Wien, Austria

A project group was commissioned to develop a content- and action-related competency grid in order to enable quality assessment and comparability of mathematics examination questions in the Austrian Matura (final examination at the end of the Secondary School Level II). Based on theoretical grounds, in the competency grid the three dimensions operating, modelling and reasoning are distinguished and described on four levels.

Keywords: Competency level model, school, final exam, operating, modelling, reasoning.

Obtaining information on the development of mathematical competency is a central concern of mathematics education (e.g., Leuders, 2014) and empirical educational research (e.g., Hartig, 2007). In Austria, an approach with the goal of a standardized competency-based written final examination – the so called 'Matura' at the end of Secondary Level II – (cf. AECC Mathematik, 2009; BIFIE, 2013a) in the context of mathematics as a general education subject (cf. Fischer, 2001; Fischer & Malle, 1985; Klafki, 1985; Winter, 1975, 1996) was applied. Examinees are expected to have both mathematical (basic) knowledge and (basic) ability, as well as general mathematical skills such as reasoning skills, problem solving skills, and also the ability to use mathematics in different situations, i.e. modelling skills. However, in PISA 2000, a lack of modelling competency was observed, when students failed to solve (real-life) problems with the help of models in a satisfying way (cf. Klieme et al., 2001). Based on this result, modelling competence was crucial for competency orientation in the curriculum enhancement of mathematics education in the German-speaking region (thus also for the *Matura* in Austria). With reference to Weinert's definition of competencies (2001, p. 27) as

> the cognitive skills and abilities which the individual possesses, or which can be learned, to solve certain problems, as well as the associated motivational, volitional and social readiness and skills in order to successfully and responsibly use problem solutions in a range of situations.

In an iterative process we developed a competency level model for the written final exam in mathematics at the end of Secondary School Level II. The process consisted of four elements: the discussion of competency specifications and developments, the discussion of mathematical tasks, task rating in due consideration of the competency model and the discussion of these ratings. Against the background of theoretical and also experience-based ideas about the current development of mathematical skills in school learning, we described the following three domains of mathematical competencies: operating, modelling, and reasoning[1] (O-M-A) on four levels.

In close cooperation with the Federal Institute of Educational Research, Innovation and Development of the Austrian School System ('BIFIE'), we developed a competency level model facilitating the description and comparison of the exam requirements, especially with regard to examination questions in the final examination in mathematics (Siller et al., 2013).

COMPETENCY LEVEL MODEL

In the competency models of the German-speaking countries Austria, Switzerland and Germany (AECC Mathematik, 2008; HarmoS, 2011; KMK, 2012), content areas (such as geometry or arithmetic), general mathematical competencies (such as reasoning) and skill levels (usually three-stage) are considered. The elements of the model in each country are therefore different when compared to one another. The competency levels are somewhat vague. Therefore, they can only be described on the basis of empirical task difficulty. To put our competency level model in a wider scientific context, we follow Leuders (2014, p. 10): "A model is discussed which (i) a priori postulates levels in acquiring a certain competence, is describing (ii) through stepped task situations and (iii) hierarchically ordered categorical latent ability variable. This allows (iv) determination about which competency pupils possess at each level."

In comparison to earlier statements, the development that has taken place in this area is evident. For example, Helmke and Hosenfeld stated in 2004 (p. 57):

> Neither are the currently available versions of the educational standards derived at the time from comprehensive and didactic accepted competency models (...) nor is there already in all relevant areas of content expertise models which meet the abovementioned requirements, particularly theoretically coherent developmental and learning psychology based levels concepts.

Thinking in (competency) levels is common in schools since curricula and teaching materials are based on this view (cf., e.g., Kiper, Meyer, Mischke, & Wester, 2004). Competency level models contribute to the diagnosis of the learners' levels of competency by the assessment of their achievements. Moreover, the models aim at describing the development of competencies. Their weaknesses, however, are embodied in the fact that it usually remains undetermined how a change to the next level can be accomplished and what conditions are necessary for this. Furthermore, a fixed sequence is assumed, which implies that neither can any steps be skipped nor regressions occur, but which assumes steady, cumulative learning.

For the present competency level model we have agreed on four stages, which can be identified in a manner analogous to Meyer (2007), who described the following four levels (Meyer, 2007, p. 5):

1) Execution of an action, largely without reflective understanding (level 1)

2) Execution of an action by default (level 2)

3) Execution of an action after insight (level 3)

4) Independent process control (level 4)

The activity theory forms the background for the didactic interpretation of such initially pragmatic levels (cf., e.g., Lompscher, 1985) with the corresponding concept of different cognitive actions and their specific dimensional structure. Nitsch and colleagues (2014) developed and empirically verified a competency structure model that describes relevant student actions when translating between different forms of representations in the field of functional relationships. For example, they could show that the two basic actions of acquirement *Identification* and *Implementation* (Construction) and the basic cognitive actions *Description* and *Explanation* differ in their cognitive demands, i.e. they are based on different facets of competency. Therefore, we used the theoretical model of hierarchical structure of cognitive actions (Bruder & Brueckner, 1989) for the description of competency levels.

DEVELOPMENT OF A COMPETENCY LEVEL MODEL

Currently existing competency models are primarily based on empirical analyses: Based on the solution probabilities of tasks (items), competency levels are modelled in the context of large-scale studies. An alternative approach is to primarily derive a model from theoretical concepts. This also requires the recognition of central instructional goals such as a sustainable understanding of mathematical relationships, which in turn presupposes a high level of cognitive activation in the teaching processes (cf. Klieme et al., 2006). This can, for example, be achieved by the following measures:

— the preparation of relationships for basic knowledge and skills learned;

— the challenge to describe mathematical relationships or solutions in application contexts;

— the creation of occasions for reasons or reflections.

Such criteria of demanding instruction should also be appropriate to form a competency level model.

THE COMPETENCY LEVEL MODEL O-M-A

Competency level models that are empirically based indicate to what extent tasks differ in their level of difficulty in terms of processing. Evidence of existing difficulties can be obtained by carefully analysing potential and actual solutions. Normative stipulations of difficulty levels imply that it is not possible to successfully process the task on a lower level. The levels of the competency model postulate what skills are needed to solve them. This does not exclude that there are multiple solution strategies, particularly for complex task definitions.

For designing the domains of mathematical competencies, we follow an orientation to Winter's basic experiences (cf. Winter, 1996, p. 37):

1) To perceive and understand phenomena of the world around us that concern or should concern all of us, from nature, society and culture in a specific way,

2) to learn and comprehend mathematical objects and facts represented in language, symbols, images and formulas as intellectual creations as a deductive-ordered world of its own kind,

3) to acquire task problem-solving skills that go beyond mathematics (heuristic skills).

While the first basic experience corresponds to mathematical modelling as a fundamental action area in learning mathematics, there are the other two basic experiences "operating" and "reasoning", which serve the second fundamental experience as well as "problem solving" for the third basic experience. In various competency models „communicating" is included to emphasize the linguistic aspects, as well as other domains of mathematical competencies.

"Problem solving" is not separated as an independent domain in the Austrian requirements for the final examination (BIFIE, 2013). "Problem solving" is defined as a more complex aspect of action and therefore includes the domains of the mathematical competen-

	Domains of mathematical competency		
Level	Operating	Modelling	Reasoning
1	Identify the applicability of a given or familiar method; Implementing/executing a given or familiar rule	Implementation of a representation change between context and mathematical representation Using familiar and directly recognizable standard models for describing a given situation with appropriate decision	Perform basic technical language reasoning Examine the application of a relationship or method and the fit of a term for a given (intra-mathematical) situation
2	Implementing/executing multi-step methods/rules, possibly with the use of computers and use of control options	Description of the given situation by mathematical standard models and mathematical relationships Recognizing and setting general conditions for the use of mathematical standard models	Understand, comprehend, explain mathematical concepts, principles, methods, representations, reasoning chains and contexts
3	Determine whether a particular method/specific rule is appropriate for a given situation, make and perform the appropriate method/rule	Apply standard models to novel situations, find a suitable fit between suitable mathematical model and real situation	Examine and complete mathematical reasoning, perform and describe multi-step mathematical standard reasoning
4	Develop/form macros[1] and join together macros already available	Complex modelling of a given situation; reflection of the solution variants or model choice and assessment of the accuracy or adequacy of underlying solution methods	Form independent chains of reasoning, technically correct explanation of mathematical facts, results and decisions

[1] aggregated mathematical rules

Figure 1: O-M-A Grid

cies Operating, Modelling and Reasoning, especially in higher levels of performance. “Communicating” is seen as an important domain of mathematical competencies for teaching mathematics, but cannot be specifically differentiated from Operating, Modelling and Reasoning and is therefore included in the other aspects.

The domain “Reasoning” is related to the suggestions of Bruder and Pinkernell (2011), who also pick up on considerations of Walsch (1972). “Modelling” served as the basis of the fundamental work of Niss (2003) and other ideas, e.g. of Boehm (2013) or Goetz and Siller (2012). There are relatively few preparations for a levelled conception of competencies in the mathematical domain “Operating”.

For example, Drueke-Noe (2012) shows that complex algorithms are required already in early grades. But for a high level of expertise, it is not only necessary to use complex algorithms, but also to find the right algorithm to apply in a given situation and to combine different algorithms where appropriate.

The result of our considerations as part of this project is a model with three domains of mathematical competencies (cf. Figure 1) that substantially captures the key aspects of mathematical work at school. The competency level model is aimed at fulfilling all essential requirements with regard to the conception of mathematical learning outcomes in Austrian mathematics education of the Secondary School Level II (cf. BIFIE, 2013a). Complex problem solving situations can be described by the interaction of the three domains of mathematical competencies.

EMPIRICAL EVIDENCE FOR THE COMPETENCY LEVEL MODEL

The question about an empirical verification of the theoretical competency level model with respect to the separation of domains of mathematical competencies and the gradation can be answered only in the context of a sufficient number of processed tasks for each area of expertise.

Data were taken from the so-called “school experiment” in 2014. Before the central final examination throughout Austria will be implemented in the school year 2014/2015, secondary academic schools and mathematics teachers were invited to voluntarily take part in a pilot study on graduating students’ math competences. In this study, the math tasks were processed under the same conditions as they would be processed at the mandatory central final examination. It is important to note that the performance in the tasks contributes to students’ final grade. For the school experiment whose data are being reported here, there were 803 students (m = 345, f = 458) from 42 classes from 9 districts in Austria. The examination consisted of two separated parts with so-called type 1 and type 2 tasks (cf. BIFIE 2013b).

Type 1 tasks “focus on the basic competencies listed in the concept for written final examination. In these tasks, competence-oriented (basic) knowledge and (basic) skills without going beyond independence are to be demonstrated.” (cf. BIFIE, 2013a, p. 23). They are coded as solved against non-solved. The various bound response formats such as multiple-choice format and a special gab-fill format enable accurate scoring. For the award of points in tasks with open and semi-open response format, solution expectations and clearly formulated solution keys are specified.

The characterization of type 2 tasks presents serious challenges to the basic principles of modern test theory. The tasks are considered “for the application and integration of the basic competencies in the defined contexts and application areas. This is concerned with extensive contextual or intra-mathematical task assignments, as part of which different questions need to be processed and operative skills are, where appropriate, accorded greater importance in their solution. An independent application of knowledge and skills is necessary” (cf. BIFIE, 2013a, p. 23). These tasks are also consistently structured in design and presentation, as well as in terms of scoring (cf. BIFIE, 2013).

A total of 16 (type 1) tasks in the competency domain of operating, 2 tasks (type 1) in the competency area of modelling, and 4 tasks (type 1) in the competency area of Reasoning were tested in the 2014 school experiment. Thus, no level analyses could be conducted.

There is a relatively high variation of the solution frequency within competency domain Operating (cf. Figure 2), which can be explained by the heterogeneity of tasks presented, especially with regard to high profile / over-training. Variation of solution frequency was also observed for the competence do-

Task	Competence domain	level of difficulty (% solved)
3	Operating	0.81
4	Operating	0.84
7	Operating	0.98
8	Operating	0.94
9	Operating	0.84
10	Operating	0.88
12	Operating	0.87
13	Operating	0.59
14	Operating	0.88
15	Operating	0.56
17	Operating	0.84
18	Operating	0.57
21	Operating	0.80
22	Operating	0.49
23	Operating	0.81
24	Operating	0.58

Figure 2: Difficulty level of the tasks for the competence domain Operating (n = 803)

Task	Competence domain	level of difficulty (% solved)
19	Modelling	0.51
20	Modelling	0.91

Figure 3: Difficulty level of the tasks for the domain Modelling (n = 803)

Task	Competence domain	level of difficulty (% solved)
1	Reasoning	0.80
5	Reasoning	0.73
6	Reasoning	0.94
11	Reasoning	0.64

Figure 4: Difficulty level of the tasks for the domain Reasoning (n = 803)

main Modelling (cf. Figure 3) as well as Reasoning (cf. Figure 4). The parameter "difficulty" was not measured, only the percentage of solution as an indicator for the level of difficulty of a task.

Two of the type 1 tasks are positioned on competency level 2 and could be analysed. A heterogeneous picture emerged for these two tasks: While task 2 could rarely be solved, task 16 was easily mastered by the students.

Can the pre-defined four levels be confirmed empirically in all the three areas of competency? This question can be answered in a first approximation only on the basis of type 2 tasks for levels 1 and 2 due to the fact that higher graduations did not appear in these exam booklets.

As can be seen in Figure 5 (in general) and Figure 6 (separated among O-M-A), the level 2 tasks seem to be more difficult in general. Thus the competency level of the task gives us a good statement about the level of difficulty.

SUMMARY AND OUTLOOK

The provided model with the three domains of mathematical competencies Operating, Modelling and Reasoning (O-M-A) distinguishes three basic mathematical operations on four levels. It is based on considerations from educational sciences and

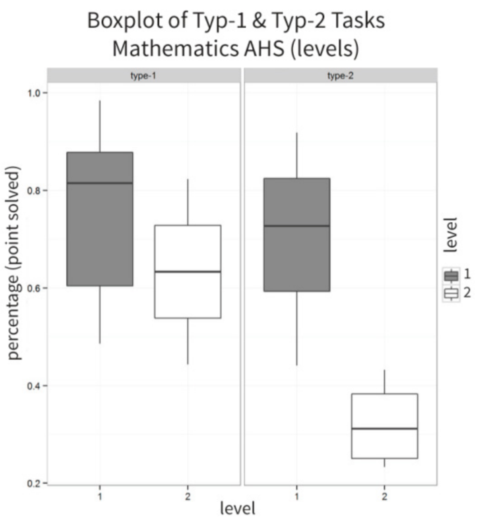

Figure 5: Empirical difficulties of type 2 tasks separated by type and level (n = 803)

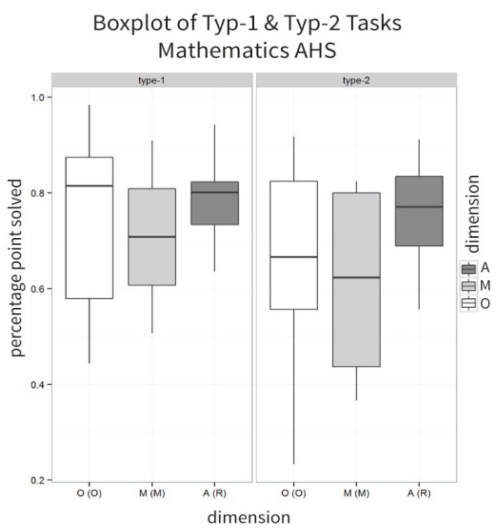

Figure 6: Empirical difficulties of the type 1 and type 2 tasks among O-M-A (n = 803)

learning theories as well as insights and experiences with regard to relevant factors for learning mathematics in school. It is part of a complex effort to gain a sound basis for competency diagnostics and performance assessment in mathematics in the German-speaking countries. It differs from other models by its consistent theoretical foundation and by the focus on potential lines of development for long-term competency building. The model O-M-A provides a normative setting for relevant levels of requirement in the three domains of mathematical competencies. This facilitates a certain comparability of type 1 and type 2 tasks provided for the final examination.

The added value of the developed model lies in several areas:

— It provides guidance both for the assessment of (written) performance and for the learning tasks in the classroom.

— It serves the purpose to reveal potential for development in the classroom.

— It allows for the identification of development potential in the task structure.

Limitations of the competency level model O-M-A lie in the coarseness of the approach. Neither can all the differences between the test tasks relevant to their level of difficulty be considered in detail (such as linguistic complexity), nor can individual developmental trajectories be mapped in learning processes. Further restrictions of the model are also indicated by the fact that of all the mathematical content and activities implied in each task only a basic competency referring to the list of basic skills (cf. BIFIE, 2013a) can be adopted. The specific situation of each school class or priorities of teachers cannot be reflected. Thus, many tasks can prove to be easier, but also more difficult than in the rating.

The competency level model O-M-A aims at describing levels of competencies by identifying the qualitative differences of each competence. The growing body of research on mathematics learning served as the theoretical background. The data and results presented so far are preliminary and did not account for not controllable influence factors such as training effects. However, they can be interpreted as a first clue that the O-M-A can be rudimentarily verified empirically. Therefore, further research is necessary to empirically test the levels of the model and to test the model against level 3 and level 4 tasks.

The model O-M-A is indefinite in explaining the attainment of the next higher level. For this reason we define it as a competency *level* model and not a competency *development* model. To answer the question as to whether this model could map potential lines of students' long-term competency development, more theoretical and empirical work is needed. So far, it cannot be applied to the development of a math learning process.

ACKNOWLEDGEMENT

The project "Competency Level Modelling" was funded with support from the Federal Institute of Educational Research, Innovation and Development of the Austrian School (BIFIE). This publication reflects the views of the authors, and the BIFIE cannot be held responsible for any use that may be made of the information contained therein. We acknowledge the contribution of Martin Schodl, who has been working in the project as a coordinator and who has helped selecting certain data.

REFERENCES

AECC Mathematik (Ed.) (2008). *Standards für die mathematischen Fähigkeiten österreichischer Schülerinnen und Schüler am Ende der 8. Schulstufe*. Version 4/07. Klagenfurt, Austria: Institut für Didaktik der Mathematik. Fakultät für interdisziplinäre Forschung und Fortbildung der Alpen-Adria-Universität.

AECC Mathematik (Ed.) (2009). *Das Projekt „Standardisierte schriftliche Reifeprüfung in Mathematik" – Sicherung mathematischer Grundkompetenzen*. Klagenfurt, Austria: Institut für Didaktik der Mathematik. Fakultät für interdisziplinäre Forschung und Fortbildung der Alpen-Adria-Universität.

BIFIE (Ed.) (2013). *Standardisierte kompetenzorientierte Reifeprüfung Reife- und Diplomprüfung: Grundlagen – Entwicklung – Implementierung*. Retrieved from https://www.bifie.at/system/files/dl/srdp_grundlagen-entwicklung-implementierung_2013-11-06.pdf [21.03.2015].

BIFIE (Ed.) (2013a). *Die standardisierte schriftliche Reifeprüfung in Mathematik. Inhaltliche und organisatorische Grundlagen zur Sicherung mathematischer*

Grundkompetenzen. Retrieved from https://www.bifie.at/node/1442 [21.03.2015].

BIFIE (Ed.) (2013b). *Haupttermin 2013*. Retrieved from https://www.bifie.at/node/2633 [21.03.2015].

Boehm, U. (2013). *Modellierungskompetenzen langfristig und kumulativ fördern – Tätigkeitstheoretische Analyse des mathematischen Modellierens als Lerngegenstand in der Sekundarstufe I*. Wiesbaden, Germany: Vieweg + Teubner.

Bruder, R., & Brückner, A. (1989). Zur Beschreibung von Schülertätigkeiten im Mathematikunterricht – ein allgemeiner Ansatz. *Pädagogische Forschung, 30(6)*, 72–82.

Bruder, R., & Pinkernell, G. (2011). Die richtigen Argumente finden. *Mathematik Lehren*, *168*, 2–7.

Drueke-Noe, C. (2012). Wer Kalküle kann, schafft eine Klassenarbeit. Stimmt das? In *Beiträge zum Mathematikunterricht 2012* (pp. 213–216). Münster, Germany: WTM.

Fischer, R. (2001). Höhere Allgemeinbildung. In A. Fischer, A. Fischer-Buck, K. H. Schäfer, D. Zöllner, R. Aulcke, & F. Fischer (Eds.), *Situation – Ursprung der Bildung. Franz-Fischer-Jahrbuch der Philosophie und Pädagogik 6* (pp. 151–161). Leipzig, Germany: Universitätsverlag.

Fischer, R., & Malle, G. (1985). *Mensch und Mathematik. Eine Einführung in didaktisches Denken und Handeln*. Mannheim, Germany: Bibliographisches Institut.

Goetz, S., & Siller, H.-St. (2012). *Was heißt Kompetenzorientierung nach acht bzw. zwölf Jahren Mathematikunterricht?*. Invited talk/Paper presented for further training of teachers, PH Lower Austria, Amstetten, 03/2012.

HarmoS (2011). *Grundkompetenzen für die Mathematik. Nationale Bildungsstandards*. Freigegeben von der EDK-Plenarversammlung. Retrieved from http://www.edk.ch/dyn/12930.php [21.03.2015].

Hartig, J. (2007). Skalierung und Definition von Kompetenzniveaus. In B. Beck & E. Klieme (Eds.), *Sprachliche Kompetenzen. Konzepte und Messung* (pp. 83–99). Weinheim, Germany: Beltz.

Helmke, A., & Hosenfeld, I. (2004). Vergleichsarbeiten – Standards – Kompetenzstufen: Begriffliche Klärung und Perspektiven. In M. Wosnitza, A. Frey, & R. S. Jäger (Eds.), *Lernprozess, Lernumgebung und Lerndiagnostik* (pp. 56–75). Landau, Germany: Verlag Empirische Pädagogik.

Kiper, H., Meyer, H., Mischke, W., & Wester, F. (2004). *Qualitätsentwicklung in Unterricht und Schule. Das Oldenburger Konzept*. Oldenburg, Germany: diz.

Klafki, W. (1985). *Neue Studien zur Bildungstheorie und Didaktik – Zeitgemäße Allgemeinbildung und kritisch-konstruktive Didaktik*. Weinheim, Germany: Beltz.

Klieme, E., Neubrand, M., & Luedtke, O. (2001). Mathematische Grundbildung: Testkonzeption und Ergebnisse. In Deutsches PISA-Konsortium (Eds.), *PISA 2000. Basiskompetenzen von Schülerinnen und Schülern im internationalen Vergleich* (pp. 139–190). Opladen, Germany: Leske + Budrich.

Klieme, E., Lipowsky, F., Rakoczy, K., & Ratzka, N. (2006). Qualitätsdimensionen und Wirksamkeit von Mathematikunterricht. Theoretische Grundlagen und ausgewählte Ergebnisse des Projekts „Pythagoras". In M. Prenzel & L. Allolio-Näcke (Eds.), *Untersuchungen zur Bildungsqualität von Schule – Abschlussbericht des DFG-Schwerpunktprogramms* (pp. 127–146). Münster, Germany: Waxmann.

KMK (2012). *Bildungsstandards im Fach Mathematik für die Allgemeine Hochschulreife*. Retrieved from http://www.kmk.org/fileadmin/veroeffentlichu ngenbeschlues-se/2012/20121018-Bildungsstandards-Mathe-Abi.pdf [21.03.2015]

Leuders, T. (2014). Modellierungen mathematischer Kompetenzen – Kriterien für eine Validitätsprüfung aus fachdidaktischer Sicht. *Journal für Mathematik-Didaktik, 35(1)*, 7–48. doi: 10.1007/s13138-013-0060-3

Lompscher, J. (1985). *Persönlichkeitsentwicklung in der Lerntätigkeit*. Berlin, Germany: Volk und Wissen.

Meyer, H. (2007). *Leitfaden Unterrichtsvorbereitung*. Berlin, Germany: Cornelsen Scriptor.

Niss, M. A. (2003). Mathematical competencies and the learning of mathematics: the Danish KOM project. In A. Gagatsis & S. Papastavridis (Eds.), *3rd Mediterranean Conference on Mathematical Education – Athens, Hellas 3–4–5 January 2003* (pp. 116–124). Athens, Greece: Hellenic Mathematical Society.

Nitsch, R., Fredebohm, A., Bruder, R., Kelava, T., Naccarella, D., Leuders, T., & Wirtz, M. (2014). Students' competencies in working with functions in secondary mathematics education – Empirical examination of a competence structure model. *International Journal of Science and Mathematics Education,* 1–26. doi: 10.1007/s10763-013-9496-7

Siller, H.-St., Bruder, R., Hascher, T., Linnemann, T., Steinfeld, J., & Schodl, M. (2013). Stufenmodellierung mathematischer Kompetenz am Ende der Sekundarstufe II. In M. Ludwig & M. Kleine (Eds.), *Beiträge zum Mathematikunterricht 2013* (pp. 950–953). Münster, Germany: WTM.

Walsch, W. (1972). *Zum Beweisen im Mathematikunterricht*. Berlin, Germany: Volk und Wissen.

Weinert, F. E. (2001). Vergleichende Leistungsmessungen in Schulen – eine umstrittene Selbständigkeit. In F. E. Weinert (Eds.), *Leistungsmessungen in der Schule* (pp. 17–31). Weinheim, Germany: Beltz.

Winter, H. (1975). Allgemeine Lernziele für den Mathematikunterricht? *Zentralblatt für Didaktik der Mathematik*, *3*, 106–116.

Winter, H. (1996). Mathematikunterricht und Allgemeinbildung. *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, *61*, 37–46.

ENDNOTE

1. The German word 'Argumentieren' is synonym to reasoning.