Mathematics communication within the frame of supplemental instruction – SOLO and ATD analysis

Annalena Holm, Susanne Pelger

Mathematics communication within the frame of supplemental instruction – SOLO and ATD analysis

Annalena Holm and Susanne Pelger

Lund University, Faculty of science, Lund, Sweden, annalena.holm@varberg.se

Teaching at Swedish primary and secondary schools is often combined with collaborative exercises in a variety of subjects. One such method for learning together is Supplemental instruction (SI). Several studies have been made to evaluate SI in universities throughout the world, while at lower levels hardly any study has been made until now. This study aimed at identifying learning conditions in SI-sessions at two Swedish upper secondary schools. Within this study, a combination of ATD (Anthropological theory of the didactic) and the SOLO-taxonomy (Structure of the Observed Learning Outcome) was successfully tried as an analysis strategy.

Keywords: SOLO, ATD, networking, mathematics communication, SI.

INTRODUCTION

The teacher's choice of education methods has a high influence on what students learn (Hattie, 2009), and education research has shown to add to a better understanding of the prospects of successful teaching (Good & Grouws, 1979; Hattie, 2009). In spite of previous educational research, there is no clear answer to the question whether one method has advantages over the other, or if whole-class teaching is more successful than "dialogue-teaching".

To strengthen the findings, researchers have argued that there is a need for more sophisticated research methods (Jakobsson, Mäkitalo, & Säljö, 2009). There is also a need for more systematic connection between various education research theories – so-called networking (Prediger, Bikner-Ahsbahs, & Arzarello, 2008). According to (Prediger et al., 2008), the reasons that theories in mathematics education research have evolved differently are (1) mathematics education is a complex research environment, and (2) various research cultures prioritise different components of this complex field. Different theories and methods have different perspectives and can provide different kinds of knowledge. Thus, different theories and perspectives can connect in different ways.

An educational concept that needs to be explored, and systematically connected with various theories, is the so-called Supplemental instruction (SI). SI is a method where groups of students are provided peer collaborative learning exercises at meetings led by SI-leaders (Hurley, Jacobs, & Gilbert, 2006). The method is used worldwide both at the university level and lower levels. To strengthen students' knowledge in mathematics, a number of upper secondary schools in Sweden have introduced SI as a complement to regular teaching.

AIM

Within the present study, SI-sessions were analysed in upper secondary school. The purpose was gaining more insight into the conditions that may facilitate mathematics learning. For the analyses two frameworks were chosen and tested: (1) the Anthropological theory of the didactic (ATD) with focus on the development of mathematical activities defined in terms of praxeologies (Chevallard, 2015; Winsløw, 2010), and (2) Structure of the Observed Learning Outcome (SOLO), which instead focuses on students' learning outcome quality (Biggs & Collis, 1982). An aim of the study was to explore whether a combination of these two frameworks could contribute to deepen the analysis of the students' discussions. The research questions of this paper are, hence: *To what extent is a combination of SOLO and ATD a suitable strategy for analysing SI-sessions? Are these two frameworks compatible and complementary?*

THEORY AND CONNECTING FRAMEWORKS

Research needs theoretical frameworks. This was stated by (Lester, 2005), who argued that a theoretical framework provides a structure when designing research studies, and that a framework helps us to transcend common sense when analysing data. Below, the two frameworks that have been important for the study are discussed. First, the concept Supplemental instruction is presented. Then follows a section where SOLO-taxonomy and the ATD-praxeology are presented. Finally, possibilities and challenges with combining frameworks are discussed.

Supplemental instruction, or SI, is an educational method used at universities in many countries. Groups of students discuss and solve problems together, and SI is a complement to regular teaching. No teacher is present at the meetings (Malm, Bryngfors, & Mörner, 2012). The groups are instead guided by an older student, who is supposed to provide peer collaborative learning exercises (Hurley et al., 2006). SI has lately been introduced in some upper secondary schools in Sweden. First year students form the groups, while second and third year students serve as SI-leaders (Malm, Mörner, Bryngfors, Edman, & Gustafsson, 2012).

Biggs and Collis (1982) developed the *SOLO-taxonomy* for evaluating learning outcomes for students at tertiary level. SOLO names and distinguishes five levels according to the cognitive processes required to obtain them. The authors argued that SOLO is useful when categorising test results in closed situations with formulated expectations. They used five levels, SOLO-1 to SOLO-5, when categorising student responses (Biggs & Collis, 1982). Later Brabrand and Dahl (2009) used the SOLO-taxonomy for analysing (1) what curricula focus on and (2) what students actually learn. By using so-called active verbs (as shown hereafter), the authors state that it is possible to understand on which level of knowledge the learning outcome is:

> SOLO 1 *(pre-structural):* student misses the point
>
> SOLO 2 (*uni-structural*): define, count, name, recite, follow instructions, calculate
>
> SOLO 3 (*multi-structural*): classify,describe, enumerate, list, do algorithm, apply method
>
> SOLO 4 (*relational*): analyse, compare, explain causes, apply theory (to its domain)
>
> SOLO 5 (*extended abstract*): theorize, generalize, hypothesize, predict, judge, reflect, transfer theory (to new domain)

Brabrand and Dahl (2009) conclude that SOLO can be used when analysing science curricula, but they question whether SOLO is a relevant tool when analysing mathematics curricula. They write:

> For mathematics it is usually not until the Ph.D. level that the students reach SOLO 5 and to some extent also SOLO 4. The main reason is that to be able to give a qualified critique of mathematics requires a counter proof or counter example as well as a large overview over mathematics which the students usually do not have before Ph.D. level. (Brabrand & Dahl, 2009, p. 543)

Other researchers, however, claim that SOLO is useful in various contexts. Pegg (2010) has described three studies where SOLO has been used to analyse primary and secondary students' learning mathematics. In addition, Pegg (2010) states that SOLO helps to describe observations of students' mathematics performance. Hattie and Brown (2004) also describe SOLO as a useful tool in mathematics education. They use a strategy where mathematics exercises are formulated by using SOLO, and they claim it is possible to use SOLO when analysing children's mathematics knowledge and when describing the processes involved in asking and answering a question on a scale of increasing difficulty or complexity.

The Anthropological theory of the didactic (ATD) is a research program for analysing and developing mathematics education, which offers a handful of tools (Chevallard, 2006; Winsløw, 2010). One of these is *the notion of praxeology,* and one of the overarching perspectives is *the paradigm of questioning the world*.

While *the paradigm of questioning the world* defines the perspective of the curriculum, the ATD-praxeology makes a helpful tool for analysing the content that is taught. A praxeology can be described as a four-tuple explaining the *components* of activities or knowledge that are taught. This four-tuple consists of: a *type of tasks* (T), a *technique* (τ), a *technology* (θ) and a *theory* (Θ) (Winsløw, 2010). These four constituents, if fully understood and used, can help to analyse what is done at school. The *type of tasks* and the *technique* form the *practice block* or the *know-how*. The *technology* and the *theory* constitute the *theory block* or the *know-why*. Hence, a technique is used to solve a task of a given type, while the technology justifies the technique, and the theory gives a broader understanding of the field. When used to describe bodies of knowledge, praxeologies can refer to "small" as well as "big" fields. Hence, a *point praxeology* is a single type of tasks that is solved by a technique; several point praxeologies can be combined into a local praxeology when they share the same technology and several local praxeologies sharing the same theory can be combined to form a regional praxeology (Winsløw, 2010).

The ATD-praxeology can be applied at various levels of education. Winsløw (2006), for example, discusses how to use the praxeology when studying advanced mathematics, while Barbé, Bosch, Espinoza, and Gascón (2005) suggest how to use ATD when studying classroom activities at upper secondary school. All together ATD is described as a theory which analyses what is taught and thus *showing the shortcomings* or even paradoxes of didactic practices. Winsløw (2010) also states that ATD is useful when proposing ambitious ways to *transform* education.

Different theories have different perspectives and can provide different kinds of knowledge. Looking at the same data from different perspectives can give deeper insights (Prediger et al., 2008). In this study, the ATD and SOLO frameworks were combined in order to study the conditions and outcomes of students' learning through SI. The purpose of combining two frameworks was to catch the advantages of each of them, and hence, to contribute to mathematics education research and networking.

METHOD

This study bases its statements on classroom observations. The phenomenon being studied was students' discussions of mathematics. The context was small groups in upper secondary school. We used a qualitative case study approach (Cohen et al., 2007) to provide an analysis of how the students in the groups dealt with the mathematical problems. (Cohen, Manion, & Morrison, 2007) describe the purpose of a case study to portray, analyse, and interpret situations through accessible accounts. As such, the case study method provided a systematic way of looking in depth, analysing and reporting how students discuss mathematical problems and how the discussions might facilitate learning.

Meetings at two upper secondary schools in south-western Sweden were observed with groups from the humanist, technology and natural science programs. The main criterion for choosing schools was their different experiences of support from the university. Another difference between the two schools was the implementation of SI. The criterion for choosing SI-groups to observe was availability. Not all groups wanted to be observed. Meetings were *videotaped* and the tapes were transcribed. The documents were coded by closed coding, i.e. a deductive *analysis* with codes from theoretical frameworks. During the whole study, the analysis strategy was developed and revised. Due to limited space not all observations can be presented here. For more comprehensive insight in the study see Holm (2014).

The first students to be observed were one group from the technology program and one group from the humanistic program. Both groups discussed the same exercise (see Table 1). The exercise was part of a former national test from the Swedish national agency for education, which in 2010 had been intended for all students in the first grade of Swedish upper secondary school. At these two particular group-sessions no SI-leaders were present as this was a first test of the frameworks. The observed sessions lasted 40 minutes at one school and 60 minutes at the other. The students were not told anything about the SOLO- and ATD-classification of the exercise.

The exercise was *pre-classified* by SOLO and the ATD-praxeology. The intention was (1) to test if it was possible to do this classification in advance before giving the exercise to the students, (2) to decide whether the two frameworks were a suitable choice when analysing student learning outcome, and finally, (3) if it was possible to correlate every SOLO-level to a specific dimension of the ATD-praxeology.

Three different ways of using the SOLO-taxonomy were found in the literature, and initially all three of them were used when classifying the exercise. One of the three was part of the original method defined by Biggs and Collis (1982), with instructions for how to analyse student achievements in elementary mathematics. The authors recommended that the children's solutions were to be analysed by deciding inter alia whether the child can handle several data at the same time and whether the child shows the ability to "hold off actual closures while decisions are made".

A second method was described by Hattie and Brown (2004). They grouped the exercises in advance, so that if a student answered a certain question the student was considered to reach a certain SOLO-level. Finally, Brabrand and Dahl (2009) used the SOLO-taxonomy by the active verbs once formulated by Biggs (2003) and compared university curricula with the table of verbs. Certain verbs were considered to point at certain "intended learning outcomes" in the curricula. Notice that the verb "calculate" and "do simple procedure" are added to SOLO 2. These verbs are mentioned in (Brabrand & Dahl, 2009) and in (Biggs & Tang, 2011). In the results section we explain why not all the three ways of using the SOLO-taxonomy were suitable for the present study.

Although the SOLO-taxonomy is widely used, in different ways, the work done by Biggs and Collis (1982) was based on closed situations, and not open situations, which are one of the main ideas of SI. Thus, it was decided that a complementary framework was needed for this study, specifically designed for mathematics education and also for open situations. Here, the ATD was found a suitable complement to SOLO.

The ATD is widely used, especially within the French, Spanish and Latin-American mathematics education research traditions (Bosch & Gascón, 2006; Chevallard, 2015). It is developed to fit education research in mathematics and other disciplines, and calls for more open situations and open questions at school in general and in school mathematics in particular (Chevallard, 2015). In this study, the analysis and development of open mathematics learning situations was, thus, done by using the ATD-praxeology, while the SOLO-taxonomy was used for the analysis of student learning outcomes.

RESULTS

The initial exercise about the volume of a cylinder was coded before it was given to the students (see Table 1). The SOLO-coding was based on the three methods described above. First, the "Hattie-Brown-method" was used, as it appeared to be near to practice.

Exercise: A roll of paper (statement)	**SOLO**	**ATD praxeology**
A rectangular sheet of paper can be rolled to make a tube (cylinder) as shown in the figure.		
Such a tube is made by rolling a square piece of paper with side length 10 cm. **The diameter of the tube will be about 3.2 cm. Find the volume of this tube (cylinder).*	2 /later chang-ed to 3	Technique $\tau 1$ (calculate the volume of a cylinder given its diameter and height)
*Show that the diameter of the tube will be about 3.2 cm if the side length of the sheet of paper used is 10 cm	2/3 /later 3	Technique $\tau 2$ (calculate the diameter of a circle given its perimeter)
If the length and width of the paper are different, you can make two different tubes (cylinders) depending on how you roll the paper. **Starting with rectangular sheets of paper with dimensions 10 cm × 20 cm, two different tubes are made. Find the volumes of the two tubes (cylinders).*	3	Combination of techniques $\tau 1$ & $\tau 2$ (first calculate diameter, then the volume)
*Compare these two volumes and calculate the ratio between them. **Investigate the ratio between the cylinder volumes using sheets of paper with other dimensions. What affects the volume ratio between the tall and the short cylinder?*	4 4	Technique $\tau 3$ (calculate the ratio of the volumes found) Technology (general statement about the ratio)
**Show that your conclusion is true for all rectangular papers.*	5	Technology (variation of $\tau 3$ using parameters, proof the general case)

Table 1: An exercise was pre-classified SOLO and the ATD-praxeology

It seemed to be easy to decide whether one or two aspects were involved in the question. However, when it came to higher SOLO-levels, it was more difficult to judge whether the aspects were "integrated". Here, the "Biggs-Brabrand-Dahl-method" was helpful as it offered additional verbs, alternative to "integrate", e.g., "compare" and "analyse", which could be used for the coding.

An example of the use of active verbs in the coding is the sub-task where students should first calculate two volumes and then compare these two volumes (Table 1):

> "Starting with rectangular sheets of paper with dimensions 10 cm × 20 cm, two different tubes are made. Find the volumes of the two tubes (cylinders)."

> "Compare these two volumes and calculate the ratio between them."

In both sub-exercises several aspects are involved. A volume is calculated by multiple parameters. But the active verbs separate the two sub-tasks, as the first requires only an algorithm: "find" (the volume), while the second requires that the student goes one step further and makes a comparison: "compare" (these two volumes). Finally, it was important to compare the coding with the "Biggs-Collis-method", as Biggs and Collis (1982) had formulated the original recommendations for how to use SOLO. In their book, however, the mathematics examples were fetched from elementary mathematics, and it was not obvious how to apply the method in the present study.

To conclude, the active verbs were found to be the most appropriate method when dealing with mathematics exercises. By using SOLO, a clear borderline could be drawn between the active verbs "do algorithm" (SOLO 3) and "explain causes" (SOLO 4), and the active verbs made it possible to identify these structural differences between exercises. The initial exercise about the volume of a cylinder was also coded using the ATD-praxeology (Table 1). This coding was based on the work done by Mortensen (2011), who has coded museum exhibition exercises – the so-called "intended praxeology". In the exercise about the cylinder, each sentence was coded. It was for example decided whether the students were supposed to deal with available "know-how" to solve a problem (the dimensions type of task & technique) or if they were supposed to deal with "know-why", i.e., a special way to justify the technique (the dimensions technology and theory).

At first, in the analysis of the described exercise (Table 1), SOLO and the ATD-praxeology were laid side by side. The exercise was coded both by SOLO and ATD. The strategy to try to correlate every SOLO-level to a specific dimension of ATD- praxeology caused problems. ATD and SOLO evaluate different dimensions. Thus, the strategy was abandoned at this early stage in the study. From now on, the two frameworks were used for different purposes: SOLO to analyse the quality of student learning outcomes, and the ATD-praxeology to analyse the didactic situations. In other words, they were considered answering different questions: *what qualities does the student outcome show?* and *which dimensions does the learning situation contain?* During the rest of the study it was discovered that the two frameworks often did not correlate.

The next step of the study was to code the group discussions about the cylinder. The sentences of the discussions were coded by the active verbs, and by the praxeological analysis. There were occasions when SOLO and ATD did correlate and there were other occasions when they did not. Table 2 shows part of one discussion and how the discussion can be analysed by SOLO and ATD. The students discussed the volume of the cylinder. They did not remember the formula and therefore they tried different strategies. Finally one student managed to solve the first exercise.

According to the analysis of the discussions of this first exercise, the SOLO-active verbs clarified the *learning outcome*. SOLO 4 for example told that students may have "explained" and/or "analysed". If an element of the situation was classified by ATD as "technology", it means that the student *dealt with a discussion concerning* "knowing why" a technique was being used. *Hence, it was possible to use the two frameworks within one study (compatible).* However, when entering into detail, the two approaches *lead to different characterisations* of students' mathematical activities (*complementary*).

— Students follow instructions "how": SOLO 2 & Technique

- Students use an algorithm: SOLO 3 & Technique
- A discussion about single tasks (point praxeologies) develops into a situation about knowing why (regional praxeologies), students may then explain why a method works: SOLO 4 & Technology
- A problem can develop into a situation that deals with knowing why, but students use the algorithm without discussing why: SOLO 3 & Technology
- The situation deals with knowing how to solve a problem by using an algorithm and students compare different solutions: SOLO 4 & Technique

Finally, it was concluded that the pre-classification did not hold. When the students did not remember the formula, they had to discuss the problem more thoroughly and thus reach other SOLO-levels and ATD-dimensions (Table 2).

DISCUSSION AND CONCLUSIONS

ATD and SOLO were combined to deepen the analysis of students' mathematics discussions. Such networking of frameworks is supported by Lester (2005) and Prediger and colleagues (2008), who argue that networking does not have to imply a total integration or unifying between frameworks. Lester (2005, p. 466) even advocates the adaptation of ideas from a range of theoretical sources to suit goals both for research and for developing practice in the classroom in a way that "practitioners care about".

The initial intention was to correlate specific SOLO-levels to specific ATD-praxeology dimensions. If this had been possible the conclusion would have been that one of the frameworks had been eliminated from this study. However, it was found that the two frameworks were both compatible and complementary. The present study thus succeeded in adapting theoretical models for analysing empirical material and in contributing to the development of strategies for analysing students' learning.

REFERENCES

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: The case of limits of functions in Spanish high schools. *Educational Studies in Mathematics, 59*(1–3), 235–268.

Biggs, J. B. (2003). *Teaching for quality learning at university: what the student does.* London, UK: Society for Research into Higher Education: Open University Press, 2. ed.

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning : the SOLO taxonomy (Structure of the observed learning outcome).* New York, NY: Academic Press.

Biggs, J. B., & Tang, C. S.-K. (2011). *Teaching for quality learning at university: what the student does*. Maidenhead, UK: Open University Press, 4. [rev.] ed.

Bosch, M., & Gascón, J. (2006). Twenty-five years of the didactic transposition. *ICMI Bulletin, 58*, 51–65.

Quotes	SOLO	ATD	Comments
e) It is the diameter times the length or height ... a) Is that so? (e) I think so. a) But no. It does not become square ... a) It is supposed to be CM3. It just gets CM2. It does not work.	1 3/4	Technique/ Technology	Student (a) and (e) try to find a relevant technique to calculate the volume of the cylinder. However, the technique is erroneous. Student (a) notices that their technique does not work. (a) tries to discuss "knowing why". They try to question the technique.
d) How do you count ... We were supposed to have the area of the circle. b) Wait what are we supposed to figure out? (reading task) d) Volume ... then we need the area of the base b) What?	3	Technique	A parallel discussion goes on between student (d) and student (b). Student (d) comments what (a) just said.
a) Yes exactly b) The area of the base? ... d) Is not the radius times the radius times pi?	3	Technique	The two groups start to discuss with one another. Student (d) takes the command and finds the technique – the "knowing how".

Table 2: Quotes from group discussion analysed by SOLO and by ATD-praxeology. Quotes are translated from Swedish and commented by the observer

Brabrand, C., & Dahl, B. (2009). Using the SOLO taxonomy to analyze competence progression of university science curricula. *Higher Education, 58*(4), 531–549.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In M. Bosch (Ed.), *Proceedings of CERME4* (pp. 21–30). Barcelona, Spain: FUNDEMI-IQS.

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counterparadigm. In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges* (pp. 173–188). Cham, Switzerland: Springer.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research Methods in Education*. London, UK: Routledge Falmer.

Good, T. L., & Grouws, D. A. (1979). The Missouri Mathematics Effectiveness Project: An experimental study in fourth-grade classrooms. *Journal of Educational Psychology, 71*(3), 355–362.

Hattie, J. A. C. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement.* London, UK: Routledge.

Hattie, J. A. C., & Brown, G. T. L. (2004). *Cognitive processes in asTTle: The SOLO taxonomy Technical report* (Vol. 43). Auckland, New Zealand: University of Auckland/Ministry of Education.

Holm, A. (2014). *Mathematics education within the frame of supplemental instruction, identifying learning conditions.* Licentiate dissertation, Lund University, Sweden.

Hurley, M., Jacobs, G., & Gilbert, M. (2006). The Basic SI Model. *New Directions for Teaching and Learning, 106*, 11–22.

Jakobsson, A., Mäkitalo, Å., & Säljö, R. (2009). Conceptions of knowledge in research on students' understanding of the greenhouse effect: Methodological positions and their consequences for representations of knowing. *Science Education, 93*(6), 978–995.

Lester, F. K., Jr. (2005). On the theoretical, conceptual, and philosophical foundat ions for research in mathematics education. *ZDM, 37(6)*, 457–467.

Malm, J., Bryngfors, L., & Mörner, L.-L. (2012). Supplemental instruction for improving first year results in engineering studies. *Studies in Higher Education, 37*(6), 655–666.

Malm, J., Mörner, L.-L., Bryngfors, L., Edman, G., & Gustafsson, L. (2012). Using Supplemental Instruction to Bridge the Transition from Secondary to Tertiary Education. *International Journal of Education, 4*(3), 31–48.

Mortensen, M. F. (2011). Analysis of the Educational Potential of a Science Museum Learning Environment: Visitors' Experience with and Understanding of an Immersion Exhibit. *International Journal of Science Education, 33*(4), 517–545.

Pegg, J. (2010). *Promoting acquisition of higher-order skills and understandings in primary and secondary mathematics.* Paper presented at the Research conference: Teaching mathematics? Make it count: What research tells us about effective teaching and learning of mathematics, Melbourne, VIC.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework. *ZDM, 40*(2), 165–178.

Winsløw, C. (2006). Transformer la théorie en tâches : la transition du concret à l'abstrait en analyse réelle. In R. Rouchier (Ed.), *Actes de la XIIIème Ecole d'Eté de didactique des mathématiques* (pp. 1–12). Grenoble, France: La Pensée Sauvage.

Winsløw, C. (2010). Anthropological theory of didactic phenomena: Some examples and principles of its use in the study of mathematics education. In M. Bosch, J. Gascón, A. Ruiz-Olarría, M. Artaud, A. Bronner, Y. Chevallard, et al. (Eds.), *Un panorama de la TAD* (pp. 117–138). CRM Documents, vol. 10. Bellaterra, Spain: Centre de Recerca Matemàtica.