Linking inquiry and transmission in teaching and learning mathematics

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Different theories assume that learning mathematics should be based on constructivist methods where students inquire problem-situations and assign a facilitator role to the teacher. In a contrasting view other theories advocate for a more central role to the teacher, involving explicit transmission of knowledge and students' active reception. In this paper, we reason that mathematics learning optimization requires adopting an intermediate position between these two extremes models, in recognizing the complex dialectic between students' inquiry and teacher's transmission of mathematical knowledge. We base our position on a model with anthropological and semiotic assumptions about the nature of mathematical objects, as well as the structure of human cognition.

Keywords: Mathematical instruction, inquiry learning, knowledge transmission, onto-semiotic approach, mathematical knowledge.

INTRODUCTION

The debate between the models of a school that "conveys knowledge" and others in which "knowledge is constructed" currently seems to tend towards the latter. This preference can be seen in the curricular guidelines from different countries, which are based on constructivist and socio-constructive theoretical frameworks (NCTM, 2000):

> Students learn more and learn better when they can take control of their learning by defining their goals and monitoring their progress. When challenged with appropriately chosen tasks, students become confident in their ability to tackle difficult problems, eager to figure things out on their own, flexible in exploring mathematical ideas and trying alternative solution paths, and willing to persevere (p. 20).

In the case of mathematics education, problem solving and "mathematical investigations" are considered essential for both students' mathematical learning and teachers' professional development. Constructivist viewpoints of learning shift the focus towards the *processes* of the discipline, practical work, project implementation and problem solving, rather than prioritizing the study of facts, laws, principles and theories that constitute the body of disciplinary *knowledge*.

Nevertheless, this debate is hiding the fact that students differ in skills and knowledge, and most of them need a strong guidance to learn; even when some students with high skills and knowledge can learn advanced ideas with little or no help. The issue of the type of aid needed, depending on the nature of what is to be built or transmitted is also missed in this debate. Consequently of this situation, the question of the kind of help that a teacher should give to a usually heterogeneous class, when we want students acquire mathematical knowledge, understandings and skills, also arises.

The family of "Inquiry-Based Education" (IBE), "Inquiry-Based Learning" (IBL), and "Problem-Based Learning" (PBL) instructional theories, which postulate the inquiry-based learning with little guidance by the teacher, seem not to take into account the described reality, namely the students' heterogeneity and the variety of knowledge to be studied. These models may be suitable for gifted students, but possibly not for the majority, because the type of help that the teacher can provides could significantly influence the learning, even in talented students.

In this paper, we analyse the need to implement instructional models that articulate a mixture of construction/inquiry and transmission of knowledge to achieve a mathematical instruction that locally optimize learning. The basic assumption is that the moments in which transmission and construction of knowledge can take place are *everywhere dense* in the instructional process. Optimization of learning involves a complex dialectic between the roles of teacher as instructor (transmitter) and facilitator (manager), and student's roles as active constructor of knowledge and receivers of meaningful information. Hiebert and Grouws (2007) state that "because a range of goals might be included in a single lesson, and almost certainly in a multi-lesson unit, the best or most effective teaching method might be a mix of methods, with timely and nimble sifting among them" (p. 374).

We support this mixed model of mathematical instruction in cognitive (architecture of human cognition) and onto-semiotic (regulative nature of mathematical objects) reasons.

Below we first summarize the main features of instructional models based on inquiry and problem solving and secondly of models that attribute a key role to transmission of knowledge. We then present the case for a mixed model that combines dialectically inquiry and transmission, basing on the epistemological and didactical assumptions of the onto-semiotic approach to mathematical knowledge and instruction (Godino, Batanero, & Font, 2007). Finally we include some additional reflections and implications.

INQUIRY AND PROBLEM BASED LEARNING IN MATHEMATICS EDUCATION

As indicated above, the acronyms IBE, IBL, PBL designate instructional theoretical models developed from several disciplines, which have parallel versions for the teaching of experimental sciences (IBSE) and mathematics (IBME). They attributed a key role to solve "real" problems, under a constructivist approach. In some applications to mathematics education it is proposed that students construct knowledge following the lines of work of professional mathematicians themselves. The mathematician faces non-routine problems, explore, search for information, make conjectures, justify and communicate the results to the scientific community; mathematics learning should follows a similar pattern.

Using problem-situations (mathematics applications to everyday life or other fields of knowledge, or problems within the discipline itself) to enable students making sense of the mathematical conceptual structures is considered essential. These problems are the starting point of mathematical practice, so that problem solving activity, including formulation, communication and justification of solutions are keys to developing mathematical competence, i.e. the ability to cope with not routine problems. This is the main objective of the "problem solving" research tradition (Schoenfeld, 1992), whose focus is on the identification of heuristics and metacognitive strategies. It is also essential to other theoretical models such as the Theory of Didactical Situations (TDS) (Brousseau, 1997), and Realistic Mathematics Education (RME) (Freudenthal, 1973; 1991), whose main features are described below.

Theory of Didactical Situation (TDS)

In TDS, problem-situations should be selected in order to optimize the adaptive dimension of learning and students' autonomy. The intended mathematical knowledge should appear as the optimal solution to the problems; it is expected that, by interacting with an appropriate *milieu*, students progressively and collectively build knowledge rejecting or adapting their initial strategies if necessary. According to Brousseau (2002),

> The intellectual work of the student must at times be similar to this scientific activity. Knowing mathematics is not simply learning definitions and theorems in order to recognize when to use and apply them. We know very well that doing mathematics properly implies that one is dealing with problems. We do mathematics only when we are dealing with problems—but we forget at times that solving a problem is only a part of the work; finding good questions is just as important as finding their solutions. A faithful reproduction of a scientific activity by the student would require that she produce, formulate, prove, and construct models, languages, concepts and theories; that she exchange them with other people; that she recognize those which conform to the culture; that she borrow those which are useful to her; and so on. (p. 22).

To allow such activity, the teacher should conceive problem-situations in which they might be interested and ask the students to solve them. The notion of *devolution* is also related to the need for students to consider the problems as if they were their own and take responsibility for solving them. The TDS assumes a strong commitment with mathematical epistemology, as reflected in the meaning attributed to the notion of fundamental situation, which Artigue and Blomhøj (2013, p. 803) describe as "a situation which makes clear the raison d'être of the mathematical knowledge aimed at".

Another important feature of the TDS is the distinction made between different dialectics: action, formulation and validation, which reflect important specificities of mathematical knowledge.

Realistic Mathematics Education (RME)

In RME, principles that clearly correspond to IBME assumptions are assumed. Thus, according to the "activity principle", instead of being receivers of ready-made mathematics, the students, are treated as active participants in the educational process, in which they develop themselves all kinds of mathematical tools and insights. According to Freudenthal (1973), using scientifically structured curricula, in which students are confronted with ready-made mathematics, is an 'anti-didactic inversion.' It is based on the false assumption that the results of mathematical thinking, placed on a subject-matter framework, can be transferred directly to the students. (Van den Heuvel-Panhuizen, 2000).

The principle of reality is oriented in the same direction. As in most approaches to mathematics education, RME aims at enabling students to apply mathematics. The overall goal of mathematics education is making students able to use their mathematical understanding and tools to solve problems. Rather than beginning with specific abstractions or definitions to be applied later, one must start with rich contexts demanding mathematical organization or, in other words, contexts that can be mathematized. Thus, while working on context problems, the students can develop mathematical tools and understanding. The guidance principle stresses also the same ideas. One of Freudenthal's (1991) key principles for mathematics education is that it should give students a "guided" opportunity to "re-invent" mathematics. This implies that, in RME, both the teachers and the educational programs have a crucial role in how students acquire knowledge. According to Artigue and Blomhøj (2013, p. 804), "RME is thus a problem-solving approach to teaching and learning which offers important constructs and experience for conceptualizing IBME".

TRANSMISSION BASED LEARNING IN EDUCATION

We consider as models based on knowledge transmission various forms of educational intervention in which the direct and explicit instruction is highlighted. A characteristic feature of strongly guided instruction is the use of worked examples, while the discovery of the solution to a problem in an information-rich environment is similarly a compendium of discovery learning minimally guided.

For several decades these models were considered as inferior and undesirable regarding to different combinations of constructivist learning (learning with varying degrees of guidance, support or scaffolding), as shown in the initiatives taken in different international projects to promote the various IBSE and IBME modalities (Dorier & Garcia, 2013). Transmission of knowledge by presenting examples of solved problems and the conceptual structures of the discipline is ruled by didactical theories in mathematics education with strong predicament, as mentioned in the previous section.

The uncritical adoption of constructivist pedagogical models can be motivated by the observation of the large amount of knowledge and skills, in particular everyday life concepts, that individuals learn by discovery or immersion in a context However, Sweller, Kirschner and Clark (2007) state that

> There is no theoretical reason to suppose or empirical evidence to support the notion that constructivist teaching procedures based on the manner in which humans acquire biologically primary information will be effective in acquiring the biologically secondary information required by the citizens of an intellectually advanced society. That information requires direct, explicit instruction. (p. 121)

This position is consistent with the argument put forward by Vygotsky; scientific concepts do not develop in the same way that everyday concepts (Vygotsky, 1934). These authors believe that the design of appropriate learning tasks should include providing students an example of a completely solved problem or task, and information on the process used to reach the solution. As Sweller, Kirschner, and Clark (2007) observe, "we must learn domain-specific solutions to specific problems and the best way to acquire domain-specific problem-solving strategies is to be given the problem with its solution, leaving no role for IL [inquiry learning]" (p. 118). According to Sweller et al., empirical research of the last half century on this issue provides clear and overwhelming evidence that minimal guidance during instruction is significantly less effective and efficient than a guide specifically designed to support the cognitive process necessary for learning. According to (Kirschner, Sweller, & Clark, 2006):

> We are skilful in an area because our long-term memory contains huge amounts of information concerning the area. That information permits us to quickly recognize the characteristics of a situation and indicates to us, often unconsciously, what to do and when to do it. (p. 76).

STUDYING MATHEMATICS THROUGH AN INQUIRY AND TRANSMISSION BASED DIDACTICAL MODEL

In the two previous sections we described some basic features of two extreme models for organizing mathematics instruction: discovery learning versus learning based on the reception of knowledge (usually regarded as traditional whole-class expository instruction). In this section, we describe the characteristics of an instructional model in which these two models are combined: the students' investigation of problem-situations with explicit transmission of knowledge by the "teacher system" [1] at critical moments in the mathematical instruction process. We consider that it is necessary to recognize and address the complex dialectic between inquiry and knowledge transmission in learning mathematics. In this dialectic, *dialogue* and *cooperation* between the teacher and the students (and among the students themselves), regarding the situation-problem to solve and the mathematical content involved, can play a key role. In these phases of dialogue and cooperation, moments of transmitting knowledge necessarily happen.

The onto-semiotic complexity of mathematical knowledge and instruction

The semiotic, epistemological and cognitive assumptions of the Onto-semiotic approach to mathematical knowledge and instruction (OSA) (Godino, Batanero, & Font, 2007) are the basis for our instructional proposal, which recognizes a key role to both the inquiry and the transmission of knowledge in the teaching and learning of mathematics (and possibly other disciplines). This model takes into account the nature of mathematical objects involved in mathematical practices whose students' competent performance is intended.

The way a person learns something depends on what has to be learned. According to the OSA, students should appropriate (learn) the onto-semiotic institutional configurations involved in solving the proposed problem-situations. The paradigm of "questioning the world" proposed by the Anthropological Theory of Didactics (TAD) (Chevallard, 2015), and, in general, by IBE models is assumed, so that the starting point should be the selection and inquiry of "good problem-situations."

The key notion of the OSA for modelling knowledge is the *onto-semiotic configuration* (of mathematical practices, objects and processes) in its double version, institutional (epistemic) and (cognitive). In a training process, the student's performance of mathematical practices related to solving certain problems, brings into play a conglomerate of objects and processes whose nature, from the institutional point of view is essentially normative (regulative) (Font, Godino, & Gallardo, 2013) [2]. When the student makes no relevant practices, the teacher should guide him/her to those expected from the institutional point of view. Thus each object type (concepts, languages, propositions, procedures, argumentations) or process (definition, expression, generalization) requires a focus, a moment, in the study process. In particular regulative moments (institutionalization) are *everywhere dense* in the mathematical activity and in the process of study, as well as in the moments of formulation / communication and justification.

Performing mathematical practices involves the intervention of previously known objects to understand the demands of the problem-situation and implementing an initial strategy. Such objects, its rules and conditions of application, must be available in the subject's working memory. Although it is possible that the student him/herself could find such knowledge in the "workspace", there is not always enough time or the student could not succeed; so the teacher and peers can provide invaluable support to avoid frustration and abandonment. These are the moments of remembering and activation of prior knowledge, which are generally required throughout the study process. Remembering moments can be needed not only in the exploratory-investigative phase, but also in the formulation, communication, processing or calculation, and justification of results phases. These moments correspond to acts of knowledge transmission and may be crucial for optimizing learning.

Results of mathematical practices are new emerging objects whose definitions or statements have to be shared and approved within the community at the relevant time of institutionalization carried out by the teacher, which are also acts of knowledge transmission.

Inquiry and transmission didactical moments

Under the OSA framework other theoretical tools to describe and understand the dynamics of mathematics instruction processes have been developed. In particular, the notions of *didactical configuration* and *didactical suitability* (Godino, Contreras, & Font, 2006; Godino, 2011). A didactical configuration is any segment of didactical activity (teaching and learning) between the beginning and the end of solving a task or problem-situation. Figure 1 summarizes the components and the internal dynamics of a didactical configuration, including the students' and the teacher's actions, and the resources to face the joint study of the task.

The problem-situation that delimits a didactical configuration can be made of various subtasks, each of which can be considered as a sub-configuration. In every didactical configuration there is an epistemic configuration (system of institutional mathematical practices, objects and processes), an instructional configuration (system of teacher and learners roles and instructional media), and a cognitive configuration (system of personal mathematical practices, objects and processes) which describe learning. Figure 1 shows the relationships between teaching and learning, as well as with the key processes linked to the onto-semiotic modelling of mathematical knowledge (Font, Godino & Gallardo, 2013; Godino, Font, Wilhelmi, & Lurduy, 2011). Such modelling, together with the teachers and learners roles, and their interaction with technological tools, suggest the complexity of the relationships established within any didactical configuration, which cannot not be reduced to merely inquiry and transmission moments.

SYNTHESIS AND IMPLICATIONS

In this paper, we argued that instructional models based only on inquiry, or only on transmission are simplifications of an extraordinarily complex reality: the teaching and learning processes. As Hiebert and Grouws (2007) write, "classrooms are filled with complex dynamics, and many factors could be responsible

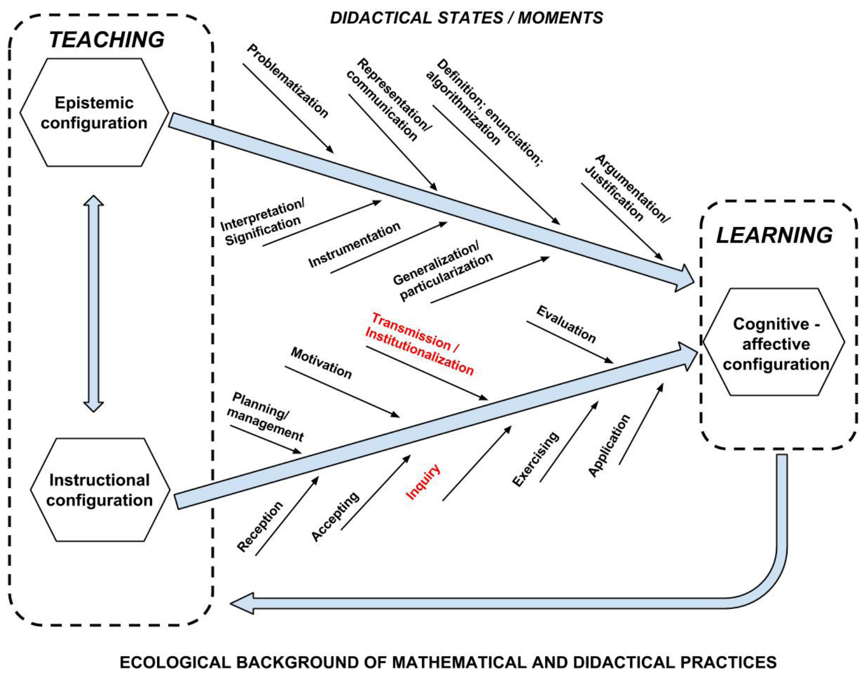

Figure 1: Components and internal dynamic of a didactical configuration

for increased student learning.... This is a very central and difficult question to answer" (p. 371).

Although we need to establish instructional designs based on the use of rich problem-situations, which guide the learning and decision-making at the global and intermediate level, local implementation of didactical systems also requires special attention to managing the students' background needed for solving the problems, and to the systematization of emerging knowledge. Decisions about the type of help needed essentially have a local component, and are mainly teacher's responsibility; he/she needs some guide in making these decisions to optimize the didactical suitability of the study process.

We also have supplemented the cognitive arguments of Kirschner, Sweller, and Clark (2006) in favour of models based on the transmission of knowledge in the case of mathematical learning, with reasons of onto-semiotic nature: What students need to learn are in a great deal, *mathematical rules*, the circumstances of its application and the required conditions for a proper application. The learners start from known rules (concepts, propositions, and procedures) and produce others rules that should be shared and compatible with those already established in the mathematical culture. Such rules (knowledge) must be stored in subject's long term memory and put to work at the right time in the short-term memory.

The scarce dissemination of IBE models in actual classrooms and the persistence of models based on the transmission and reception of knowledge can be explained not only by the teachers' inertia and lack of preparation, but by their perception or experience that the transmission models may be more appropriate to the specific circumstances of their classes. Faced with the dilemma that a majority of students learn nothing, get frustrated and disturb the classroom, it may be reasonable to diminish the learning expectations and prefer that most students learn something, even only routines and algorithms, and some examples to imitate. This may be a reason to support a mixed instructional model that articulates coherently, locally and dialectically inquiry and transmission [3].

REFERENCES

Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM, 45*(6), 797–810.

Brousseau, G. (2002). *Theory of didactical situations in mathematics*. Dordrecht, The Netherlands: Kluwer.

Chevallard, Y. (2015). Teaching Mathematics in Tomorrow's Society: A Case for an Oncoming Counterparadigm. In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges* (pp. 173–188). Cham, Switzerland: Springer.

Dorier, J. L., & García, F. J. (2013). Challenges and opportunities for the implementation of inquiry-based learning in day-to-day teaching. *ZDM, 45*(6), 837–849.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht, The Netherlands: Reidel.

Freudenthal, H. (1991). *Revisiting mathematics education.* Dordrecht, The Netherlands: Kluwer.

Font, V., Godino, J. D., & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educational Studies in Mathematics*, *82*(1), 97–124.

Godino, J. D. (2011). *Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas*. Paper presented at the XIII Conferência Interamericana de Educação Matemática, Recife, Brazil.

Godino, J. D., Contreras, A., & Font, V. (2006). Análisis de procesos de instrucción basado en el enfoque ontológico semiótico de la cognición matemática. *Recherches en didactique des mathématiques*, *26*(1), 39–88.

Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM, 39*(1–2), 127–135.

Godino, J. D., Font, V., Wilhelmi, M. R., & Lurduy, O. (2011). Why is the learning of elementary arithmetic concepts difficult? Semiotic tools for understanding the nature of mathematical objects. *Educational Studies in Mathematics*, *77*(2), 247–265.

Hiebert, J. S., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester, Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 371–404). Reston, VA: National Council of Teacher of Mathematics.

Kirschner, P. A., Sweller, J., & Clark, R. E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), 75–86.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: Author.

Radford, L. (2013). Three key concepts of the theory of objectification: knowledge, knowing, and learning. *Journal of Research in Mathematics Education*, *2*(1), 7–44.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, meta-cognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York, NY: MacMillan.

Sweller, J., Kirschner, P. A., & Clark, R. E. (2007). Why minimally guided teaching techniques do not work: A reply to commentaries. *Educational Psychologist*, *42*(2), 115–121.

Van den Heuvel-Panhuizen, M. (2000). *Mathematics education in the Netherlands: A guided tour.* Freudenthal Institute. CR-ROM for ICME9. Utrecht, The Netherlands: Utrecht University.

Vygotsky, L. S. (1934/1993). Pensamiento y lenguaje. In *Obras escogidas* (vol. II, pp. 9–287). Madrid, Spain: Visor.

ENDNOTES

1. This system can be an individual teacher, a virtual expert system, or the intervention of a "leader" student in a team working on a collaborative learning format.

2. This view of mathematical knowledge is consistent with that taken by Radford's objectification theory. Radford (2013) writes: "Knowledge, I just argued, is crystallized labor – culturally codified forms of doing, thinking and reflecting. Knowing is, I would like to suggest, the instantiation or actualization of knowledge" (p.16). He adds: "Objectification is the process of recognition of that which objects us – systems of ideas, cultural meanings, forms of thinking, etc." (p. 23). In our case, such crystallized forms of work are conceived as cultural "rules" fixing ways of doing, thinking and saying faced to problem-situations that demand an adaptive response.

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