Discriminatory networks in mathematics education research

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This paper is written in an organisational language developed in the context of mathematics education by Dowling (2009, 2013) – social activity method (SAM) – as a commentary on Radford's (2008, 2014) discussion of theoretical networking. An exemplar is given of SAM's approach of recontextualising, and thus learning from, what it finds of interest elsewhere – here, Chevallard's Anthropological Theory of the Didactic (ATD). The approach puts emphasis on the autonomy and emergent quality of well-formed research activity. SAM is not, however, solipsistic: it is designed to recursively self-organise in relation to what it encounters elsewhere but on the explicit basis of its own principles. By biasing a reading of ATD, SAM's organisational language develops in the form of a discriminatory research network.

Keywords: Anthropological Theory of the Didactic, deformance, discriminatory research networks, recontextualisation, Social Activity Method.

INTRODUCTION

Writing about theoretical networking presents a formidable challenge. This paper looks at the relation between two research programmes in the domain of mathematics education research, Social Activity Method (Dowling, 1998, 2009, 2013 – hereafter SAM) and the Anthropological Theory of the Didactic (hereafter ATD; Bosch & Gascón, 2014) together with one meta-theory of theoretical networking (Radford, 2008, 2014). This already involves three specialised assemblages of principles and tacit knowledges: to introduce all three would exceed the space available. This limitation is addressed by considering the other approaches as an illustration of how, *from SAM's point of view,* theoretical dialogue might be achieved. For this reason, it is the principles of SAM that are given most emphasis: these are then used to *select* principles from the other approaches. This means that the principles of ATD and Radford's meta-theory must, fundamentally, be misread – what I shall refer to as a (I hope, productive) *deformance* (Dowling, 2009) of them.

SAM has in common with some other research in mathematics education an interest in the specificity of social activity in the context in which it is produced and reproduced (see especially Dreyfus & Kidron, 2014, p. 87). Its focus is on the strategies that lead to emergent alliance; ordered relations of action in which people may come to recognise themselves as working together e.g. as *in SAM* or *ATD.* I first introduce the central *Domains of Action Schema* of SAM. This provides principles for further application of the method in forming a regard on both ATD and Radford's work. One part of this schema – the esoteric domain – is then considered in greater detail to allow a discussion of the continuities and discontinuities between SAM and ATD. A new schema is then generated to bias a reading of ATD from the regard of SAM.

The question I address is: what can a strongly institutionalised research programme in mathematics education, SAM, make of another such strongly institutionalised approach, ATD? How does this allow *SAM* to learn and thus deform itself? It needs the greatest emphasis that SAM *makes no assumptions at all about what ATD might or might not learn* because SAM assembles only its own principles. From the point of view elaborated here there can be no literal connection of similars: any metonymic chain between signifiers of two research programmes involves *recontextualising work.* A secondary question is: what light does this shed on the need for meta-theories to conceptualise theoretical networking such as the one proposed by Radford?

For the purpose of clarity and to summarise the position and rationale of the paper: well-formed research activities are incommensurable – they are emergent and not graspable as such, even by themselves. The

	Content (signifieds)	
Expression (signifiers)	I+	I-
I+	*esoteric*	*descriptive*
I-	*expressive*	*public*

Figure 1: Domains of Action (from Dowling 2009, p. 206)

term "continuity" between theories can refer only to those metonymic chains of signifiers that are of interest *to the recontextualising regard of the theory in question* – hence also the possibility of discontinuity. To claim otherwise, I argue, is counter to a fundamental socio-semantic principle: that sense is made locally in the context of an assembled practice not outside of it. There is, therefore, no possibility of "connection" in terms of similar "component parts". Such a claim would also involve an infinite regress: the notion of similarities or points of contact between theories begs the question of what is the theory that allows such similarity to be discerned. I formalise this as a *general argument* later in this paper.

INTRODUCING THE DOMAIN OF ACTION SCHEMA

The Public Domain

Radford's (2008, 2014) discussion of "*networking theories*" in mathematics education research recontextualises some aspects of Lotman's (2001) *semiotics* to introduce "the *semiosphere* as a theory networking space". Of particular interest is the resulting delimitation of theoretical work as "*bounded*" by the principles that grant its "*autonomy*". Radford (2008, p. 319) produces a description of the mathematics research semiosphere that is in "constant motion"; accelerating as information is transmitted and received with new technologies. Autonomy of a theory within the semiosphere is given by a *hierarchical* order of *principles, methodology* and *research questions* in which the *system* (Radford, 2008, p. 320) of principles is in regulative control. The potential for networking theories is then a question of their *closeness of principle*. Some theories are too far apart to work well together, others may have surprising affinities yet to be articulated. Generally, we may be experiencing a drifting apart: networking might stabilise this, at least for a time.

This paper is written in SAM: the selection of, and extracts from, Radford's paper are motivated by its common interest in the terms given emphasis in the paragraph above such as boundedness and system. But these are expressions not specialised in SAM; and neither is their content – see the axes of Figure 1 below. This schema relates expression to content but moves beyond semiotics by further taking into account whether this is specialised or unspecialised in the *institutionalisation* of the activity. My summary of Radford's position is in the *public domain* of SAM – involving unspecialised, weakly institutionalised (I-) expression and content (Dowling, 2009, p. 206) *from the regard of SAM*. Radford's language is a highly specialised one in its own terms; but these specialised terms – and the way in which they interlink – are not recruited in the institutionalisation (denoted I+) of SAM. Figure 1 expresses SAM's *self-reference*: as a research activity it articulates specialised expression and content in its *esoteric* domain, for example "domain of action".

The Esoteric Domain

A central concept in Radford's work is that of autonomy. I will retain this word but *recontextualise it* from SAM's regard in the *esoteric domain* of SAM. Figure 1 schematises this as a socio-semantics rather than a semiosphere – *institutionalisation* (recognisable regularity of practice) occurring as research *activity* where flows of strategic semiosis (gestures, images, words) are *assembled* in more or less stabilised emergent *alliances*. The principles of action in the esoteric domain regulate what can be recognised/realised in the public domain. Weakly institutionalised terms such as autonomy and semiosphere are alienated in favour of I+ terms such as those given emphasis in this section. This is a *deformance*: the "encounter" (Radford, 2008, p. 317) read through the principles of SAM. Yet the *expressive domain* ensures that self-reference need not become solipsism: the "identity" (Radford, 2008, p. 319) of the self-reference changes in its engagement with the other.

The Expressive Domain

The deformance involved in expressive domain action can be illustrated with respect to the expression "networking theories". *(a) Network*. Eco (1984, p. 81)

	Semiotic Mode	
Mode of Action	Discursive	Non-Discursive
Interpretative	*theorem/enunciation*	*template/graph*
Procedural	*procedure/protocol*	*operational matrix/operation*

Figure 2: Modalities of the Esoteric Domain Apparatus (Dowling, 2013, p. 333)

characterises the semiosphere (in his terms the *global semantic universe*) as a *labyrinthine rhizomatic net.*

> The main feature of a net is that every point can be connected with every other point, and where the connections are not yet designed, they are, however, conceivable and designable. A net is an unlimited territory [...] the abstract model of a net has neither a center nor an outside. (Eco, 1984, p. 81).

A network is not a net (fishing, internet, tuber or any other). The metaphoric expression nonetheless points to potentially productive specialised content. Perhaps its most significant aspect is that a network cannot be described as a whole or from a global point of view; because any attempt at such a description is immediately re-inscribed as new connectivity. The concept of connectivity here is semiotic: the deferred and anticipatory action of one signifier on another. This occurs *even if they are the same.* For example, the signifier <institutionalisation> in SAM points to the schematised content of Figure 1. The same signifier is part of ATD; but its sense there – forged in dialogue with Mary Douglas' *How Institutions Think* – is not organised as a relational space. No *literal* connection of this similar is then possible, only a transformative one. (*b*) *Theory.* At the nodes of the network, Radford has "theories". Radford is careful not to impart undue reliance on the discursive by putting emphasis on the composition of the "triplet" to include principles, methodology and the "template" of research questions. Yet from SAM's regard there is still some danger of the term being read as implying potential representational adequacy (the global all-seeing net). For this reason the phrase *research activity* or *approach* has been preferred to allow full room for the "practice turn" in theory.

ASSEMBLAGES OF MATHEMATICAL MODES

In its most recent development SAM has considered the esoteric domain of school mathematics to be constituted as an *assemblage* of strategies, a term recontextualised from Deleuze (Deleuze & Parnet, 2007 [1997], p. 69; Turnbull, 2000, p. 44). As a sociology, SAM is concerned with the distributional consequences of the ways alliances emerge through strategic action in the social: these indicate (never quite fix) the norms of who can say, think, or do what; here in school mathematics.

An assemblage is specified by SAM as a relational schema – Figure 2 – that can be contingently recruited in the (re)production of school mathematics. The dimension *semiotic mode* distinguishes discursive (explicitly articulated principles, methods and symbols, for example formulae) from non-discursive modes of mathematical engagement (diagrams, or equipment such as a pair of compasses). The dimension *mode of action* opposes interpretative and procedural activity: in the former case where there is work to be done in making sense of the semiotic mode (formulae, diagrams), in the latter case where there are rules or sequences to be followed (discursively ordered algorithms, non-discursive techniques for manipulating the compasses or computer software appropriately). This establishes four *general* strategies: *template, operational matrix, procedure and theorem.* Further, the second term of each strategy in the table denotes *local* rather than generalising action.

The schema suggests competence in that discipline (or anything else) is not *acquired as such* but is constituted by the development of a pragmatic ability to *contingently* deploy an effectively inter-linked mixture of strategies in local context – upon which action the assemblage and those whose alliances will be distributed by it will develop or change. SAM therefore has no "epistemological" concerns in contrast, for example, to ATD. Figure 2 is an introduction to the technology for generating empirical description in SAM – see the many further schemas in (Dowling, 2009). These pin down *modes of action.* This is not a speculative space: it arose from an empirical engagement with a number of mathematical settings (Dowling, 2013). For recent further work in SAM see (Burke, Jablonka, & Olley, 2014; Dowling, 2014; Dudley-Smith, 2015; Burke, 2015).

	Discursive Saturation	
Mode of Action	DS-	DS+
Operationalising	*technique* (τ)	*technology*(θ)
Orientation	*type of tasks* (*T*)	*theory* (Θ)
	skill	***discourse***

Figure 3: Praxeological Modes

A RECONTEXTUALISATION OF ATD

Dowling (2014, p. 528) has noted that Chevallard's work also makes use of a "complementary" concept of recontextualisation – didactic transposition – although with a primary focus on the contextualisation of *cultural* sense-making in pedagogic settings. The schema of the assemblage is potentially in dialogue with ATD's vision of schools as providers of *discoveries* along the way of *research and study paths* (Chevallard, 2015) contingent to the opening up of a body of questions found to be of interest as the research unfolds. In what follows the "amalgam" of *praxeologies* (Artigue et al., 2011, p. 2) is recontextualised by SAM in a *deformative re-ordering*.

Consider the praxeological components [T/τ/θ/Θ] of type of tasks, technique, technology and theory (Artigue, Bosch, & Gascon, 2011; Chevallard & Bosch, 2014). ATD notices a key relation between *praxis* and *logos*: one of both imbrication and tension. Yet empirical studies have shown that this is sometimes denied: thus, for example, in the university some action (Bosch, 2014) is seen to hive off [θ/Θ] from [T/τ]. This has proved a highly fruitful distinction: thus, for example, Job and Schneider (2014) use this framework to make a productive separation of the pragmatic praxeology of the development of calculus and the rather monumentalising deductive praxeology of analysis imposed on mathematics undergraduates – with school mathematics very much a hotchpotch of both. However, in ATD the amalgam [T/τ/θ/Θ] is conceived as containing "ingredients" (Artique et al., 2011, p. 3) – suggesting to a casual reader elements to be enumerated. From SAM's regard a prophylactic against such a misreading is to suggest that the idea of a praxeology can be schematised.

First, it is possible to distinguish what I will call *operationalising* and *orientation*. Orientation concerns what one is about in a specific context: practically as embodied in *type of tasks*, logo-centrically as informed by a *theory*. The former involves low discursive saturation (DS-) as it is embedded in the situated interests or (Maussian) habitus of context. The latter is discursively saturated (DS+), i.e., context free. Operationalising involves techniques – in SAM's terminology "DS- skills" or ways of doing – as well as DS+ "technological discourse" (Bosch & Gascón, 2014, p. 69).

In Figure 3 this produces four *strategies* rather than components. In SAM's research activity the development of schemas such as Figure 3 allows a *particular kind* of regulated engagement with the empirical (without exclusion of others such as ATD). One orienting strategic mode of this is given discursively by the *theory*-logos Θ; self-referentially in SAM's case, particularly the semiotics imbricated in the *raison d'être* of the operationalising *technology*-logos θ of its schemas. Yet much is tacitly acquired: the DS- orientation of SAM's emergent *type of tasks T* – a concern with emergent alliance, the strategies achieved that enable a stabilised commonality of action – is difficult to explain to novitiates outside a context of apprenticeship. Operationalising is also composed of strategies of practical *technique* τ. Certainly these can be aggregated in homology with ATD: the DS- modes identified by ATD as [T/τ] can be identified as *skill*, the DS+ strategies of [θ/Θ] as *discourse* (Dowling, 2009, p. 95); but the recontextualisation now sees each as a strategic mode rather than an element of an amalgam.

Once relationised in this way, SAM and ATD (from the deforming regard of SAM) have the same objective: to describe the empirical deployment of strategies in the assemblage of Figure 2 *and* in the praxeological modes of Figure 3. These common objectives are not *translatable* but they are *transformable*. Specifying the schema of Figure 3 allows a development *of SAM*. The insistence that this is a recontextualisation preserves the autonomy of ATD. Both may then point – in a dialogue of potential complementarity – to the principles for a resistance to the closed and syncretic esoteric domains typical of school disciplinary subjects precisely of the kind Job and Schneider (2014) identify. In learning it is then both *operationalising* and the *orientation* of the student to the regularities of practice *in both the DS- and DS+* that would establish apprenticeship

In ATD the theory of didactic transposition acknowledges that school is a specific context of pedagogic relations. In SAM this is expressed as a matter of recontextualising action conceived as a general socio-semantic process of structuration, i.e. in constituting the esoteric domain of a specialised social activity such as school mathematics as a cultural arbitrary. Both approaches put great emphasis on an interrogation of how mathematics is institutionalised differently according to circumstance. Thus in recent programmes for ATD (Chevallard & Bosch, 2014) the T of *the current milieu of the student* (in reference to its sociality outside the school) is given appropriate emphasis – this is so often tragically downplayed by policy makers. As Radford (2008, p. 322) observes, research questions derive from the principles that allow their articulation. The focus in ATD is on the provision of appropriate activity (and the elimination of the inappropriate) to open to the student the possibilities of what mathematics *might become* for the student in their specific context. To ATD the school may (and often does) block this possibility but this is incidental to the possibility. SAM also sees content as constituted through institutionalisation; within the research programme identified by Jablonka, Wagner and Walshaw (2013), the content of school mathematics is itself always-already recruited in processes of social reproduction – the *particular alliances* (and, of course, oppositions) formed in the schoolroom always *different* to those formed in research (for example, mathematics research).

GENERAL ARGUMENT

This paper has considered the way in which SAM might stand in productive relation to other theoretical frameworks and to itself. From the autonomous and self-referential regard of SAM this must be a matter of the principle of recontextualisation, as that is what organises its regard. The self-reference is fundamental; but it is not a solipsism unless foolishly demanding that its categories replace all others to totalise the net. Both development and renewal are possible via an openness to the empirical and to theoretical antecedents. The following *general argument* rejects the idea that there is a "landscape of strategies for connecting theoretical approaches" (Prediger, Bikner-Ahsbahs, & Arzarello, 2008, p. 170) in favour of the deformative determination of autonomous self-reference. To formalise the situation, let the operator → refer to the recontextualising regard of an approach, ABC, to mathematics education research. Further, let Δ refer to its development, ⇒ denote consequence and let ES_i be a particular empirical setting:

> If one has SAM → ES_i and, elsewhere, ABC → ES_i then recognition of commonality would require a *general unifying framework*, GUF, such that GUF → (SAM → ES_i, ABC → ES_i) to integrate an answer to "perspectives *of what*?". This would deny that ES_i is constituted as an artefact of SAM or of ABC (a refutation of this denial is the many (justified) observations in *Networking of Theories as a Research Practice in Mathematics Education* (Prediger & Bikner-Ahsbahs, 2014) that the data was not collected appropriately for the theoretical framework concerned). Rather networking occurs as SAM → (ABC → ES_i) ⇒ ΔSAM with possible *answerability* of the form ABC → (SAM → (ABC → ES_i)) ⇒ ΔABC *& etc*. In each case the recontextualisation is either misrecognised through literalised equivalence (including elements of "similarity") or constituted as a deformative chiasmus (Merleau-Ponty, 1968), that is, realised as (re)new(ed) embodied practice in response to the objectifying regard of the other. For obvious reasons SAM cannot totally catch its own tail: SAM → (SAM → ES_i) also ⇒ ΔSAM; hence the importance of the dialogic (even if with yourself), a potentially unlimited recursion (or freedom).

In terms of their key diagram (Prediger & Bikner-Ahsbahs, 2014, p. 119), there is no role here for understanding, comparison, synthesis or integration, no "relationships between parts of theoretical approaches" (ibid, 118). It is not a question of attempting to find "similarities and differences" (ibid, 119) but to be open to deformative encounters – allowing these to prompt further self-organisation. It is the possibility of *complementarity*, not commonalities, that defeats "isolation", and the principle of *recontextualisation* that annihilates "global unifiers" who put forward GUFs. In Lotman's (2001, p. 143) semiotics, as in SAM's social-semantics, the principle of *asymmetry* is paramount – information-enriching activity *deforms*.

CONCLUSION – SOME GENERAL THEMES

The main theme of the paper has been to provide a framework which allows a discussion of the continuities and discontinuities between SAM and ATD. It sets out an agenda for a commentary SAM → (ATD → ESi) in invitation of a counter-commentary from ATD. The framework claims *autonomy* in the regards of ATD and SAM, but also the possibility of dialogue as described in the *general argument*.

I have to this point left one consequence of this implicit. There is a need to *take mathematics out of the theoretical framework of mathematics education research*. From SAM's regard, mathematics (however institutionalised) is the *empirical setting* of research. Yet, contrary to both SAM and ATD, many research programmes seem to wish to make it part of their *theoretical framework* by including considerations from a (notional) *mathematics-itself*. For SAM, the separation is required because the truth claims of a particular practice (for example, the often rather strange modalities of school mathematics) have their *own specificity*. A further consequence arises from the relationality of SAM's approach. The coherence of a theoretical framework is not a matter of the signification of individual theoretical terms; as if these can be translated by single substitutes to stand on their own account – and thus be 'connected' *as such*, or be absorbed into another theory. A theory's coherence rests on the relationality of its content, not on a collection of atomised concepts. The *general argument* above suggests the importance of dialogue between the esoteric domains of autonomous research activities. The development takes place as a coherent deformance of the principles that enabled a particular position in argument. Above all, therefore, we should see theoretical frameworks as a space for the becoming of the subjectivity of the individuated researcher. As such they must *de-stabilise* existing identities in order to forge new ones. The development of a good research programme will offer the potential subject of research action an on-going deformance of their own certainties.

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