Introduction to the papers of TWG17: Theoretical perspectives and approaches in mathematics education research

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INTRODUCTION

The 'theory working group' (under various names) has been a feature of CERME since CERME4. An early and constant focus has been 'networking theories', exploring ways of using different theories in mathematics education research (MER) into learning and teaching mathematics. The CERME9 'Call for papers' included:

- The need to go beyond a specific theory when researching a phenomena
- Benefits and/or strategies and/or difficulties in connecting theories
- Conditions for a productive dialogue between theorists
- Difficulties and strategies when gathering results from different frameworks
- Linking theoretical and methodological approaches
- The epistemological dimension in theories
- Steps towards (local/global) theoretical convergence in MER

In our 12 hours together at CERME9 we discussed 19 research papers and two posters; our task in these five pages is to introduce you to the ones (almost all of them) which have been accepted for these Proceedings. We arrange the papers into five groups in the next section. These groups are not 'strongly defined' but are, we believe, useful for communication. The closing section examines issues arising from the papers as a whole and future possibilities. For reasons of space, we refer to the papers by the surname(s) of the author(s) alone.

SUMMARY OF THE PAPERS AND POSTERS

The two papers, by Chevallard, Bosch & Kim and by Dudley-Smith, are grouped in relation to the question 'What is a theory?' Yves Chevallard is the founder of the Anthropological Theory of the Didactic (ATD) and Chevallard and colleagues address the above question directly via the ATD:

> ATD conduces to focus the research effort on examining the implicit, unassuming or even wanting parts of technologies and theories. It then appears that a theory is made up of two main components, that we may call its "emerged part" and "immersed part" ... a theory is thus a hypothetical reality that assumes the form of a (necessarily fuzzy) set of explicit and implicit statements about the object of the theory. A theory is

> in truth the current state of a dialectic process of theorisation ...

Dudley-Smith does not directly address the above question but uses 'Social Activity Method' to interpret (deform/recontextualise) theories in order to explore theoretical networking. It is difficult to summarize this paper, but the following words of the author express a key idea:

> well-formed research activities are incommensurable – they are emergent and not graspable as such, even by themselves. The term "continuity" between theories can refer only to those metonymic chains of signifiers that are of interest *to the recontextualising regard of the theory in question*– hence also the possibility of discontinuity.

The papers by Castela, by Zaragoza and by Roos and Palmér are grouped under the heading 'theories in mathematics education'. Castela considers theoretical diversity and networking theories from the points of view of the ATD and of Bourdieu's theory of social fields. The processes of developing theoretical knowledge is shaped through praxeologies that take place in a community. Further to this "A field is characterised by a game that is played only by its agents, according to specific rules". Using these two approaches, Castela argues that networking will result from researchers from different paradigms working together on the same objects. Zaragoza presents a structuralist definition of a theory as a net of 'theory-elements' connected via a 'specialisation'. A *theory-element* is determined by: the portions of reality it conceptualizes; the *laws* which apply; and potential and actual models. Specialisation concerns the models and laws related to theory-elements. Zaragoza applies these ideas to networking theories and the ATD. Roos andPalmér explore the use of Wenger's 'communities of practice' construct in ten published mathematics education studies. The paper documents differences over these ten papers with regard to: foci on pre-existing or designed communities of practice; foregrounding/backgrounding individuals/groups; constructs (e.g., practice, identity, ...) used. Roos and Palmér conclude, "if a researcher says that (s)he has been using Wenger's social theory of learning, we can be quite sure that we do *not* know exactly what that use of Wenger's theory might imply".

The papers by Holm, by Monaghan and by Şay and Akkoç and the poster by Shvarts and Zagorianakos are grouped under the heading 'connecting theories'. Holm reports on an attempt to use both the SOLO taxonomy and the ATD in order to better understand the advantages of peer collaborative learning exercises for group investigation. The analysis shows that these two frameworks evaluate different dimensions of students' behaviour and relating SOLO-levels to characteristics of ATD praxeology was not possible but the two theories are complementary in terms of understanding student activity. Monaghan focuses on tool use in mathematics and how different theories in mathematics education view tool use. Tool use is important in activity theory (AT) but the consideration of tool use in AT studies varies with the unit of analysis. AT places human agency at the centre of activity and, in contrast to actor network theory (ANT), undervalues non-human agency. Monaghan attempts to 'synthesise' AT and ANT with regard to tool use. Şay and Akkoç examine teachers' social and social-mathematical norms and their instrumental orchestrations in technology-enhanced learning environments in a study designed to investigate how orchestration types and norms affect each other. They found teacher-centred orchestrations in classes where the dominant norm was 'teacher as the mathematical authority' and student-centred orchestrations in classes where the endorsed social norms put students into the centre. Shvarts and Zagorianakos explore the complementarity of activity theory and phenomenology through a detailed analysis of perceptual action by an eye-tracking methodology. While activity theory predicts the development of the perception of visual models through involvement into cultural practice, the data showed that it is the child who makes sense from the presented practice, at the levels of her operative intentionality and the intentionality of act.

The papers by Florensa, Bosch and Gascón and by Kidron are grouped under the heading 'epistemological aspects of theories'. Florensa and colleagues argues that didactics involves both the problem of the development of knowledge and the problem of the diffusion, the use and the transposition of knowledge. Using the ATD, the paper considers means to analyse learning and teaching practices within real institutional environments. The construct 'reference epistemological model' is used to explore extant and new praxeologies through the elaboration of alternative mathematical organisations that could be close to or

very distant from the institutional contents that are taught and learned. Kidron considers "mathematical objects not as absolute objects, but as entities which arise from the practices of given institutions" and this "leads us to analyze the role of both, the epistemological dimension and the socio cultural dimension, in theories". The paper provides "an example of networking that demonstrates how the social dimension might influence the epistemological analysis".

The remaining papers (and one poster) have been grouped under the heading 'issues in mathematics education related to theories'. The papers by Bingolbali and Bingolbali and by Godino, Batanero, Cañadas and Contreras focus on teacher-learners. Bingolbali and Bingolbali argue that student-centred teaching (SCT) consists of two components: mixed teaching methods and principals. They state six principles of SCT: valuing students' prior knowledge into consideration; handling students' difficulties with appropriate methods; developing students' skills; providing effective feedback; creating communicative classrooms; integrating assessment into instruction. Godino and colleagues argue that the inquiry-transmission polarity of instructional models is a simplification of a complex reality. The paper outlines semiotic, epistemological and cognitive assumptions of the onto-semiotic approach to mathematical knowledge and instruction which recognizes a key role to both inquiry and transmission models. The paper by Kent and Foster also challenges a polarity in mathematics education, conceptual versus procedural understanding in mathematics. The paper asks if it would be appropriate to describe a learner in possession of an algorithm for responding satisfactorily to such prompts as displaying conceptual understanding. They relate this question to Searle's 'Chinese Room' thought experiment and draw on Habermas' theory of communicative action to develop implications for addressing the problem of interpreting learners' mathematical understandings.

The papers by Ertas and Aslan-Tutak and by Perez focus on the teacher. Ertas and Aslan-Tutak report on tests of mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK) given to senior student mathematics teachers and senior mathematics students. The performance of the student teachers was significantly higher than that of the mathematics students in the test on MCK. The paper discusses the reasons behind this unexpected finding and notes the challenges to measure MPCK. Perez presents the notion of adaptive conceptual frameworks employed to conduct design-based research with the aim of developing ICT supported mathematics instruction. The paper employs three frameworks: one used when the researcher engages with the teachers; one used to understand outcomes and to plan the next design cycle; one for organizing and supporting the teachers' professional development. Perez uses ideas from 'networking theories' to consider interactions between the frameworks.

The papers by Koichu and the poster by Seidou focus on the learner. Koichu focuses on problem solving and introduces a 'confluence framework' which "consolidates ideas taken from several frameworks", mainly John Mason's theory of shifts of attention. The central premise of the framework is that a key solution idea to a problem can be constructed by a solver as a result of shifts of attention that come from individual effort, interaction with peer problem solvers or interaction with a source of knowledge about the solution. Seidou's framework is Brandom's 'inferentialism' which prioritises inference over reference or representation. The paper reports on students' language 'moves' (how claims are put forward) while reasoning in a geometric sorting activity, "The open-ended aspect of this task creates favorable conditions for a fruitful game of giving and asking for reasons".

The papers by Siller, Bruder, Hascher, Linnemann, Steinfeld and Sattlberger and by Lindenskov, Tonnesen, Weng and *Østergaard* focus on policy. Siller and colleagues report on a project that developed a competency grid to assess the quality of Austrian end of school examination questions. The competency grid has three dimensions (operating, modelling and reasoning) and four levels related to students' mathematical actions: "activity theory forms the background for the didactic interpretation of such initially pragmatic levels". Lindenskov and colleagues report on efforts to develop early "interventions for marginal student groups". The work was inspired by critical theories in mathematics education and practical intervention approaches from various countries. The paper investigates possible contextual influences on networking theories. The paper presents a 'program logic model' for early interventions.

ISSUES ARISING FROM THE PAPERS AND FUTURE POSSIBILITIES

To highlight the productivity and limitations of the work done in the TWG, we propose to consider the questions that have been raised by the papers, their reviews during the discussions, and also issues that have been disregarded or overlooked.

The issue of networking theories was omnipresent in previous 'theory' TWGs since 2005. It has been approached, here, in a different (maybe more mature?) way. The efforts have focused on some basic epistemological and methodological reflections (the nature of theorising, for example) rather than on the description and study of networking strategies. Some new questions have also been opened to a broader debate, with a view to developing research topics in the years to come. One such issue can be called "the question of questions": it relates to the way in which teachers' and students' difficulties can be made sense of by different theories and how the research problems thus arrived at depend on the approach taken. In this context, the issue of mathematics education as a discipline seems especially relevant: what is the place of the didactics of mathematics as a discipline in the arts and sciences realm? How is it related to the didactics of other disciplines and to the other sciences? Can it lead the development of teaching and of teacher education?

Another "big" question that deserves to be addressed is the relationship between local and global theorizations. Many proposed "theories" in (mathematics) education seem to be content with trying to account for a deliberately limited number of didactic phenomena: in this respect, they can be termed "local" constructs for which we have to make clear the didactic dimensions they take into account as well as those they (necessarily or not) overlook. This is especially important since what is called a "theory" or a "theoretical approach" in a given research vein may vary in its degree of development, from limited models to more extensive theoretical constructions. For this reason, it is essential to be constantly aware that theories are living entities in continuous development, and that most "global" theories started with a local or limited scope. The main aim, purpose or ambition adopted by a given theorisation process may result, in this context, as a crucial variable to be taken into account, an issue closely related to the notional and methodological tools elaborated to control and ensure the solidity and productivity of the intended theoretical construction.

The balance between group and individual advancement in the development of research areas is also an interesting topic to approach, in terms for instance of the cultures and craft methodologies elaborated within communities, which are often not easy to disseminate through the traditional channels of science communication (papers, surveys, doctoral dissertations, conference proceedings, etc.). Finally, and related to these last issues, the contrast between findings across different approaches is a question that has never been directly addressed, especially when these findings appear to be, if not contradictory, at least not directly compatible. More generally, a more straightforward, frank, and even antagonistic approach to the problem of theoretical diversity could prove fruitful.