Representations of creative mathematical thinking in collaborative designs of c-book units

Ioannis Papadopoulos, Berta Barquero, Andrea Richter, Maria Daskolia, Mario Barajas, Chronis Kynigos

To cite this version:

Ioannis Papadopoulos, Berta Barquero, Andrea Richter, Maria Daskolia, Mario Barajas, et al.. Representations of creative mathematical thinking in collaborative designs of c-book units. Konrad Krainer; Nada Vondrová. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Feb 2015, Prague, Czech Republic. pp.2381-2387, Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education. <hal-01289279>

HAL Id: hal-01289279
https://hal.archives-ouvertes.fr/hal-01289279
Submitted on 16 Mar 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Representations of creative mathematical thinking in collaborative designs of c-book units

Ioannis Papadopoulos¹, Berta Barquero², Andrea Richter², Maria Daskolia³, Mario Barajas² and Chronis Kynigos³

1 Aristotle University of Thessaloniki and CTI & Press Diophantus, Thessaloniki, Greece, ypapadop@eled.auth.gr
2 University of Barcelona, Barcelona, Spain, bertabf@gmail.com, a.richterboix@gmail.com, ma.barajas@gmail.com
3 University of Athens and CTI & Press Diophantus, Athens, Greece, mdaskol@ppp.uoa.gr, kynigos@ppp.uoa.gr

The study presented here focuses on the representations of creative mathematical thinking (CMT) held by two Communities of Interest (CoI) before and after designing educational resources with an innovative tool affording diverse expressive media for creativity in classrooms. Our analysis revealed similarities but also distinct differences in the way these communities operationalized their CMT representations in their products. Both CoI (Greek and Spanish) appreciated the novel affordance of diverse expressive media to foster CMT in students through open, real life, interdisciplinary problems amenable to multiple solutions. However, the CMT affordances appreciated by the CoI members were distinctly different with respect to contextual factors involving personal and schooling issues as well as influences by the research culture of the two corresponding teams.

Keywords: Creative mathematical thinking, technology.

INTRODUCTION: CMT AND TECHNOLOGY

Various definitions regarding mathematical creativity have been proposed during the last decades. Poincaré (1948) viewed mathematical creativity as the interplay between a period of conscious and unconscious work, in which any choices in terms of mathematical reasoning and proof are made in the pursuit of elegance and usefulness. Recently, Sternberg and Lubart’s (2000) identified mathematical creativity as the ability to predict some ‘non-predictable’ conclusions that are at the same time useful and applicable. More recently discussion on math creativity has been led towards sorting out whether to approach it in terms of a product or a process (Shriki, 2010), as a general ability (associated with applying analogous problem-solving processes from one field to the other) or as some domain-specific skill (exemplified within a particular disciplinary or other activity field and taking into account the logico-deductive procedures already developed in it) (Lev-Zamir & Leikin, 2011). More particularly, the three most prominent strands in the research tradition of math creativity are as follows: the first and most popular is situated in the so-called the ‘genius’ approach of creativity (Sriraman, 2005); the second, represented by Silver (1997), is based on the claim that mathematical creativity is susceptible to instructional and experiential influences and gives equal emphasis on problem posing as on problem solving, with preponderance on the use of ill-structured and open-ended problems; finally, a more recent but rather under-researched approach is that of ‘techno-mathematical literacies’, the focus of which is on identifying and addressing mathematical creativity in out-of-school contexts and in out of mathematics (non-disciplinary) situations, such as looking for and applying mathematical concepts and relations in technology-based workplace practices (Noss & Hoyles, 2013).

Only the latter of these approaches tightly addresses the use and role of digital media for CMT. Although often implied, the development of CMT with the use of exploratory and expressive digital media has not often been centrally addressed in providing users with an access to and a potential for creative engagements with mathematical concepts (Healy & Kynigos, 2010). During the last decades several digital tools, applets and authoring systems have appeared designed to foster CMT affording dynamic manipulation, interconnected representations (inc. mathematical formalism), simulations of phenomena and situations embedding mathematical rules, visualisations of data representations and handling of probability. Some of
Representations of creative mathematical thinking in collaborative designs of c-book units (Ioannis Papadopoulos and colleagues)

The designer communities have been orchestrated within the frame of approaches with a constructionist, argumentational, inquiry-based, problem-based, experimentation origin, all connected but not centrally matched with creativity. Nevertheless, even though research points to the potential of expressive digital media to foster CMT, the uses of these same media in educational practice at large are conversely frequently instrumented towards contexts of traditional lecturing and demonstration of exercise solutions (Ruthven, 2008). This looks like a paradox situation where digital media are being used very differently with respect to their potential for providing users with access to mathematical ideas hitherto obscure and inaccessible, and at the same time allowing them to creatively engage with experiential constructionist, dialogical and social generation of mathematical meanings, understandings and social norms has long been elaborated (Hoyles & Noss, 2003).

Our paper therefore addresses a real wide-scale problem and it is within that context we attempt to contribute to our understanding of teachers designing of affordances for CMT. No doubt, the clarification of shared understandings of the essence of CMT and the spreading of respective activity in education are slow and non-linearly developing processes. New media affording collaborative designs for CMT may provide us with the means to generate socio-technical environments (Fischer, 2011) more conducive to addressing these issues and designing for learning cultures cultivating CMT. This paper studies designers’ representations of CMT (in the context of a European project called ‘MC Squared’) as they became operationalized in the collaborative production of materials with an innovative e-book author aiming to afford CMT (we call it the ‘c-book’, ‘c’ for creativity) to the end users. The designer communities have been orchestrated by four research teams so as to ensure the diversity among the members in relation to the disciplinary backgrounds, expertise, history and membership in different communities of practice which are defined in literature as Communities of Interest (CoI) (Fischer, 2011). Four such CoI (English, French, Greek and Spanish) were thus formed in MC2 project. The CoI were put together to include a variety of expertise involving designers and developers of digital resources for math education, publishers of mathematics educational materials, researchers in (and outside) math education or in creativity, teacher educators, school math teachers and students.

Given the diverse approaches to CMT outlined above, and the relative lack of connection to math activity with expressive digital media, the concept remains fuzzy in the literature. So, the aim of research is to illuminate how a shared understanding of CMT can be generated in CoI using technology specially built to afford collaborative design for CMT. Such research may contribute in crafting ways to orchestrate and moderate teacher communities of practice potentially playing a role in spreading the use of expressive media for CMT related activities in classrooms. Our research focus was thus twofold: (a) to illuminate the designers initial representations of CMT in two (of the four) CoI, and (b) to illuminate the process of operationalization of these representations during collaborative designs of such technologies for students which would be done by examining how the CoI members evaluated their product in terms of CMT affordances. In the sections to follow we briefly outline our research method, and then discuss the CMT representations of the two CoI so far revealed by our analyses (the Greek and Spanish respectively) in two ways. Firstly as they were expressed in their responses to two semi-structured questionnaires before they were engaged in the design, and subsequently, as they were manifested in their evaluation of the CMT affordances of their collaboratively produced c-book units. Finally, through a synthetic analysis we aimed to identify common patterns and differences in the ways the two CoI approached the CMT criteria with regards to their task at hand with the c-book technology.

DESCRIPTION OF THE STUDY AND METHODOLOGY

The decision to work with four (here two) CoI was a methodological one for two reasons: The first one was that we aimed to obtain diversity in relation to the expertise of the CoI members within each CoI. The second was that each CoI would correspond to a particular educational context. This decision would contribute to deepening our awareness of the role of the context diversity both by the exercise of de-contextualising our findings and by using our synthetic knowledge to later try to develop methods to re-contextualize generic findings (Lagrange & Kynigos, 2014) by distinguishing between commonalities and differences in the operationalized representations of the members of the two CoI.
In order to identify the CoI members’ initial representations of CMT, an exploratory approach was chosen independently by the two research teams, mainly based on the use of questionnaire as the main instrument for data collection. Each research team developed its own questionnaire and followed a different but nevertheless comparable procedure. In the Greek case the questionnaire was administered to the 17 participants in the Greek CoI. It consisted of: (a) open-ended questions aiming to explore the CoI members’ personal definition of CMT, and (b) a list of 17 statements concerning the nature and characteristics of CMT, based on a 6-point Likert type scale (ranging from 1=completely disagree to 6=completely agree). The Spanish research team developed a different questionnaire, which was administered to the 17 members of the Spanish CoI. This questionnaire consisted of: (a) a list of 26 statements (which respondents were asked to respond to by using a 5-point Likert scale, ranging from 1=completely disagree to 5=completely agree) concerning the nature of creativity and CMT, the characteristics of creativity in their professional background and the profile of a creative student and a creative math teacher; (b) three open-ended questions to collect features, criteria and examples of tasks and activities that could foster CMT. The qualitative data from the open-ended questions were subjected to thematic analysis and the criteria of either math ideas or some concrete mathematical ‘objects’. This is in line with a ‘constructionist’ approach to creativity giving emphasis to the learners’ creative expression and learning through the active exploration, modification and creation of digital artefacts (Daskolia & Kynigos, 2012). The next two themes that emerged are in accordance to what literature identifies as the criteria of novelty/originality (Liljedahl & Sriraman, 2006) and usability/applicability (Stenberg & Lubart, 2000). The former relates CMT to ‘mathematical productions’ that are new/unusual, and/or new or unexpected ways of applying mathematical knowledge in posing and solving mathematical problems. The latter was perceived by the Greek CoI as the purposeful combination of math knowledge from different math domains or with knowledge from other scientific areas. Some other themes identified in the Greek CoI representations of CMT were those pertaining to fluency, flexibility and imagination. The literature views all three of them as characteristics of a creative mathematical process (Silver, 1997; Leikin, 2009), interconnected but not necessarily all present at the same time (Baer, 1993).

The analysis of the close questions showed that almost all Greek CoI members seemed to agree that mathematical creativity entailed that is an ability which can be fostered through interaction with other people in a collaborative context (Mdn=6, IQR=1) and within a milieu which is rich in many alternative - even contradictory- ideas (Mdn=6, IQR=1), which allows openness to other disciplinary fields (Mdn=6, IQR=1), and includes problems inspired from real life (Mdn=6, IQR=0.25). Most Greek CoI members agreed that CMT has to be based on a deep and well-rooted mathematical background, and that it can emerge from the people’s ability (a) to use many, different and unusual ways

Finally, comparative analysis was employed to trace similarities between the criteria proposed by the two CoI.

**INITIAL REPRESENTATIONS OF CMT IN THE TWO COI**

The analysis of the data gathered from the Greek CoI responses to the open-ended question asking them to provide a definition of CMT revealed a set of themes ascertaining that the participants conceptualised CMT both in terms of process and product.

The first theme matched CMT with the ‘construction’ of either math ideas or some concrete mathematical ‘objects’. This is in line with a ‘constructionist’ approach to creativity giving emphasis to the learners’ creative expression and learning through the active exploration, modification and creation of digital artefacts (Daskolia & Kynigos, 2012). The next two themes that emerged are in accordance to what literature identifies as the criteria of novelty/originality (Liljedahl & Sriraman, 2006) and usability/applicability (Stenberg & Lubart, 2000). The former relates CMT to ‘mathematical productions’ that are new/unusual, and/or new or unexpected ways of applying mathematical knowledge in posing and solving mathematical problems. The latter was perceived by the Greek CoI as the purposeful combination of math knowledge from different math domains or with knowledge from other scientific areas. Some other themes identified in the Greek CoI representations of CMT were those pertaining to fluency, flexibility and imagination. The literature views all three of them as characteristics of a creative mathematical process (Silver, 1997; Leikin, 2009), interconnected but not necessarily all present at the same time (Baer, 1993).

The analysis of the close questions showed that almost all Greek CoI members seemed to agree that mathematical creativity entailed that is an ability which can be fostered through interaction with other people in a collaborative context (Mdn=6, IQR=1) and within a milieu which is rich in many alternative - even contradictory- ideas (Mdn=6, IQR=1), which allows openness to other disciplinary fields (Mdn=6, IQR=1), and includes problems inspired from real life (Mdn=6, IQR=0.25). Most Greek CoI members agreed that CMT has to be based on a deep and well-rooted mathematical background, and that it can emerge from the people’s ability (a) to use many, different and unusual ways
to solve a problem ($Mdn=5$, $IQR=0.25$), (b) involving many and diverse mathematical representations for approaching a math problem ($Mdn=5$, $IQR=1.5$).

In the case of the Spanish CoI, thematic analysis was also employed for the analysis of the data from the open-ended questions. The analysis revealed the key traits which could trigger CMT in students: a) allowing for multiplicity in the approaches and techniques to resolve questions (16 respondents out of 17); b) promoting multiple representations of a particular mathematical concept (16 / 17); c) engaging students in math problem situations close to their reality (13 / 17); d) situating math problems in interdisciplinary contexts, by combining math knowledge with other disciplines and showing its functionality in other domains (13 / 17); and e) including communication tasks within the activities to share students proposals and opinions (10 / 17).

Concerning the nature and characteristics of CMT, most Spanish CoI members consider creativity being a quality that can be developed through instruction ($Mdn=4$, $IQR=1$) and that interaction with people with different perspectives can enrich the creative process ($Mdn=5$, $IQR=1$). Most of them consider themselves as creative professionals to a great extent ($Mdn=4$, $IQR=0$), and acknowledged that mathematics help promoting creativity in other disciplines ($Mdn=4$, $IQR=1$).

In describing a creative student, they emphasised as requisites the ability to: (a) formulate questions and initiate investigations ($Mdn=4.5$, $IQR=1$); and (b) find different ways to solve problems combining different tools and representations ($Mdn=4.5$, $IQR=1$). The Spanish CoI members claimed that, in order to foster CMT, the teacher has an essential role, by holding a deep mathematical background ($Mdn=5$, $IQR=1$), and by encouraging the advent of diverse students’ responses when approaching the mathematical questions ($Mdn=4$, $IQR=1$).

Comparing the two cases it seems that both CoI shared several criteria, which the literature identifies as closely related to CMT. To mention some: the belief that CMT can be fostered in the classroom, through appropriate education (Silver, 1997), the recognition that the interaction between the learning actors is important (Leikin, 2009) and the connection of creativity with mathematics (Silver, 1997). The promotion of CMT is understood through tasks of an exploratory nature (Mann, 2006), through open problems amenable to multiple solutions (Lev-Zamir & Leikin, 2011), through situations that demand the combination of various tools and/or representations, or through interdisciplinary tasks (Perry-Smith & Shalley, 2003).

**EXTRACTING THE COI CRITERIA OF CMT**

The CoIs’ initial representations of CMT allowed researchers to build a shared understanding and ‘vocabulary’ on how to address CMT within their CoI, which helped them reflect on and develop their own criteria for assessing the c-book units’ CMT potential impact to prospective students. The two CoI have designed and produced a series of c-book units until now. However, from a research point of view we were interested in identifying whether and how these CMT representations were employed by the two CoI immediately after they had completed their first c-book unit. In the following paragraph, we offer a short description of these two first c-book units, namely ‘Windmills’ and ‘Viral Behaviour of Social Networks’.

The main idea of ‘Windmills’ was to challenge students to explore, identify and use the mathematical concepts underlying the construction and operation of Greek windmills, an integral part of the Greek islands’ scenery, quite familiar to the students. The CoI intention was to design a c-book unit that would invite students to foster ‘unexpected’ mathematical ideas, surprising even to the initial designers of the activities. The construction of the c-book unit was initiated with the CoI members developing and exchanging through CoICode (the c-book author collaboration tool), Turtleworlds (a Logo-based Software), Geogebra (Dynamic Geometry Software) and DME Draw3D widget [3] instances (3D grid using cubes as building blocks). The rationale was to start addressing simpler and gradually move to more complex mathematical concepts. This ‘low threshold – high ceiling’ rationale adhered to the idea of making the c-book unit appropriate for students within a large range of school grades (lower to higher secondary). The first four pages included activities with Turtleworlds targeting the construction of geometrical figures – parts of a windmill (triangles, parallelograms, regular polygons, solid shapes) and use them to create a windmill. The following pages used GeoGebra to engage students in exploring the operation of ready-made models of windmills and identify the underlying mathematical concepts (linear functions, direct proportional amounts, multi-branch functions, periodic functions,
co-variation of geometric magnitudes). Finally, students would have to repair a deserted windmill to explore geometric figures.

The Spanish c-book unit focused on fitting mathematical tools and models to describe and understand the ‘Viral Behaviour of Social Networks’. Mathematical modelling was thought central to allow students to:

- a) formulate assumptions on different phenomena related to social network users,
- b) use mathematical tools to analyse relations and patterns on real data,
- c) look for, fit and test mathematical models to forecast social networks behaviour,
- d) use math (and its models) to obtain responses about reality, and
- e) be able, if necessary, to reformulate assumptions and models.

The CoI wished to put forth an interdisciplinary approach where math (mainly arithmetic and pre-functional modelling) would provide tools for analysing social phenomena. The c-book unit consists of 3 parts: The first part focuses on introducing students to a sequence of questions about how the number of friends evolves depending on the degree of friendship, in order to explore exponential properties acting under social networks. The second part presents the ‘theory of the six degree of separation’, by asking students to estimate the number of friends a teenager can have and, then, forecast how this evolves depending on the degree of friendship. Finally, the third part allows students to work on their own data by asking them to search their friends and connections in Facebook and estimate up to what degree of friendship they are connected with any other person in the world. The c-book unit was developed in 14 pages and includes widgets from 3 different widget factories: GeoGebra, DME (probability tree, answers box, tables, etc.) and a special Cinderella widget to visualize and compare graphs of points using linear and algorithmic scale.

After completing the two c-book units, the CoI members were invited to evaluate the potential of the units as educational resources fostering users’ CMT. Each research team conducted a separate analysis of the data gathered from their CoI evaluation. This analysis examined whether the criteria used by each CoI were in alignment with their initial CMT representations, with the criteria identified in the literature review, and with the educational traditions of the specific CoI. A comparative analysis between the two cases brought evidence about commonalities and differences between the two CoI. The findings of this analysis are as follow:

a) A focus on exploring ‘real’ problems and questions: Both CoI emphasised the use of real-life situations (close to students’ reality) in their c-book units. This is recognized as a characteristic of mathematical creativity and many mathematicians describe it as an invaluable aspect of their craft (Sriraman, 2005). The same criterion was traced in the initial CMT representations of both CoI (a, c, d [4]).

b) Promote openness, exploration, and diversity in the approach taken: According to their initial representations the two CoI appreciated the fact that the activities included in the c-book units were based on open questions allowing the students to suggest their own answers. Silver (1997) suggests that such open questions encourage the development of students’ creative fluency, associating fluency with exploration as a way of thinking. On the first hand, there is an explorative nature in the questions included in both c-book units. The students are prompted to explore a problematic situation using different mathematical tools and also including simulation of different mathematical models. On the second hand, both c-book units include specific widgets to collect the diversity of students’ proposals, in a more open format as windows to write down their answers to be collected or as interactive widgets. For instance, the Turtleworlds widget allows students to build and work with their windmill construction with different blade’s forms; the Cinderella widget facilitates that each user can work with their own Facebook data, combining the point table with the graphical representation in different scales (a, c, d).

c) Enable multiplicity of mathematical approaches/representations/techniques/solutions: According to both CoI, one of the strongest didactical qualities of the c-book units was the one of multiplicity. This refers to the case of embedded activities that promote multiple solutions for a problem or multiple paths toward the solution of a problem (Leikin, 2009). The technology involved in the two c-book units facilitates the students to work with, and combine, different representations of mathematical concepts. One example might be the variety of numerical and graphical representations (trees, point graphs, etc.) of geometrical sequences (a, c, d).

d) Promote inter/intra-disciplinarity: According to Perry-Smith and Shalley (2003), a person’s exposition to diverse contexts and functional areas may lead to the production of different and unusual ideas.
manifestation is an indication of novelty/originality, which is considered an inherent characteristic of CMT. The Greek c-book unit deals with mathematics through some pure engineering and environmental issues, such as the preservation of old windmills, the use of alternative energy sources, etc. The Spanish c-book unit connects mathematics with social and technology issues such as the use of Facebook. Both c-book units combine a variety of mathematical topics belonging to different mathematical domains. ‘Windmills’ constructions invite students to apply knowledge from trigonometry (angles, construction of triangles, etc), functions, coordinates, and graphs, whereas the ‘Social Networks’ promotes the construction of models based on exponential and logarithmic properties, geometric sequences, and combines diversity of representations: probability trees, point tables and graphs with ordinary and logarithmic scale (a, c, d).

e) Promote a progressive modelling process: Mathematical modelling penetrates the whole activity in the Spanish c-book unit. The c-book unit starts with an initial generating question that was broken into some derived questions placed along the unit (and complemented with the most appropriate widget instances). Each of its three phases leads to consider different mathematical models (more complex at each step) that appears from the analysis of real data about social networks, at the same time models evolve thanks to its evaluation and contrast with reality (a, b, d).

f) Involve a constructionist aspect: In the case of the Greek c-book unit the students are invited to create their own windmill or to complete half-constructed ones. This constructionist aspect is very much based on the previous experience and educational tradition of the Greek CoI members. All of them were involved in two wide-scale initiatives of the Ministry of Education in Greece. In the first, the aim was to expose mathematics teachers (amongst others) to constructionist epistemology, technologies and activity designs. The second project concerned the digital enhancement of the traditional textbooks which was also penetrated by the flavour of constructionism (a, b, d).

CONCLUSIONS
The study was carried out in the context of a growing realisation that large scale use of digital media in educational practice is widely dis-aligned from CMT affordances. Understanding and promoting CMT representations in designer communities has had a dual purpose of suggesting a strategy to spread CMT cultures and illuminating our own understanding of the fuzzy notion of how CMT is represented amongst designers. We thus tried to identify some initial CMT representations amongst diverse designers and then to illuminate how these representations were operationalized in the process of collaborative design. Both CoI appreciated the potentiality for the c-book units to foster CMT in students appreciating the potential for integrating real-life problems that are open and amenable to exploration, multiple solutions and the combination of various disciplines and/or various mathematical topics. But equally important were the operationalization of representations of CMT linked to a specific context. The Greek CoI associated the chance to foster students’ CMT with the ability of the c-book technology to support constructionist activities. The Spanish CoI made a corresponding association of this technology with its ability to support a progressive modelling process.

ACKNOWLEDGMENT
The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007–2013) under grant agreement n° 610467 – project “MC Squared”, http://mc2-project.eu. This publication reflects only the authors’ views and Union is not liable for any use that may be made of the information contained therein. The c-book technology is based on the widely used Freudenthal Institute’s DME portal and is being developed by a consortium of nine partner organisations, led by CTI & Press ‘Diophantus’.

REFERENCES


ENDNOTES


3. **Widgets** – c-book-widgets (widgets for short) are small pieces of software which can be included into c-books via the c-book environment in order to allow interactive content. **Widget instance** – a widget that has actually been inserted into a c-book page is called a widget instance. They can still be configured by the c-book author in order to fulfill the specific needs. For example, for visualizing the graph of a function, the c-book author may specify the ranges to be used, etc.

4. Letters a, b, c, and d refer to the four ways for organizing the evaluation criteria used by the CoI members as they are described in p. 4.