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How do research mathematicians teach Calculus?

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We investigate Calculus teaching at university mathematics departments and in particular research mathematicians’ teaching practice in the context of lectures. We are interested in how lecturers draw mathematics students into mathematical culture. In this paper, we focus on the teaching of a lecturer of a large cohort of students that we analyse using grounded techniques and the Teaching Triad construct (Jaworski, 1994). In spite of the lecture format, the analysis suggests that this lecturer’s teaching is characterized by the way he supports students’ engagement in the lecture and the way he familiarises them with mathematical production. The use of the Teaching Triad brings to our insight what sensitivity to students could mean in this traditional setting and what mathematical challenge could be at the university level.

Keywords: University teaching, sensitivity to students, mathematical challenge.

INTRODUCTION

Teaching at the University level deals commonly with large groups of students and the lecture is the instructional activity within which teaching usually takes place. Arguably the lecture instruction has received a strong criticism and has been widely maligned by mathematicians and mathematics educators alike (Weber, 2004). It seems that lecture is considered as a unified, single teaching paradigm and thus a small number of studies investigated mathematics teaching in this context (Speer, Smith, & Horvath, 2010). For example, Weber (2004) addressed the nature of lecturing showing that it is not as a single teaching paradigm as it thought to be and Jaworski, Treffert-Thomas and Bartsch (2009) pointed out the tensions that a lecturer experiences in satisfying student needs and mathematical values. Yet, Speer and her colleagues (2010) in their systematic literature review argued that university mathematics teaching is “an unexamined practice” and drew attention to the need for empirical research that examines and describes the work of teaching university mathematics in detail.

Seeking to readdress this scarcity of empirical research and to gain better understandings of the actual teaching at this level, our study aims to explore characteristics of first year university mathematics teaching in Greek mathematics departments. The topic in focus is Calculus which is a first year compulsory course taught in a lecture format. Research concerning the learning of Calculus at the university level has shown that students experience difficulties in aligning with advanced mathematical processes and concepts when they enter into university (Artigue, Batanero, & Kent, 2007; Nardi, Biza, & González-Martin, 2009). A question is how teaching at this level and in the particular context of a lecture could deal with the above difficulties while drawing mathematics students into mathematical culture. In this respect, there is a growing body of research seeking to characterize elements of teaching practice that takes students into account (some examples will be discussed below in the theoretical background section) but not at the university level. Being aware of the complexity of teaching that has been identified by many researchers into secondary level teaching (e.g., Potari & Jaworski, 2002) and does not end with the transition to the university level, we attempt to investigate first year Calculus teaching in the, most usual for this level, format of a lecture. In particular, we focus on one lecturer’s teaching actions and the rationale behind these actions to identify the characteristics of his teaching and we attempt to realise the nature of the identified characteristics in this particular context.

THE THEORETICAL BACKGROUND

Exploring characteristics of teaching deals inevitably with the fundamental question “what is teaching”
addressed by some philosophical studies (e.g., Hirst, 1971). Some other studies refer to the debate whether teaching is a practice or a means to introduce students to another practice – the mathematical practice (e.g., Noddings, 2003). We see teaching as an activity following Pring (2000) who claims that: “An action might be described as ‘teaching’ if, first, it aims to bring about learning, second, it takes account of where the learner is at, and, third it has regard for the nature of what has to be learnt” (p. 23). Adopting a sociocultural perspective, teaching is considered not only as a product of the constructive activity of the individual teacher but also as a social practice; a complex nexus of social inter-relationships (Jaworski, 2002). Learning mathematics at this level is seen as enculturation in advanced mathematical practices (Artigue, Batanero, & Kent, 2007). Enculturation, in the sense described by Bishop (1991), is an interpersonal process so the role of people who have special responsibility for this process is emphasized. This “cultural group” of people are for us the mathematicians who teach at university level. Bishop (1991) quoted Wilder (p. 6) who wrote for the mathematicians: “Those people who do mathematics – the ‘mathematicians’ – are not only the possessors of the cultural element known as mathematics but, when taken as a group in their own right, so to speak, can be considered as the bearers of a culture, in this case mathematics”. This view offers us a base to interpret lecturer’s attempts to draw students into mathematical culture and characterize his practice since “introducing children into the culture of a mathematical practice is basically a social process” (Van Oers, 2001, p. 73).

Speer and colleagues (2010) made a distinction between instructional activities and teaching practice. According to this distinction the lecture, the context of our study, is an instructional activity while teaching practice concerns what teachers do when they are planning, teaching and reflecting on their lesson. Thus, lecture is the usual instructional activity at university level yet in lecturing different teaching practices may take place. We investigate a lecturer’s teaching practice i.e. his/her teaching actions (what s/he does intentionally) and the rationale behind these actions, seeking to characterize this practice and gain deeper insights into university teaching.

Our research tool in the endeavour to interpret the nature of teaching characteristics is the Teaching Triad (TT). TT is an analytic framework that emerged from an ethnographic study of investigative mathematics teaching at secondary level (Jaworski, 1994). Its main goal was to capture essential elements of the complexity of mathematics teaching by analyzing classroom interactions. Jaworski (2002) describes that the triad consists of three “domains” of activity in which teachers engage: management of learning (ML), sensitivity to students (SS) and mathematical challenge (MC). ML describes how the teacher organizes the classroom learning environment (e.g., groupings, planning of tasks, setting norms). SS describes teacher knowledge of students and attention to their needs and in particular the ways that he/she interacts with individual students and guides group interactions. Sensitivity to students has been shown to relate to both affective (e.g., offering praise, encouraging students to participate) (SSA) and cognitive (e.g., judging appropriate questions, inviting explanation) (SSC) domains. MC describes the challenges offered to students to engage mathematical thinking and activity. This includes tasks set, questions posed, and emphasis on metacognitive processing. The above elements are closely interrelated as the study of Potari and Jaworski (2002) indicated. The authors claim that a balance between sensitivity to students (in both cognitive and affective domains) and mathematical challenge is an indicator of effective mathematics teaching in the sense that students can be involved in rich and meaningful mathematical activity. The triad has also been used to characterize teaching, for example in a study of interactions in university mathematics tutorials (Jaworski, 2002, Nardi, Jaworski, & Hegedus, 2005) but it has not been used to characterize lecturing so far. It is a question for example, what sensitivity to students could mean in lecturing large groups of students or what mathematical challenge describes in this context. It is in our interests to re-examine the elements of the Triad and define the potential meaning they gain at this level.

Nardi and colleagues (2005) also characterized teaching approaches in small group tutorials from the perspective of the tutor offering a theoretical perspective on the links between mathematics and pedagogy. Lobato, Clarke & Ellis (2005) developed a theoretical reformulation of telling, characterizing teaching actions according to their function and Anghileri (2006) characterized teaching by identifying a hierarchy of interactions that relate to teaching practices that can enhance mathematics learning. Grandi and Rowland (2013) analyzed the function of teacher interventions while Drageset (2014) characterized teachers’ com-
ments. The above studies characterized approaches to teaching and gave us the theoretical underpinnings to formulate specific criteria for coding the lecturer’s teaching actions.

METHODOLOGICAL ISSUES:
DATA AND ANALYSIS

This paper is a part of an ongoing study with aim to investigate the first year Calculus lecturing in two mathematics departments. Six lecturers participated in the wider study. Calculus is a proof-based, first year course in both departments and it is taught in parallel in two or three classes of approximately 100 students each (4 hours for theory and 2 hours for solving exercises per week for one semester – all in a lecture format). In this paper, we analyse data from one lecturer who is an experienced active research mathematician and university teacher, with the ultimate goal to draw students into mathematical culture. Data for this lecturer were collected through observations (3 two-hour lectures), interviews (3 interviews right after each lecture) and group discussions (5 three-hour group discussions). During the observation of the lectures, field notes were also kept and after class interviews with the lecturer conducted by the first author. Moreover, in group discussions among some of the 6 participants of the wider study and mathematics education researchers, more general issues about university teaching were discussed. The lecturer of this study participated in all group discussions. All lectures, interviews and discussions were audio-recorded (one of the lectures was also video-taped) and transcribed.

In data analysis, grounded approaches (Strauss & Corbin, 1998) and the Teaching Triad (Jaworski, 1994) were used. The analysis was conducted in three phases. In the first phase, each two-hour lecture was divided into episodes according to the accomplishment of teaching a theorem. Each episode was coded with descriptive codes according to what the lecturer did during this episode and from these descriptive codes teaching actions were identified. Codes for teaching actions were merged or refined after continuous comings and goings through the whole data and were supplemented by categories taken from Anghileri (2006), Drageset (2014) and Grandi & Rowland (2013). This process enabled us to form criteria for characterising the codes of the teaching actions. Finally, the analysis resulted in 12 codes. The rationale of teaching has been investigated through the analysis of interviews and discussions which were also coded according to the subject under discussion and in this way the lecturer’s teaching practice (i.e. teaching actions and rationale) has been identified. In the second phase, teaching actions were grouped into categories and in this way the characteristics of this lecturer’s teaching practice were identified i.e. characteristics are categories of teaching actions and their rationale. In the third phase we used the Teaching Triad to gain deeper insight into the identified characteristics. Thus characteristics of teaching were analysed further by using the TT to explore potentials of TT’s elements in this level and in this way we gained insights into the nature of these characteristics per se.

In the next section, first we give an example of analysis to illustrate the analytical process and the related emerging issues and then we give a brief account of the results from the analysis of the whole data for this lecturer.

RESULTS

A teaching episode that appears to be typical of the way the lecturer interacts with students follows as an example of analysis. In this episode, which is about the convergence of series of real numbers, the lecturer tries to facilitate students to arrive at a conjecture.

An example of analysis: A teaching episode and its analysis

At the beginning of the lecture after the definition of a convergent series, the lecturer gave the geometrical series

\[ \sum_{k=0}^{\infty} \frac{x^k}{1-x}, |x| < 1 \quad (S1) \]

as an example of a convergent series (a familiar example from secondary school). Subsequently, he gave the example of divergent series:

\[ \sum_{k=1}^{\infty} (-1)^k \quad (S2) \]

Then the dialogue below followed in Table 1.
### An episode
(The lecturer facilitates students to arrive at a conjecture)

<table>
<thead>
<tr>
<th>Teaching actions</th>
<th>Descriptive Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exploration</td>
<td>Challenging students to be engaged in mathematical exploration</td>
</tr>
<tr>
<td>Focusing</td>
<td>Prompting students to focus on the examples ([S1] and [S2])</td>
</tr>
</tbody>
</table>

L: Now, can you hypothesize, when a series may converge?

You can base on the series S1 and S2.

(No response)
The lecturer wrote again the two examples (S1 and S2) on the board (one next to the other) with the words “converges” and “does not converge” respectively next to each example.

L: Can you see any differentiation in these two cases S1 and S2?

They are two specific cases of course but... is there any difference between them?

S1: Yes. The base [of the v] in the first case (S1) is a positive number.

L: Not necessarily. If x is a negative number the sign [of x^k] changes.

(he moves his hand left and right showing the alternation of the signs over an imaginary number line).

S2: Counting starts from different numbers

L: No, it has nothing to do with... We could start from the same number. (He changed (in S1) the counting from k = 0 to k = 1 on the board). Now on the board the two series were:

\[ \sum_{k=1}^{\infty} x^k \text{ (S1a) and } \sum_{k=1}^{\infty} (-1)^k \text{ (S2)}. \]

L2: Here you are! Now the series ([S1a] and [S2]) seem comparable. Finite number of terms at the beginning of a series does not have any influence on its behaviour [related to its convergence].

S3: Is the difference that the 1st series (S1a) has a variable?

L: It’s not a matter of variable. OK. If I set here... I don’t want to confuse you. Let me take a specific geometrical series.

(He writes the series \( \sum_{k=1}^{\infty} \frac{(-1)^k}{10^k} \text{ (S1b)}. \))

I took units everywhere. This number... I made it to look about the same.

S1: Is the difference that the 1st case (S1a) has the condition \(|x|<1|?

S4: Is the difference that in the 2nd case (S2) the absolute value is 1 (|-1|^k = 1)?

L: This observation (of S4) may imply something. It’s a correct observation.

Your fellow student said we should take these terms (the sequence in S2) and we’d consider their absolute value. This absolute value will always be 1. This might be a difference. Whilst in this case (S1b) the absolute value of the sequence in the series is 1/10^k. And?
In the above example we see some of typical teaching actions of this lecturer’s teaching. In particular, the lecturer mainly poses a problem; calls students to express their observations; uses examples familiar to them; evaluates students’ observations by giving comments and by simplifying the examples; asks more focused questions when students do not follow; uses students’ relevant observations as a basis to direct discussion and summarizes the main results.

In terms of the TT, we see expressions of his sensitivity to students and the mathematical challenge he offers to them. For example, he tries to motivate students to participate by asking questions and reassures students that their contributions are acceptable (SSA); highlights and builds on their ideas and comments (SSC). He also addresses mathematical important questions; encourages students to think deeper to identify relations (e.g. by reflecting on familiar examples) and engenders students’ interpretations of relationships and representations (MC). Moreover, he organizes the content of the lecture to support students’ reflections (e.g. he brings an example and later he uses it as a basis for exploring relationships) and he establishes norms of a working group (e.g. he always uses “we” in his talk) (ML). Analysing all the data from this lecturer, two specific goals were identified. The first goal was mainly affective: the lecturer tried to stimulate students to become confident in their engagement with the advanced mathematical content:

“What I do consciously is that I try to tone up students psychologically ... Speaking emphatically, I believe that, in order for someone to be engaged in the learning process he must have the appropriate psychology. I think that the 50% of learning is the learner’s psychology.” (Translated from Greek)

The second goal was mainly mathematical: he tried to initiate students into advanced mathematical thinking and mathematical production: “What I try to do is to teach students how to think mathematically... And I have to make them understand how someone thinks and produces mathematics.” (Translated from Greek).

These goals were carried out with specific teaching actions. These teaching actions were related and grouped forming the characteristics of this lecturer’s teaching which have been analysed further with the TT. Table 2 presents how all teaching actions are grouped into characteristics and how the characteristics are related to the three domains of the Teaching Triad.

We related the actions “explaining”, “directing discussion” and “summarizing” because they characterise lecturer’s guidance to students by informing or providing suggestions to them. In terms of the TT, we see this guidance as an indication of ML and SS as the lecturer gives explanations, suggestions or points out an idea with sensitivity to students’ needs.

We related the actions “evaluating”, “drawing on students’ experiences” and “checking for consensus” because they characterise lecturer’s supporting of students engagement. The lecturer evaluates students’ contributions in a way that rewards their involvement even if the contribution is invalid and he is not moving on without their consensus. In terms of the TT, we see supporting students’ engagement as an expression of lecturer’s SS as the lecturer opens up students’ ideas, builds on them and confirms that these ideas contribute in the learning process.
We related the actions “focusing” and “simplification” because they both characterise discarding any irrelevant features. The lecturer simplifies examples a strategy which he uses in his research: “This is the way I produce work.” So, discarding any irrelevant features is a process of mathematical production at least for him. As he states in an interview simplifying also allows him “to include more students in the lecture”, so to be more in accord with students’ needs. Thus, in terms of the TT, we see discarding any irrelevant features as an expression of MC which, in this case, is related with SS.

We finally related all the actions through which the lecturer introduces students to problem-solving techniques by creating conditions for investigations (“posing a problem” and “exploring”) and by encouraging connections (“relating” and “translating mathematical ideas”). In terms of the TT, we see that using problem-solving techniques is a characteristic of MC as the lecturer challenges students to be engaged in mathematical exploration and to relate and represent mathematical ideas – both important processes of mathematical thinking and production.

Summing up, we could claim that this lecturer’s teaching is characterized by the way he takes students into account showing sensitivity to their needs and by the way he tries to draw them into mathematical production offering mathematical challenge.

DISCUSSION

In this paper, we studied the teaching practice of a lecturer teaching large cohorts of students and we identified characteristics of his teaching which we interpreted in terms of sensitivity to students, mathematical challenge and management of students’ learning. In particular, this specific lecturer supported students to participate in the processes of advanced mathematical thinking and production bringing experiences from his research activity into his teaching.

In terms of the Teaching Triad, we see that in this case, mathematical challenge at the university level is directly related with the mathematical production. This is a different dimension of mathematical challenge than the ones may be found in secondary level and we believe that it is worth to explore what other dimensions mathematical challenge could receive at this level. We also see that in spite of the lecture format, this lecturer’s teaching is characterized by his sensitivity to students since he builds on their ideas to support their engagement in the learning process. This is far from obvious in lecturing large group of students.

We tried to shed light on what takes place during the dominant instructional activity of the lecture. Our results showed that the lecture is not a unified teaching paradigm. This lecturer’s teaching practice contrasts with the practices thought of as usually adopted in Calculus lectures and offers evidence that teaching can exist in a lecture format that is sensitive to the students and resembles mathematical production.
Collegiate teachers seem to work in different ways in their classrooms under the instructional context of a lecture. For this reason, as Speer and colleagues (2010) state their practice and reasoning is worthy of study because it can help others (teachers and researchers) understand how and why teaching happens in certain ways.

Obviously, many lectures at the university level are still taught in a transmissive way in the sense that the lecturer conveys information and the students listen and passively take notes. However, this study provides evidence that there are alternative ways for teaching in the instructional context of the lecture and might be used as a lecturers’ self-awareness springboard towards improving teaching.

REFERENCES


