Asking productive mathematical questions in kindergarten

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Although the trends seem to be shifting, researchers have given far less attention to the work of teaching mathematics in kindergarten than to children's learning. This paper aims at contributing to this under-developed area by focusing on one particular task of teaching mathematics: asking productive mathematical questions. From analysis of a situation that involves Lego play, we attempt to decompose the different kinds of mathematical questions asked and thus contribute to the further conceptualization and understanding of this particular task of teaching mathematics in kindergarten.

Keywords: Kindergarten, mathematics, teaching, productive mathematical questions.

INTRODUCTION

Children's learning of mathematics has been studied for decades; the teaching of mathematics to children below school age has been studied much less. Whereas numerous theories have been developed in order to describe different aspects of mathematics learning, few theories describe mathematics teaching. Almost three decades ago, Lortie (1975) called for a language to describe the work of teaching, and a language and theory of teaching is still called for – especially in kindergarten.

When developing a practice-based theory of mathematical knowledge for teaching (MKT), Ball, Thames and Phelps (2008) focused on "recurrent tasks of teaching". In the work of teaching mathematics, teachers are faced with different challenges, and these are referred to as tasks of teaching. An example is "asking productive mathematical questions" (Ball et al., 2008, p. 400). Teachers are continually challenged to ask questions that stimulate further mathematical thinking among children. Is this challenge similar in kindergarten? Identifying and investigating such tasks of teaching might provide a common foundation for further conceptualizations of knowledge needed for teaching mathematics (Hoover, Mosvold, & Fauskanger, 2014), and this paper represents an attempt to further investigate the task of asking productive mathematical questions in a Norwegian kindergarten context. We approach the following research question: How can the task of asking productive mathematical questions be manifested in a Norwegian kindergarten context?

Based on studies in the US, Ginsburg and Amit (2008) argued that teaching mathematics in kindergarten is similar to teaching mathematics in school. Other studies suggest that the work of teaching mathematics differs across kindergarten contexts (Mosvold, Bjuland, Fauskanger, & Jakobsen, 2011). Further investigations of the work of teaching mathematics are thus needed, both in order to develop more comprehensive theories of teaching mathematics in kindergarten and to learn more about similarities and differences in the work of teaching mathematics in different kindergarten contexts. In our attempt to approach this challenge, we focus on the challenge of posing productive mathematical questions. This is arguably a central task of teaching mathematics – also in the kindergarten context – and we aim at contributing to the further unpacking of this task. We draw upon exemplary data from a Norwegian kindergarten context, where a kindergarten teacher interacts with six children in an activity involving Lego play. The activity is analyzed with a focus on tasks of teaching as conceptualized by Ball and colleagues (2008). In the following section, we present some trends from research on teaching mathematics in kindergarten as well as previous research and theories related to asking questions.

THEORETICAL BACKGROUND

Traditionally, research on early years mathematics has had a strong emphasis on children and their learning and understanding of mathematics. In the last decades, however, research on the early childhood mathematics teacher has flourished. Some studies focus on the knowledge and beliefs of the teachers (e.g., Schuler et al., 2013), whereas other studies investigate the actual work of teaching mathematics in a kindergarten context (e.g., Carlsen, 2013). In mathematics education, a large amount of research has focused on the knowledge needed for – or used when – teaching mathematics (see e.g., Rowland & Ruthven, 2011). Relatively speaking, much research has focused on mathematics teachers in school; far less research has focused on teaching mathematics in kindergarten. The study by Ginsburg and Amit (2008) represents one of few examples of the latter, and these researchers argue that teaching mathematics in kindergarten is mostly similar to teaching mathematics in school.

When investigating mathematics teaching, there are different possible approaches (for an overview, see Thames, 2009). One possibility is to identify and describe issues regarding the mathematical content of what is being taught; another possibility is to identify the work of teaching that is distinctively mathematical. Our study, although related to both of these two, represents a somewhat different approach in that we analyze the work of teaching in an attempt to identify the mathematical demands. In doing this, however, we also investigate the nature of the mathematical tasks of teaching that are involved in the work of teaching – in particular related to asking productive mathematical questions.

A goal with mathematics teaching at all levels is that children (or adults) learn to think mathematically; some describe this as a process of mathematizing. In order to reach this a goal, an environment needs to be created where conjectures can be put forward and discussed without fear of being ridiculed, and children need to be engaged in such mathematical discussions (Lampert, 1990). When creating this kind of environment, the kinds of questions mathematics teachers ask are of importance. By asking the right mathematical questions, the teacher can create a supportive atmosphere in which the children further develop their mathematical thinking and start thinking like mathematicians (Mason, 2000). In a mathematics classroom, teachers ask different kinds of questions. Some questions are open and aimed at stimulating further inquiry, whereas other questions are more closed – oftentimes serving as control questions (cf., Carlsen, Erfjord, & Hundeland, 2010). There also seem to be cultural differences in the questions mathematics teachers ask in classrooms. In their study of questions asked in 1st grade mathematics classrooms in Japan, Taiwan and the US, Perry, VanderStoep and Yu (1993) found that teachers in the Asian countries asked more questions about problem solving strategies and conceptual knowledge than their colleagues in the US. Subsequent studies seem to confirm these findings (e.g., Hiebert et al., 2003), and international assessments like TIMSS and PISA show that children from these Asian countries outperform children from most countries in the Western world.

In a kindergarten context, Carlsen and colleagues (2010) found that the kindergarten teachers' frequent use of questions enabled children's participation in the learning activities. They did, however, also find that kindergarten teachers often asked questions that were not true questions. This coincides with a more recent study where Carlsen (2013) found that a kindergarten teacher mainly used structuring questions in her orchestration of a mathematical activity involving the telling of a fairy tale. From these studies, it can be argued that the questions asked by kindergarten teachers oftentimes serve as a means of reaching joint attention, and numerous studies in psychology contend that joint attention is of vital importance in children's learning. When Bruner and his colleagues started investigating this issue in the late 1950's, they mainly analyzed newborn babies or young children with a focus on their eye gaze (Bruner, 1995). A narrow understanding of the concept of "joint attention" would thus simply be whether or not an individual is looking where someone else is looking (Sigman & Kasari, 1995). A broader definition includes responsive and initiating behaviors as well as facial expressions and gestures. Baldwin (1995) defines joint attention with the mutual awareness in mental focus that two or more individuals have when looking at the same thing. A key issue then is that the mutual awareness must be in the mental focus – not only that two people stare at the same thing. Sigman and Kasari (1995) argue: "joint attention must involve an integration of information processing and emotional responsiveness" (p. 190). Studies like that of Tomasello and Farrar (1986) show that joint attention has a cen-

tral role in children's early language learning, but it is arguably important in early years mathematics learning as well.

Given that joint attention has a central role in young children's learning, a natural follow-up is to ask about the role of the adult in this. The process of reaching joint attention has been referred to as a tutorial process (Wood, Bruner, & Ross, 1976), where an adult or "expert" helps somebody who is less skillful. A crucial feature of such interactions, it can be argued, is the adult's ability to make joint attention (Bruner, 1983). In earlier works, Bruner focused on joint attention in relation to language development and learning (Bruner, 1983); later he described the role of the adult in terms of scaffolding (Bruner, 1995). In this study, we follow Baldwin's (1995) understanding of joint attention in that it includes a mutual awareness of mental focus. We suggest that the questions posed by kindergarten teachers – and thus also the task of asking productive mathematical questions – could be understood in terms of reaching joint attention. When we investigate the task of asking productive mathematical questions, we therefore suggest that the process of joint attention needs to be an integrated aspect.

METHODS

In order to investigate kindergarten teachers' mathematical questions, we asked a kindergarten teacher for permission to video record an everyday activity that involved something he associated with mathematics. The kindergarten teacher decided to organize a situation of Lego play with six children from his kindergarten class (his class included a total of 18 children aged 3–6 years). The kindergarten teacher and the children sat around a table that was filled with classical Lego bricks of different shapes and colors; the children could play freely with the bricks. The first author recorded the session as illustrated in Figure 1.

The kindergarten teacher had 17 years of experience, and he finished his education before mathematics was introduced as a required course in Norwegian kindergarten teacher education. The six participating children were between 3,11 and 5,4 years of age (the decimals represent months). We refer to the children by fictitious names; the kindergarten teacher is referred to as "Teacher". The Lego play activity lasted 22 minutes, and the researcher had the role of a passive observer. Neither the kindergarten teacher nor the children seemed to take much notice of the researcher or his camera. Afterwards, the video was transcribed verbatim, and the transcripts were coded by the use of conventional content analysis (Berg & Lune, 2012), where the unit of analysis was the kindergarten teacher's questions.

RESULTS AND DISCUSSION

The activity begins with the kindergarten teacher gathering the children around the table to introduce the Lego activity. He explains that they are going to play with the Lego bricks, and someone (the first author) is going to record the activity in order to learn more about what they are doing. The children eagerly start playing with the Lego bricks. Shortly after, a first question is posed to the teacher:

8. Kaja: (holds up one red and one blue brick and turns to the teacher) Have you found one of those or one of those?

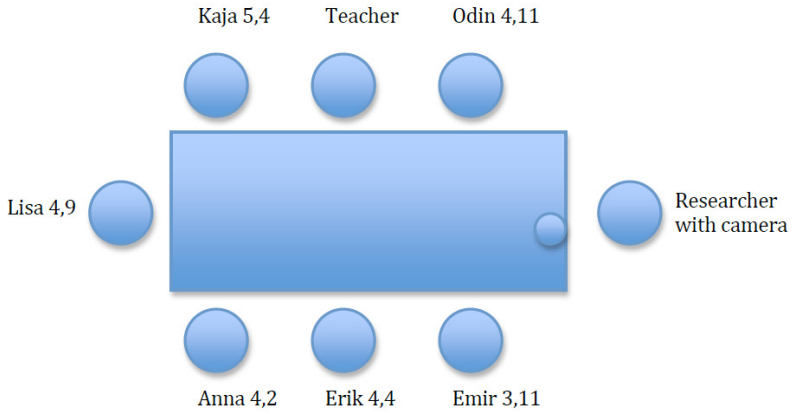

Figure 1: The placement of the participants in the Lego-activity

9. Teacher: One of those? (scratches his chin) What do you mean with "one of those"?
10. Kaja: Two of those (holds up the bricks again)
11. Teacher: Yes... (Odin hands the teacher a red brick, and the teacher holds it up) What does it look like?
12. Kaja: Table?
13. Teacher: Do you see what they look like?
14. Kaja: Triangle!
15. Teacher: Triangle, yes. But look, I'll show you something funny. If you ... put them together (puts two blue and one red brick together on the board). If we had one more of those, what would it have become then?
16. Kaja: Taaaaaable ... round [shape] (looks at the teacher)
17. Teacher: A round shape, or simply a circle.

Among the mathematical tasks of teaching, Ball and colleagues (2008) listed "responding to students' why-questions". The question posed by Kaja in the beginning (8) is not such a why-question, but we notice how the teacher uses the question as a starting point for posing another question (9) in order to direct the children's attention towards the mathematical concept that can be used to describe the bricks. His question can be described as an invitation to use more precise concepts than "one of those", and this could serve as an example of a productive mathematical question in a kindergarten context. When Kaja first responds by holding up the bricks instead of providing a more precise concept (10), the teacher asks her to describe what the brick looks like (11). As a response to this question, Kaja eventually says: "triangle" (14). The teacher confirms her answer and repeats what she said (15) – although the brick is not a triangle but a triangle-like shape with one round edge.

Even though there are six children in the activity, Kaja (5,4) is the only active participant in this part of the discussion; Odin (4,11) also contributes by finding the desired bricks, but his contribution is non-verbal. When Kaja responds to the teacher's follow-up question by saying that it would become a table (16), he introduces the concepts of "round shape" and "circle" as alternatives (17). This can be seen as an example of the task of using appropriate mathematical language, and we can also say that the dialogue in this excerpt indirectly contains the challenge of "choosing and developing useable definitions" (Ball et al., 2008, p. 400). Instead of using a more correct mathematical definition for a triangle – where the sides have to be straight line segments – the teacher decides to accept Kaja's description of the shape as a "triangle" (15).

Later in the discussion, when Kaja finished building her circle shape with two red and two blue bricks, Lisa (4,9) asks for the same kind of bricks:

41. Lisa: I need one more red and two more blue.
42. Teacher: Triangles like that? (points towards Lisa's board) But isn't it a little bit strange that ... they. How many are there here? (picks up Kaja's board to show)
43. Kaja: One, two, three, four (counts out loud while pointing)
44. Teacher: Four triangles. But isn't it a little bit strange that those triangles make a ...
45. Lisa: Circle
46. Kaja: Circle
47. Teacher: How is it that a circle can become a triangle? Or, triangles become a circle?
48. Kaja: We just ... (points to the board)
49. Lisa: The triangles have such round there.
50. Teacher: That edge ... side is a little bit round, yes.

We notice that the teacher follows up on Lisa's request by directing her attention using the word "triangle" and pointing (42) to reach joint attention. The teacher continues to ask questions while pointing at Kaja's board – which is finished – focusing on the quantity. We assume that the teacher is aware that the children know how to count, and his question about how many bricks can then be seen as an invitation to count. Ball and colleagues (2008) identified a similar task as: "connecting a topic being taught to topics from prior or future years". Kaja counts and finds out that four "triangles" are needed in order to build a circle (43).

Within a sociocultural kindergarten tradition (like the Norwegian kindergarten), the kindergarten teacher's knowledge and ability to ask questions are significant. Carlsen, Erfjord and Hundeland (2010) argued that the teacher's questioning is of vital importance for children's learning, and they found that almost half of the questions posed were open questions where the children are encouraged to present a solution themselves. The teacher in our study also

poses a lot of questions, and through this questioning we gain knowledge of the children's development of number concept as well as their knowledge of shapes.

The teacher continues to pose questions, and his next question can be seen as an effort to re-focus the children's attention towards the seeming paradox that four "triangles" can make a circle (44). Both Kaja and Lisa are now referring to the shape as a circle (45 and 46) rather than table or round shape – indicating that they have adopted the kindergarten teacher's use of a more precise concept. When asked about how the "triangles" can become a circle (47), they do not respond verbally but points to the board instead (48). This situation could have been used to reach a more precise definition of a triangle, but the kindergarten teacher does not go in that direction. Instead, this part of the dialogue ends by the teacher confirming that the "triangles" have a round edge (50). Instead of stating that these particular "triangles" are in fact circular sectors – or quadrants – and four of them put together thus make a circle, the teacher leads the children into a mutual reflection about the shape of the bricks and how they can be used to create a new compound shape (circle). This also illustrates another task of teaching related to selecting representations for particular purposes – in this case using the Lego bricks to discuss shapes. Presentation and discussion of non-examples like this can be important in children's formation of solid concept images that go beyond the prototypical triangle (Levenson, Tirosh, & Tsamir, 2011). In this case, the children identified these particular Lego bricks as triangles although they are in fact non-examples. The children rely on visual reasoning, and the kindergarten teacher could have used the situation as a starting point for discussing more examples and non-examples and helped the children towards where they notice differences between shapes (ibid). From these examples, we have seen that the task of asking productive mathematical questions includes a number of intertwined challenges – or tasks – for the teacher.

CONCLUDING DISCUSSION

Ball, Thames and Phelps (2008) identified "asking productive mathematical questions" as one of the recurrent mathematical tasks of teaching. We argue that asking productive mathematical questions is indeed a relevant task of teaching mathematics also in a kindergarten context. From our analysis of this play situation in a Norwegian kindergarten, however, we suggest that the task of asking productive mathematical questions is highly complex. In this concluding discussion, we point at three issues that add to the complexity. First, there are different types of questions that might be asked to facilitate children's further reflection and exploration of mathematics. Second, there are different possible purposes that underlie the asking of questions. Third, the task of asking productive mathematical questions is often intertwined with other tasks of teaching, and the kindergarten teacher needs to address these tasks instantly as they appear.

In their analysis of kindergarten teachers' questions, Carlsen and colleagues (2010) found that the teachers posed open questions, asked for arguments, invited to problem solving, re-phrased children's utterances, and made conclusions. These types of questions could also be identified in the play situation analyzed here. When we have focused specifically on the mathematical demands, however, we can observe the following aspects in the questions:

- encourage use of more precise mathematical language
- confirm use of more precise mathematical language
- describe what children observe with their own words
- mathematical reflection about a more compound problem
- mentally complete an observed pattern or unfinished shape
- encourage reflection about observed patterns and connections
- invite to count

This list represents an attempt to decompose the task of asking productive mathematical questions, but it can also be seen as an attempt to identify the teacher's underlying purpose in asking these types of questions. It is difficult, however, to make conclusions about purpose from observations of activities and conversations only. In this study, our focus was

on tasks of teaching that could be observed from discussions between the kindergarten teacher and the children. We have thus analyzed the observed work of teaching without bringing in the voice of the teacher concerning his intentions. Introducing the teacher's voice from a follow-up interview could have been interesting, however, but that would be beyond the scope of this paper – where our focus was strictly on unpacking the observed tasks of teaching. Interviews with the kindergarten teacher could, however, provide further information about teachers' beliefs and knowledge that would also be relevant to investigate.

Although we have investigated the data with a focus on the task of asking productive mathematical questions in particular, we have also seen that several other tasks of teaching are oftentimes intertwined in this task. When asking children to use more precise mathematical language, for instance, the kindergarten teacher could also face the task of choosing and developing usable definitions. Being faced with the apparent paradox of how four "triangles" could make a circle, the kindergarten teacher would have to make decisions about whether or not he should go into a discussion about the proper definition of a triangle. If he were to go into such a discussion with these children, however, he might also have had to deal with the concept of polygons and straight line segments, and this would probably be beyond the topics he intended to teach. On the other hand, avoiding the more precise definition at this stage could lead to misconceptions that would have to be dealt with later on, and this illustrates the complexities involved in the work of teaching mathematics in kindergarten.

An additional challenge that can be seen in several parts of this dialogue is related to joint attention. At all stages, but perhaps in particular with smaller children, the teacher is faced with the challenge of reaching joint attention. Several questions, comments and even gestures made by the kindergarten teacher can be seen as acts of reaching joint attention, and we argue that the issue of joint attention is embedded in all tasks of teaching mathematics in kindergarten.

In this paper, we have tried to contribute to the unpacking of one particular task of teaching mathematics: asking productive mathematical questions. We have seen that this task is complex, as we have already discussed, and we believe that it is also context specific. Asking productive mathematical questions to kindergarten children probably involves other kinds of challenges than asking such questions to children in lower secondary school, but the task of teaching is still relevant at all levels. We thus support the argument made by Hoover and colleagues (in press) that mathematical tasks of teaching "can serve as a common foundation for conceptualizing and measuring mathematical knowledge for teaching" (p. 101) – also in kindergarten – but we suggest that further studies are needed in order to investigate and unpack the tasks of teaching mathematics in different contexts and at different levels.

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