E-Dynamic.Space: A 21 ST century tool to stage-manage and build experience in the field of the history of mathematics and its teaching

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This study aims to answer the question of how can the history of mathematics resort to a digital tool – E-Dynamic.Space – designed by teacher-students and intended to serve as a workbench not only to create supportive knowledge from historical material, which has proved to support the understanding of mathematics but, also to orchestrate both, their actual learning of the tangent line problem and their future mathematics teaching experience. It therefore explores aspects for the design of the teaching activities, and it analyses the 'whys' and 'hows' of including the historical dimension into the teaching experience. It is part of a bigger research project that looks at how can teacher-students favour from a historical informed pedagogy of mathematics that uses a personalised learning environment as a means to learn.

Keywords: Digital tool, PLE, orchestration, tangent line problem, teacher education.

INTRODUCTION

Digital technologies are the landmarks of the 21st century, ubiquitous and bearers of social identity for the majority, especially young people (Boyd, 2014). In what way can digital technologies support 'history in mathematics education', which is understood here as the learning of mathematics supported by the integration of elements from the history of mathematics. Researchers of mathematics education call for more research of digital technology (e.g., Hoyles et al., 2010; Trouche & Drijvers, 2014), and this paper extends their call to the integration of supportive knowledge created by teachers (Kuhn, submitted at the 7th ESU in History and Epistemology of Mathematics) using web-based tools, which have their own possibilities and difficulties. In this paper, digital technologies are understood in a broader sense, not only as mathematical software but also as web-based tools and social media, which brought together by the end user in a flexible digital environment will constitute what I will call from now on personalised learning environment (PLE) (Buchem, 2010; Kuhn, 2014a[1]).

Teachers have to keep up to date with young students' mind-set and expectations, and the advancement of digital technologies. On the other hand teacher-students need support in being prepared in a sensible manner for their job in the near future and to move confidently in this new ecology of digital resources (Luckin et al., 2012). There is evidence that shows how designing and developing a PLE will improve the digital skills of the end-user (Wild et al., 2008), teacher-students in this particular case.

How can teachers explore the affordances[2] of digital tools, take advantage and build experience in the digital world and in the field of history and mathematics, integrating them for the learning of mathematics and further teaching? Looking for possible answers I propose a PLE, E-Dynamic.Space (Kuhn, 2014a) as a 21st century self-management tool, designed and populated with new content created by teacher-students to support them in the design and organisation of the learning experience.

My proposal aims to address not only how to stage-manage the learning of mathematics but also to explore how teacher-students can create supportive material from historical sources, which has proved to support the understanding of mathematics (Kuhn, submitted op. cit.), using the E-Dynamic.Space as a tentative tool for constructing meaning or in words of Noss & Hoyles (1996), webbing[3] in the process of grasping and understanding the tangent line. The tangent line has been chosen as a starting point that will set the ground of a number of concepts to develop (as a mid term goal of the project) in order to craft a more unified and connected way of teaching the background concepts of calculus for GCSE and A-levels in the UK.

In a first stage of the project I will focus on the design of some of the teaching activities I propose for teacher-students and in a second stage, not addressed in this paper, I will look at how they can transfer these skills to their classroom practice and improve the learning experience of their pupils. Empirical evidence indicates that following the work of teacher-students during "a time long enough to be able to catch real changes (a) during a program, (b) immediately after the program, and (c) one or more years later, can assist in providing valid feedback mechanisms for professional development programs" (Trouche et al., 2013).

SOME THEORETICAL ASPECTS FOR THE DESIGN

Troublesome Knowledge

The introduction of analytic geometry revived the tangent line problem in early 17th century. Descartes in his 1637 work La Géométrie described the problem of finding a tangent line to a given curve at a specific point as:

> (...) the most useful and general problem in geometry that I know, but even that I have ever desired to know. (Smith & Latham, 1925, p. 95)

Reading this sentence in combination with my interest in the calculus as a rich topic, both historically and conceptually, made me wonder why would an intellectual of the calibre of Descartes find this problem so useful and worth knowing. Although I was motivated and thrilled to know more I encountered difficulties while finding my way into Descartes' ideas. I found myself confronted with some trouble, or maybe with troublesome knowledge? But what exactly is troublesome knowledge and what it has to do with the history of mathematics in mathematics education? The notion derives from a research project in the UK looking to identify key factors leading to high quality learning environments in higher education, very much in line with the aim of my own research. The idea is associated with threshold concepts, conceptual gateways that have the potential to open up new conceptual spaces transforming the way learners understand the subject matter (Meyer & Land, 2005). Threshold concepts, although usually attached to particular concepts, sometimes they are not necessary concepts in any rigorous sense but different ways of thinking and practicing with a threshold-like nature, all of them providing entrance in one sense or another to a new or different conceptual landscape (Meyer & Land, op. cit.). Transformative ideas, and it is in this sense that I am using the term.

These ways of thinking and practicing, often lead to what Perkins (2006) describes as *troublesome knowledge*, knowledge that is conceptually complex, alien or counter intuitive, thus challenging students' beliefs and intuitive knowledge but at the same time, developmental productive. This is in line with Barbin's idea within mathematics education, of *depaysement* or reorientation, challenging student's perceptions, making the familiar seems unfamiliar. History shows also how mathematics is a human understanding, a history of human beings disabling or extending established ideas, allowing the learner to see mathematics as much more than disconnected algorithms or discrete chapters, integrating the subject in a sociocultural context.

Why and how to use this historical knowledge in mathematics education?

The previous section introduced, in a general way, some of the reasons for using history of mathematics in teacher education. Adding to this Jankvist (2009) answers this question in a more focused and didactical oriented way connecting it with content knowledge, suggesting that history can be used as a goal or as a tool. In particular he refers to a cognitive tool for the learner (teacher-student in this particular case). In this latter sense he implies the idea of epistemological obstacles (Jankvist, 2013). Brousseau (1997) highlights in this regard that knowledge exists and it makes sense only because it represents an optimal solution in a system of constraints. For him, history can be illuminating in finding those systems of constraints. Sierpinska (1994) suggests: "epistemological obstacles are not obstacles to right or correct understanding: they are obstacles to some change in the frame of mind." (p. 121)

Dimensions of knowledge in teacher training that can profit from the history of mathematics

One of the aims of our community for the history and pedagogy of mathematics is to find ways in which teacher-students can profit from the history of mathematics for their learning/teaching experience. In each profession there are core skills and knowledge to be mastered. In mathematics education, Ball and colleagues (2008) have developed a theoretical framework, Mathematics Knowledge for Teaching (MKT), proposing the kind of knowledge demanded by the teaching profession. This framework has been explored recently by Clark (in press), cited by Jankvist and colleagues (2012). She contextualised it in the history of mathematics exploring how the history can add to teachers' MKT. In this work I will use three of the six dimensions of the model: knowledge of content and curricula (KCC); knowledge of content and students (KCS), and horizon content knowledge (HCK), in order to see how teacher-students' knowledge can potentially profit from and be enhanced by the history of mathematics. This choice responds in part to a call that Jankvist (op. cit.) has made to address the absence of clearer links to general mathematics education research frameworks. This theoretical construct –the MKT- seems to have productive implications for teacher education (Jankvist, op. cit.).

Epistemological obstacles and conceptual development, and its association with the Mathematics Knowledge for Teachers

Tracing the historical development of a particular concept, following Brousseau (op. cit.) is a way to understand the constraints of each time, hence to understand some of the epistemological obstacles involved in the development of an idea. Connecting epistemological obstacles with the didactical situation of teacher-students is possible through the idea of conceptual development, which has been researched for didactical purposes by different authors (e.g., Vosniadou, 1994). The general consent is that for conceptual change to happen there must be, in the student, a cognitive conflict or a 'stuck place' in words of Meyer & Land (2005), a difficult stage in the conceptual development as it confronts them with different epistemological obstacles (Brousseau, 1997) blocking any transformation in the cognitive realm. Teachers are responsible to identify the sources of those obstacles and free them up making the change possible. In this regards, teachers ought to develop knowledge of content and students (KCS) (Ball et al., 2008).

This 'stuck place' is similar to what happens to the collective culture of mathematicians throughout the development of an idea. Teachers can look closely at these epistemological obstacles in order to find inspiration and knowledge to identify possible sources of obstacles in their students. This kind of understanding can also improve teachers' knowledge of content and curriculum (KCC) allowing them to make a historical informed decision in relation to the breadth and depth they should teach in the different key stages. All of the above seems to add to a wider kind of knowledge, one that goes beyond the basic knowledge teachers need to deploy in class. Following Ball & Bass (2009), it is called horizon content knowledge (HCK), and they describe it as " (...) an awareness – more as an experienced and appreciative tourist than as a tour guide – of the large mathematical landscape in which the present experience and instruction is situated" (p. 6). This kind of knowledge "confers a comprehensible sense of the larger significance of what may be only partially revealed in the mathematics of the moment." (p. 6). There is evidence (Mota, 2008; van Maanen, 2009) that this knowledge will profit from the history giving teacher-students a wider breath of the mathematical cultural context of a particular idea to be taught.

Having explored the whys of using history in teacher education and looking at how the mathematics knowledge for teaching can be enhanced by the use of historical material, let us look at how can this material be integrated in the teaching experience. Taking into account the varied background of Bath Spa University PGCE students (PGCE responds to Post Graduate Certificate in Education and it is a one year program for students with different backgrounds that want to become teachers), I decided to follow Tzanakis and Arcavi's (2000) idea of *historical packages* in which a mathematical topic (in my case the tangent line problem) from the curriculum is taught by means of historical materials in a relatively short period of time; similar to Jankvist's approach with historical modules.

How can a teacher-student get involved with the history of mathematics in order to gain a deeper understanding of the epistemological development of the concept and also take advantage of the affordances of the PLE and its available tools? One way to do this is through the activities proposed below for which the didactical intention is underpinned by the idea of webbing described previously. There is also a wider mathematical aim and it is to explore in depth the development of the tangent line problem in order to gain a deeper understanding and a wider vision, in epistemological terms about the historical process of the definition of the derivative in terms of the limit; for that the tangent line is key. In words of Whiteside (1961): "It will be illuminating therefore, to discuss the particular methods invented to resolve the tangent-problem, and this will yield a truer perspective on the *elegant general treatments which were later abstracted from the particularised methods of the mid-century* [emphasis added, p. 348]." History shows that the starting point of that definition was neither limits nor the differentials or fluxions. It has been a process of successive abstraction (Lehmann, cited in Swetz et al., 1995), which is what we aim to trace with this module.

THE HISTORICAL TOUR: FROM EUCLID TO FERMAT

In this section I will describe briefly what teacher-students will explore during the sessions. The online sheets and the web-based tools are allocated in the PLE, which they will further populate with their own creations. The didactical intention is that the learner generates new supportive knowledge as a product of webbing while exploring the historical material, making sense of new chapters of the tangent line's history. Teacher-students will course from Euclid to Fermat and reflect around the systems of constraints of each period identifying the epistemological obstacles and the change in the collective frame of mind. In doing so they will become the appreciative tourist of the larger mathematical landscape as they advance in their epistemological tour.

We need to bear in mind that there is this unavoidable risk – clearly explained by Fried (2001) – of doing 'Whig history'. In order to address this issue (though not sure to completely avoid it) an initial reading of his paper (2001) is assigned to the group.

Time and allocation of session is to be determined

As an integrative and final activity for webbing the learning of the topic and also intended to develop the epistemological understanding of the concept, students will create an interactive timeline with at least two of the resources created by them through out the module. They need to add the group reflections where pertinent and illustrative, as well as the relevant comments posted in Padlet. Highlighting new frames of mind is important in this task. The interested reader is invited to follow the link[9] to explore the PLE with man examples of sheets and resources, as well as a time line crafted by the author to explore the affordances of the tool.

DATA COLLECTION AND THE PROCESS OF WEBBING THE UNDERSTANDING OF THE TANGENT LINE PROBLEM

As suggested by Barbin and colleagues cited in Fauvel & van Maanen (2000), we can evaluate the effectiveness of introducing a historical dimension into teacher education through an examination of each of the processes involved in the development of understanding, namely, the change in how teachers perceive and understand mathematics which generally is reflected in the way they subsequently will teach, and finally in the understanding and perception of their pupils about mathematics. None of those processes can be captured in a quantitative approach, instead a qualitative and holistic method is much more desirable for understanding more in depth how to best integrate historical material into the teacher experience. Therefore qualitative data will be gathered (with the proper software, e.g., Camtasia) in their online public and private spaces. There is also a reflective logbook with didactical prompts (still under development) for each student to document their learning; the process of webbing the tangent line problem making sense of the different frame of mind and the historical development of the concept studied. The prompts will trigger in the student the cognitive processes that will help them to describe their main struggle when trying to elaborate the resources. In particular the timeline is considered a rich intellectual artefact with the potential to uncover partial understandings of the student in relation to the epistemological advancement of the concept. What resources they choose, what they consider to be an illustrating example and how they justify it will reflect students' process of constructing meaning throughout the task. An important aspect of the learning experience will be the idea of extending the web of ideas and intellectual resources (at the beginning of the journey) and re-structuring it as a result of the connections made for the learner to be able to find and construct meaning through the sequence of activities he/she is doing including discussion and reflection.

Author	Resources	Task + Question + Reflection
Euclid	Book III, def. 2 and prop. XVI Online version of Oliver Byrne[4]	Go through the definition and work out the proposition in your group, post your work in padlet[5] for a common discussion. Have a look at other posts and comment on at least one+reflect
Apollonius	a. Module of the MAA: 'Tangent line then and now'	Read the extract about Apollonius method. Interact and explore the GeoGebra example and discuss with your peers your thoughts, difficulties and any 'aha' moment.
	b. Treatise of conic section. Heath translation 1896. Online[6]	Go through proposition I.33 and discuss, try to make sense of it with your peers. Find the analogue elements with the MAA method and document the process in your logbook. Pay special attention to any difficulty in understanding any of the parts, documenting it for further thinking in the group discussion.
	c. Working with online sheets in GeoGebra[7]	Work in pairs and interact with the sheets for finding the tangent line to a parabola. Produce your own example in GeoGebra, record the steps in the sheet and post it to GeoGebra Tube. Do one of the sheets posted by your peers, comment your experiences in Padlet (difficulties + ahas + findings + remarks). Reflect on the system of constraints you think could be present in that period and what implications do they have in the method you just did. Read and comment on one post in the wall
Descartes	a. Look at the video[8] by Jeremy Gray about the history of the calculus. b. The History of mathematics (Fauvel, J & Gray, J.) Section 11.A10	Watch with particular care the section where the method of Descartes to find the tangent line is explained. Read section 11.A9 to complement. Make notes for a discussion session with the group about the steps of the procedure. Try it your self with a simple curve ($y=x^2$) and document the process. Work in a small group for a richer and more reflective discussion.
	Online GeoGebra sheets in the PLE	Go to GeoGebra and do the sheet with Descartes' method. Take notes about things that were important for you to accomplish the task, key ideas. Think about your own difficulties along the exercise and write them down for a common discussion. Try to write about the new mathematical features introduced in his method and compare it to Apollonius' one. Reflect about the system of constraints in Descartes' time and think about the new frame of mind introduced then. Think about the implications and epistemological difficulties of a more general method comparing it to the Greeks (section 11.A10)
Fermat	The history of mathematics (Fauvel, J & Gray, J). A copy of the relevant text of module 9 of the Open University course: Topics in the history of mathematics	Read section 11.C – 11.C1b. Tinker with the method. Try to create your own example working with a curve that you feel comfortable with (use pencil and paper). Make notes about your process and document any difficulties. Think about the *adequate* method he used, trace the history of the term and give meaning to it in that context. What was the problem then? Can you see why Fermat could work with a wider range of curves? Go to GeoGebra, work through the sheet. Once you have understood the method create your own sheet with its animation and uploaded it to GeoGebra Tube. Try out one of the sheets of your peers and comment on his/her work. Reflect on the steps taken by your peer.

Table 1: Sources and questions in relation to the tangent line problem

A session will be dedicated to reflective writing and how to do it in a way it can enhance their own learning process. The three dimensions, specified above, of the MKT framework will be explained in detail and their reflections will be stated in terms that make reference to these dimensions so consistent evidence can be collected (still an idea under development). The prompts given in the activities are focused in the mathematical features that have been shifting from time to time and are intended to bring students to reflect on how those changes have transformed the tangent line problem into what we know today giving them a wider background knowledge or in words of Ball & Bass (op. cit.) enhancing their HCK. Important ideas to grasp along the learning experience are the optimal solutions in a system of constraints stated above by Brousseau and the change in the mind frame argued by Sierpinska.

The module has not been tried out yet therefore a real and fruitful discussion will be part of a next piece of research, where the data will be collected and analysed, and hopefully the analysis will shed some light to the rich discussion in relation to the benefits of the integration of history for the mathematics education community.

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ENDNOTES

1. Available at: http://portal.sinteza.singidunum.ac.rs/paper/114

2. Affordances are in this context related to the digital world and it refers to the potentialities and constraints of different modes that digital tools allow. What is possible to represent with the resources of a mode and what is not.

3. Defined by Noss and Hoyles (1996) as the fundamental motor for the construction of meaning.

4. The first six books of The Elements of Euclid, described as one of the oddest and most beautiful books of the 19th century. Available at: http://publicdomainreview.org/collections/the-first-six-books-of-the-elements-of-euclid-1847

5. Padlet is a web-based tool. It affords to have a collaborative discussion and upload files to it (https://padlet.com/)

6. https://archive.org/stream/treatiseonconics00apolrich#page/n9/mode/2up

7. http://hom.wikidot.com/calculus-1 (by Gabriela Sanchis, under Creative Commons Attribution ShareAlike 3.0 licence)

8. https://www.youtube.com/watch?v=OTMkCLtflHY

9. http://www.symbaloo.com/home/mix/13ePQJ81NS