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Was Euclid in Iceland when he was supposed to go?

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In a seminar on new thinking in school mathematics, held in Royaumont, France, in 1959, one of the main speakers, Jean Dieudonné, summarized the new school-mathematics programme he had in mind in the sentence: Down with Euclid. The purpose of the article is to analyse the context in which this quote was expressed and connect it to geometry teaching in Iceland where Euclidean geometry instruction seldom had a firm ground. Euclidean geometry in an amended version gained new interest in Iceland by the introduction of the New Math in the 1960s.

Keywords: Royaumont seminar, Dieudonné, Euclidean geometry, New Math.

INTRODUCTION

One of the most renowned phrases connected with the Royaumont seminar in November 1959, where the reform movement, entitled New Math, was launched world-wide, was 'À bas Euclide' (Down with Euclid), attributed to Jean Dieudonné, who belonged to the Bourbaki-group. This seminar led to substantial alterations in mathematics teaching and geometry teaching in particular. In the following, some consequences of this reform movement will be considered with special respect to geometry teaching in Icelandic schools. The research questions are:

> In what context was the above quote expressed?

> What context did the New-Math geometry meet in Iceland?

The research method is historical: i.e., a careful analysis of a range of documents. The history is traced by referring to scholars' published work, legislation, regulations, reports, articles and mathematics textbooks, and the remembrance of the author of this article. Textbooks were analysed, their forewords as well as their mathematical content, and information about their lifetime was sought in official reports.

The importance of this study is contained in an analysis of some important seeds for development of school mathematics, sowed at Royaumont more than half century ago, but also in an analysis of an example of a dissemination process of mathematical ideas from scientifically established 'metropolis' of the time to a not yet scientifically productive one in the 'periphery' (Alpaslan, Schubring, & Günergun, 2015).

BACKGROUND

The New Math movement and the Royaumont seminar

In the aftermath of WWII, reforms of mathematics and science teaching were considered in many countries. A notable arena was the Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques, CIEAEM, the International Commission for the Study and Improvement of Mathematics Teaching, founded in 1950. Among its members were the Swiss psychologist Jean Piaget, mathematicians Jean Dieudonné and Gustave Choquet from France, and some outstanding secondary school teachers. The main concern of the CIEAEM was a growing attention to the student and the process of teaching, the relevance of psychology in mathematics education, the key role of concrete materials and active pedagogy, and Piaget's research of the relation between mental and mathematical structures as introduced by the French Bourbaki group of mathematicians, including Dieudonné, called *Mathématique Moderne*, Modern Mathematics (Furinghetti, Menghini, Arzarello, & Giacardi, 2008).

The actions of the CIEAEM, containing important germs of didactic research, were paralleled by the

New Math movement in the United States. World War II had focused national attention on the growing need for trained personnel to serve an emerging technological society (Osborne & Crosswhite, 1970), and several important school mathematics projects were launched. The actions by CIEAEM and the New Math movement had roots in common with the Bourbaki School: set theory, functions, relations and logic were to be placed in the new curricula, supported by the methodology of discovery. The reform movements gathered at a seminar on school mathematics reform in November 1959, held by OEEC, the Organisation for European Economic Co-operation, at Royaumont, France. The member countries were invited to send three delegates each, and the seminar was attended by representatives from all the invited countries except Portugal, Spain and Iceland. A questionnaire was sent out before the seminar and replies were reported from most countries, also from Iceland (OEEC, 1961, pp. 7, 135–140, 213–219).

Down with Euclid!

Among the guest speakers at the Royaumont seminar was Jean Dieudonné from the CIEAEM. In his speech, reproduced in full in the seminar's report (OEEC, 1961, pp. 31–46), he described the diverse curriculum that first year students at university should master: on one hand to be familiar with a certain number of elementary techniques in which it takes a long time to achieve proficiency, and on the other hand be already fairly well trained in the use of logical deduction and have some idea of the axiomatic method (p. 32). In the universities, new developments in analysis had been incorporated in the curriculum. Under the new overcrowded curriculum, most students emerged with the haziest notions about it.

Easing this squeeze could only be done in one way: The curriculum of the secondary schools had to be reorganized to eliminate any undue waste of time. Some elements of calculus, vector algebra and a little analytic geometry had recently been introduced for the last two or three years of secondary school, while such topics had been relegated to a subordinate position, the centre of interest remained as before pure geometry taught more or less according to Euclid with a little algebra and number theory (pp. 33–34). In Dieudonné's opinion, the day of such patchwork was over. Much deeper reform was required, and if he were to summarize the whole programme he had in mind in one slogan, it would be: *Down with Euclid*! Recently, it had become possible to reorganize the Euclidean corpus placing it on simple and sound foundation – separating what is fundamental from a chaotic heap of results with no significance except as scattered relics of clumsy methods or an obsolete approach. The whole course could actually be taught in three hours: one of them occupied with the description of the axiomatic system, one by its useful consequences and possibly a third one by a few mildly interesting exercises (p. 35).

Actually, the whole system could easily be replaced by an axiomatic system producing two-dimensional linear algebra. The present process of teaching geometry was fantastically laborious, no complete system of axioms was ever stated and it was completely impossible to check the correctness of any proof. Dieudonné suggested the following list to take the place of Euclidean geometry (pp. 37–38):

a) Matrices and determinants of order 2 and 3.

b) Elementary calculus (functions of *one* variable).

c) Construction of the graph of a function and of a curve given in parametric form (using derivatives).

d) Elementary properties of complex numbers.

e) Polar coordinates.

Dieudonné's guiding principles were two: Firstly, that a mathematical theory could only be developed axiomatically in a fruitful way when a student has already acquired familiarity with the corresponding material, i.e. with constant appeal to intuition. The other principle was that when logical inference is introduced in some mathematical question, it should always be presented with absolute honesty without trying to hide gaps or flaws in the argument (p. 39).

In his outline of a modern curriculum, Dieudonné recommended to limit the teaching of mathematics up to the age of 14 to experimental work with algebra and plane geometry and to make no attempt at axiomatization. He referred to recent research and experimentation in educational circles, especially in Belgium, concerning the methods by which this teaching of geometry as a part of physics could be conducted. This development should be highly encouraged, provided it did *not* put the emphasis on such artificial playthings as triangles, but on basic notions such as symmetries, translations, compositions of transformations etc. (pp. 40–41).

The language and notations universally in use, such as $\in$ and $\Rightarrow$, should be introduced in these experimental mathematics as soon as possible, and objects should be called by their proper name like 'group' and 'equivalence relation' whenever such an object was naturally observed in some algebraic or geometric setting. This did not at all imply to develop in advance the abstract theory of those objects. The laws of arithmetic could also be developed, starting from the 'Peano axioms' (p. 41).

Dieudonné proposed detailed programme for age 14 with the idea of a graph of functions; age 15 when a statement of axioms of two-dimensional linear algebra should be given with both algebraic and geometric interpretation, by emphasis on the various linear transformations and the groups they form; age 16 with deeper study of the groups of plane geometry, and in particular the use of angles and of trigonometric functions; and age 17 with three dimensional geometry by use of matrices and determinants. The programme should lead up to and connect directly with the then present programme of the first years in the university (pp. 42–45).

GEOMETRY IN ICELAND

Earlier times

Iceland was a tributary of Denmark since the 14th century. Its population was 40.000–50.000 until the 19th century. There was no army and therefore no military academy and no need to teach geometry for that purpose. The sole Learned School adhered to Danish regulations of the Royal Directorate of the University and the Learned Schools. A certain number of graduates from the school had priority for grants at the University of Copenhagen, (Þorláksson, 2003, p. 382) until Iceland's sovereignty in 1918.

In 1822, Björn Gunnlaugsson (1788–1876), who had studied mathematics at the University of Copenhagen, came to work at the school, to stay there until 1862. In his inauguration speech, Gunnlaugsson emphasized the utility aspect of mathematics education. During summers of 1831–1843, he travelled around the country to make geodetic surveys as a basis for a scientifically drawn map which served as a basis for maps of Iceland into the 1900s. Gunnlaugsson taught the prescribed syllabus of arithmetic, algebra and geometry by textbooks that were stipulated for the learned schools by the Royal Directorate, but remarked in his reports of 1823 that he gave exercises in land-surveying in order to enhance the students' interest. Gunnlaugsson wrote his own textbook in geometry in Icelandic. The yearly cohort of learned-school graduates was 10–12 students which was not enough for such a publication. Danish legislation of 1871 prescribed streaming at the learned schools. The Icelandic school could only offer one stream, language and history stream, and mathematics teaching became severely reduced. Students who wanted to study mathematical subjects at tertiary level, such as engineering, had thereafter to spend an extra year in Denmark. That situation remained until 1919 (Bjarnadóttir, 2007, pp. 87–90, 108, 110–170).

The Jul. Petersen's secondary school geometry textbook

The Danish geometry textbook, *Lærebog i elementær Plangeometri* [*A Textbook in Elementary Plane Geometry*] (Petersen, 1870) was adopted in 1877 in the Icelandic Learned School for the lowest grade where the average age of students was 14 years but could be in the range 13–16 yrs. It remained on the reading list into the 1970s – in translation from 1943 – with breaks in the 1920s while a textbook by Daníelsson (1920) was in use in the late 1930 and in the 1960s during the influence of the New Math reform movement (*School reports for the Reykjavík School*, 1846–1976).

The content of Jul. Petersen's plane geometry textbook is probably typical of European textbooks in Euclidean geometry. In chapter one, several fundamental concepts are listed and the postulate that one and only one line may be drawn through two points. The structure: *Fundamental concepts and their postulates – Definitions – Theorems with proofs*, is explained. In next two chapters, enough definitions and the parallel postulate are presented in order to be able to present theorems and their proofs. Chapter four is devoted to triangles and chapters five to seven to constructions using circles and triangles. Chapters eight to twelve concern angles and arcs, trapezes and parallelograms, the loci of points, similar triangles, and measuring area, with appropriate definitions, theorems, proofs and constructions. One might interpret Dieudonné's speech so that the first three chapters sufficed as geometry teaching. Against that, one could say that objects for exercising proofs on were then lacking, such as the triangles, Dieudonné's 'artificial playthings'.

Even if this textbook managed to survive in the Reykjavík School for a century, it had notable criticism. In Denmark, the textbook was intended for the so-called *Mellemskole* [middle school] for age 11–14 (Hansen, 2002, p. 40). A reviewer said about Petersen's 1905 edition:

> … one reads between the lines the author's disgust against modern efforts, which … deals with making children's first acquaintance to the mathematics as little abstract as possible by letting figures and measurements of figures pave their way to understanding of the geometry's content … Working with figures … aids the beginner in understanding the content of the theorems, which too often has been completely lost during the effort on 'training the mind'. If the author knew from a daily teaching practice, how often pupils' proofs have not been a chain of reasoning but a sequence of words, he would not have formed his introduction this way … for the middle school it [the textbook] is not suitable.[1] (Trier, 1905)

In Petersen's obituary in 1910 it said:

> … People began to realize that the advantages of these textbooks were more obvious for the teachers than for the pupils … the great conciseness and left-out steps in thinking did not quite suit children. These books were excellent when the whole syllabus was to be recalled shortly before examination, but if the students were to acquire new material one had to demand a wider form for presentation. (Hansen, 2002, p. 51)

A student at the University of Copenhagen, Finnur Jónsson, later philology professor, wrote in 1883, criticizing Reykjavík Learned School and its regulations:

> … the new regulations have [prescribed] … that the [geometry] study is to commence in the first grade; in order to grasp it, more understanding, more independent thought is needed than first-graders master … [I] tutored two [first grade] boys in geometry … and for both of them it was very difficult to understand even the simplest items; but the reason was that they did neither have the education nor the maturity of thought needed to study such things, which is very natural. (Jónsson, 1883, p. 116; underlining KB)

The pupils of the Learned School were sons of farmers, priests and other officials who also made their living from farming, so the majority of the pupils came from farming communities where there were no primary schools. The novices were prepared for school at home and by priests in Latin, Danish and basic arithmetic, and had seldom met geometric concepts. Land was e.g. not measured in square units. Dieudonné's first guiding principle was that a mathematical theory could only be developed axiomatically in a fruitful way when a student had already acquired familiarity with the corresponding material, i.e. with constant appeal to intuition. This is in accordance to Jónsson's remark that the pupils did not possess "the maturity of thought" needed to study deductive geometry as presented in Jul. Petersen's textbook. The young pupils had not acquired familiarity with the corresponding material with the appeal to intuition that Dieudonné recommended.

Daníelsson's high school geometry textbook

The textbook *Um flatarmyndir* [*On plane geometry*] by Ó. Daníelsson (1920), intended for novices at the six-year Reykjavík High School, around age 14, may be interpreted as strictly adhering to the Euclidean tradition. It began by a section on limits to prepare proving the existence of irrational numbers. Next section was a list of definitions. The author admitted in his foreword that his experience was that students were relieved when that section was completed. The third section was on parallel lines, followed by exercises whereof there were five on computing angles, one of them in the hexadecimal system, and all exercises after that through chapter six out of fifteen, were on proving on the basis of the definitions and theorems introduced. Following exercises were alternatively on constructions and proving, and computations by recently proved formulas, such as Heron's formula on area. Eventually, *On plane geometry* was transferred up to the upper level. Geometry was again required for novices in 1937. From that time, Danish textbooks were translated, among them Peterson's *Geometry* in 1943 (*School reports*, 1846–1976).

1 Translations of quotes were made by the author of this paper.

The first two grades were dropped from the high school level in 1946 and after that there was no Euclidean geometry below the age of 16. There was shortage of trained mathematics teachers who had to seek their training abroad, traditionally in Denmark with which all connection was broken during World War II. Only five high school teachers in the whole country had graduate degree in mathematics in 1959. Training of engineers at University of Iceland began in 1940 due to the broken connection to Denmark. Mathematics had not been taught before at the university. Mathematics teachers might be trained in engineering or natural sciences. In the 1960s, the high schools had to cope with up to ten times as many students as at Daníelsson's time, beginning their mathematical training at the age of 16 by studying Petersen's (1943) *Plane Geometry* (Bjarnadóttir, 2007; *School reports*). The experience of the author of this article in 1959–1960 was that the main emphasis laid by a geologist teacher was on construction, the scattered relics of clumsy methods, according to Dieudonné, and the axiomatic structure of the content was scarcely visible.

Other school levels

The first primary school legislation in 1907 contained requirements on knowledge in computations of area and volume of common objects. These requirements were repeated in national curriculum documents issued in 1929 (*Námsskrá fyrir barnaskóla*, 1944) and 1960 (Menntamálaráðuneytið, 1960). By the introduction of the New Math, a draft national curriculum was made, but when it came to geometry, the authors claimed that experience was lacking to build geometry on (*Drög að námsskrá*, 1970). So, indeed, the only geometry taught at compulsory school level before the introduction of the New Math was mensuration, computing area and volume.

The studies at the University of Iceland were tailored after the Technical University in Copenhagen. The mathematical subjects were mathematical analysis and linear algebra and no Euclidean geometry, but they surely built on the high school training.

The New Math in Iceland

For the Nordic countries the Royaumont Seminar was a catalysing event. The Nordic participants agreed upon organising Nordic cooperation on reform of mathematics teaching. A committee, *Nordiska kommittén for modernisering af matematikundervisningen* (The Nordic Committee for Modernizing Mathematics Teaching), abbreviated as NKMM, was established. The committee produced model textbooks which were then translated into the various Nordic languages. Iceland did not have a member in that committee but learned about its activities through personal contacts of high-school and university mathematics teachers G. Arnlaugsson and B. Bjarnason with Svend Bundgaard who was guest speaker at Royaumont. Arnlaugsson and Bjarnason were the leaders of the introduction of New Math in Iceland (Bjarnadóttir, 2015). Their choices of textbooks for mathematics-teacher training witness that they were aware of Dieudonné's recommendations.

Bjarnason chose a Danish textbook: *Matematik 65* (Christiansen and Lichtenberg, 1965), for a special course to train high school mathematics teacher students, the first time a course of its kind was run in the academic year 1966–67 for only three students. Other courses were part of a programme for engineering students. Section V of *Matematik 65* concerned questions from geometry. The authors remarked that around the last turn of the century, David Hilbert had succeeded in composing such a system of axioms that could follow Euclid's thought and solve all Euclid's unsolved problems at the same time (pp. 309–311). They also mentioned Gustave Choquet's system of a so-called transformation geometry with few but strong axioms: 5 undefined concepts (plane, point, line, distance function and order relation) in a set-theoretical presentation; and 4 axioms (10 in total with sub-axioms): axioms of incidence, axioms of order, axioms for affine structure, and a folding (symmetry) axiom (Christiansen and Lichtenberg, 1965, pp. 312–320; Choquet, 1969, pp. 17–75). Choquet was also member of CIEAEM and guest speaker at Royaumont. Dieudonné may have referred to his work in that recently it had become possible to reorganize the Euclidean corpus, putting it on simple and sound foundation.

For the training of teachers at primary level, one of the three teacher students was entrusted to give a course in the New Math style in 1967. Bjarnason and Arnlaugsson chose a Danish textbook on geometry (Anderson Bo, Nielsen and Damgaard Sørensen, 1963) which was built on basic notions such as symmetries, translations, compositions of transformations, etc., as Dieudonné suggested. For the more advance students, an American textbook by Schaaf (1965) was chosen. These three books for training mathematics teachers at different stages were not in use for a long time, however. Not many educators were ready to interpret them, and the educational system was in flux. The training of compulsory school teachers was transferred to tertiary level and reorganized, as was the training of high school teachers. Arnlaugsson and Bjarnason became principals for new modern high schools and did not work further on promoting the New Math (Bjarnadóttir, 2015).

Some products of the NKMM for the primary and lower secondary school levels were translated into Icelandic. Primary level mathematics textbooks, written by Agnete Bundgaard, Svend Bundgaard's sister, and E. Kyttä, were translated year by year, beginning in experimental edition in 1966 (Bundgaard and Kyttä, 1967–1972). It had not reached the geometry in volume 5 when a draft national curriculum (*Drög að námsskrá*, 1970) was published, and people therefore did not know how to present New-Math geometry for primary level. For the lower secondary level the NKMM *Rúmfræði* [*Geometry*] by Bergendal, Hemer and Sander (1970) was translated with foreword by Arnlaugsson. It was based on set theory with e.g. lines defined as sets of points, but no axiomatization. Both products provided teachers with new ideas about geometry for compulsory level while both were in use for less than a decade.

During a six-year period, 1967–1973 various texts with new topics were tried in Iceland in order to replace Petersen's Geometry in high schools, such as NKMM-texts with emphasis on vectors, functions and their derivatives, and *Book T4* in the British SMP series (School Mathematics Project, 1966), which aimed at linking algebra and geometry by vectors and matrices to present transformations and their combinations, with a final chapter on algebraic structure of matrices and examples of groups of transformations. These texts included trigonometry. Influences from Royaumont were thus channelled to Iceland through various routes: from the Nordic countries, the United States and from Britain.

SUMMARY AND CONCLUSIONS

We may understand from Hansen (2002) and Dieudonné's presentation at Royaumont that it was customary in Europe to teach axiomatic Euclidean geometry to young children, even 11–13 year old. Dieudonné's reaction, such as to limit the teaching of mathematics up to the age of 14 to experimental work with algebra and plane geometry, and to make no attempt at axiomatization, must be considered in that context. This was less the case in Iceland. It may though be spotted in Jónsson's (1883) criticism on the teaching of geometry in the Reykjavík School. However, from *School reports for the Reykjavík School* (1846–1976) one may gather that Eucliean geometry was most of the time transferred from the beginners' stage at age 14 up to age 15 or 16. While the Reykjavík School was small, only enrolling 25 students a year, and Daníelsson was the head teacher in 1919–1941, Euclidean training may have been considerable, but less so later when the number of students increased out of proportions to trained mathematics teachers. The axiomatic structure of geometry was thus not much visible in the peripheral Iceland before the New Math reform as only few teachers were capable to interpret Petersen's century old textbook successfully with respect to an axiomatic system in the 1960s.

Within a six years period, 1967–1973, geometry, modernized in the spirit conveyed by Dieudonné and Choquet, had been implemented in the teacher training and at all school levels. Euclid might thus be interpreted to have arrived in Iceland in Choquet's modified versions, at least in the teacher training, at the time of the claim 'Down with Euclid'. The conclusion is therefore that Euclidean geometry was revived in Iceland by the New Math movement.

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