Solving a problem by students with different mathematical abilities: A comparative study using eye-tracking

Miroslawa Sajka, Roman Rosiek

Solving a problem by students with different mathematical abilities: A comparative study using eye-tracking

Miroslawa Sajka and Roman Rosiek

Pedagogical University of Cracow, Faculty of Mathematics, Physics and Technical Science, Cracow, Poland, msajka@up.krakow.pl, rosiek@up.krakow.pl

The main purpose of this study is to compare the problem solving processes of mathematically gifted and underperforming students by utilizing eye-tracking methodologies. We have found the following differentiators between the groups: (a) time of the analysis of the problem's wording, (b) the number and placement of fixations, (c) the number of fixations while analysing the text of the problem. We also prove that total amount of time of solving a problem is not an important differentiating parameter; speed is not a characteristic of mathematically gifted students.

Keywords: Mathematics education, problem solving, eye-tracking, gifted students, comparative study.

INTRODUCTION

The use of eye-tracking technology for the analysis of the learning process has become more and more widespread in recent years. Examining visual attention provides information not only on where the gaze is directed and how, but also constitutes a basis for further analysis and reflections on the ways of solving problems, reasoning, attention, and mental images (Just & Carpenter, 1976; Zelinsky & Sheinberg, 1995; Ball, Lucas, Miles, & Gale, 2003; Yoon & Narayanan, 2004).

Lai and colleagues (2013) reviewed 81 papers dedicated to the use of eye-tracking technology in research related to the analysis of the learning process, describing 113 studies carried out in the period of 2000–2012. The authors distinguished the following themes of studying eye movements and learning: patterns of information processing, effects of instructional design, reexamination of existing theories, individual differences, effects of learning strategies, patterns of decision making, and conceptual development. This research refers to the mainstream of examining patterns of information processing, and strategies, and individual differences during the process of solving mathematical problems.

In the field of didactics of mathematics, studies are still being undertaken (e.g., Andra et al., 2009; Chesney, McNeil, Brockmole, & Kelley, 2013; Merkley & Ansari, 2010; Schneider, Maruyama, Dehaene, & Sigman, 2012; Susac, Bubic, Kaponja, Planinic, & Palmovic, 2014).

Some research results indicate that the measurement of eye movements provides insights into otherwise unavailable cognitive processes and may be used for exploring problem difficulty, student expertise, and metacognitive processes (e.g., Susac et al., 2014). The authors have found that the number of fixations of the eyes represents a reliable and sensitive measure that can give valuable insights into the participants' flow of attention during equation solving. The authors claim that the more efficient participants developed adequate strategies, i.e., "knew where to look." They found a correlation between the number of fixations and the participants' efficiency in equation solving. What is more, they observed that the measures derived from eye movement data were more objective and reliable in comparison to the participants' reports.

Examining the differences between the performance of novices and experts during the process of mathematical problem solving is also the interest of other researchers who use eye-tracking as a research method. They have found quantitative and qualitative differences in the way of looking at a geometry problem (Epelboim & Suppes, 2001) and reading mathematical representations (Andra et al., 2009) by novices and experts in terms of eye movements.

An in-depth knowledge on effective strategies of reading mathematical problems has important didactic consequences. Students need to learn how to read mathematical problems, but this knowledge should be recognised by researchers and teachers beforehand. What is more, it can be useful for the authors of tasks, textbooks, and other didactical materials.

RESEARCH METHODOLOGY AND DESIGN

The aim of the research

The main purpose of this study is to find the differences and similarities in the process of solving the same problem by mathematically gifted students and non-gifted students, by utilizing eye-tracking methodologies.

The objectives of this study refer to the following comparisons in the two test groups of students:

> Aim 1 (A1). Comparison of the total time of solving the problem,
>
> Aim 2 (A2). Comparison of the time of analyzing the wording of the problem,
>
> Aim 3 (A3). Comparison of the number of *fixations* (the stopping of the eyeball at a certain point on the screen) while working on the problem,
>
> Aim 4 (A4). Comparison of the number of *fixations* while analyzing the wording of the problem.

Equipment used

The participants' eye movements of the left eyeball were recorded by the eye tracker SMI Hi-Speed 1250 as well as iViewX™ software. The sampling rate was set to 500 Hz, monocular. The data obtained in the experiment were processed by SMI BeGaze software.

All students attended the experiment in the same physical conditions, in the same air-conditioned room with the same intensity of lighting.

All of the study participants passed the calibrations with an accepted angular accuracy of less than 0.5° and were included in the eye-tracking experiment of solving the science problem. All respondents sat at a distance of 50 cm from a screen the size of 30 cm × 47,5 cm.

The participants' eye movement data, question responses and mouse clicking were recorded by *SMI Experiment Center 3.4* software. In addition to providing answers by using mouse clicks, all respondents were also asked to verbally confirm the selected choices. There was no time limit in regard to the duration of the experiment.

Study participants

The research was carried out in June 2014. The experiment included 52 fifteen-year-old students attending the last grade of junior high school (gymnasium) in Poland, all of which had already taken the final external exam after finishing junior high school.

The sampling of experiment participants was diversified in terms of abilities and mathematical skills, where 18 students were finalists in a regional science competition and therefore recognized as gifted in the field of science. The remaining 34 students attended various lower high schools in Cracow, having mixed abilities and mathematical skills.

Each participant of the experiment was interviewed twice with the use of a questionnaire, both before and after the experiment.

Problem

The problem was provided in the Polish language, as shown in the subsequent figures using the data generated by the *BeGaze* software. Figure 1shows an English translation of the screen.

This problem can be solved by children at the age of 12, but it is more appropriate for lower high school students (13–15 years old). The problem is nonstandard in comparison to typical school tasks. The main difference and difficulty in solving it lies in the application of a methodological approach based on analytical thinking, using reduction. If a student considers how many days pass until half of the pond is overgrown with duckweed, the answer to the problem appears evident.

What is more, the formulation of the problem activates "System 1" according to the psychological *dual*

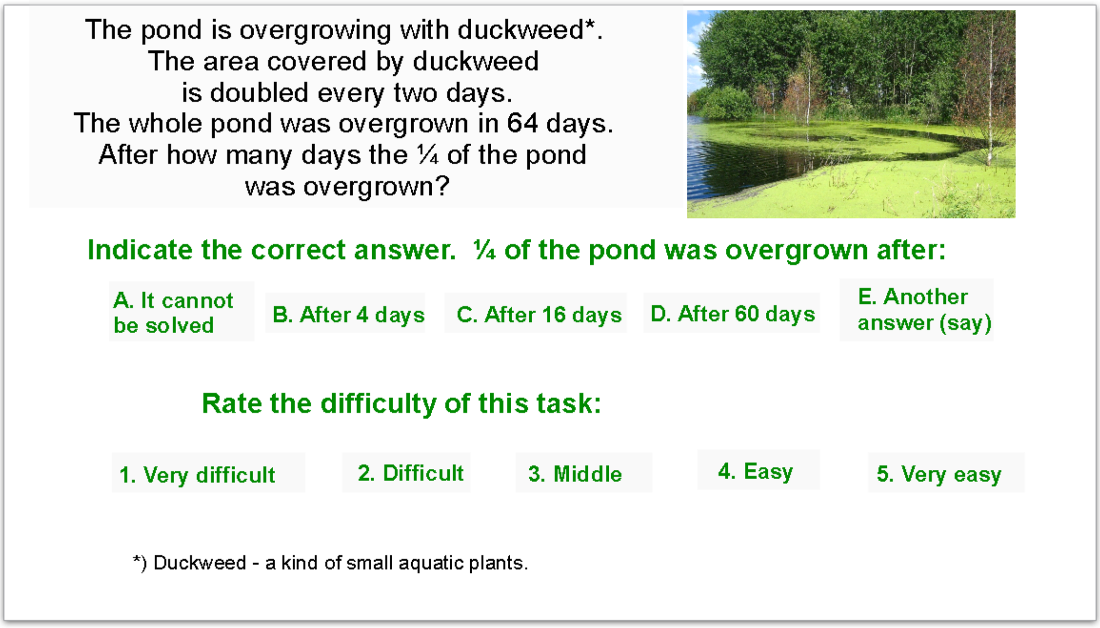


Figure 1: English translation of the Problem (slide 1)

process theory (Kahneman, 2011; Stanovich & West, 2000), and students have to overcome it. Answer C, "after 16 days," is a System 1 trap which forces quick, intuitive judgements with low mental effort which are frequently wrong. The Problem is analogous to the "lily problem" described by Kahneman.

The slide showed as Figure 1 was followed by two more slides with additional questions, the first of which suggested a method of reduction and provided graphical representations of the pond, as well as some hints and questions. The aim of slide 2 was to verify the answer provided to slide 1 and to suggest the proper method. After familiarizing themselves with this slide, all students were asked to check and correct their previous answer and rerate the difficulty of the problem.

The final slide was provided to the students in order to check their understanding of the method required to solve the problem. The students were asked to determine how many days it would take for the pond to be overgrown to 1/8 of the pond. They were also asked to assess six different methods of problem resolution, shown on the slide.

In this article we are focused only on the analysis of the answers to the initial Problem (slide 1).

Methodology

The analysis of all the answers to the questions presented in the three slides allowed us to select the students who correctly understood the whole problem and solved it perfectly. A ranking of the 52 participants was generated, taking into consideration the following criteria:

1. The correctness of answers to the whole problem (all questions on the three slides),
2. The mathematics score achieved on the final external exam after finishing junior high school.

The selection criteria for the comparative study was made by choosing a group of the best and worst performing participants from the ranking. However, we analysed the results of the groups in the context of general results.

Data for the analysis

The following sixteen "Areas of Interest" (AOIs) for obtaining the participants' data were defined within the slide area:

> Text of the problem, Picture – lake, Indicate the answer, Cannot be resolved, After 4 days, After 16 days, After 60 days, Another answer, Assign the difficulty, Very difficult, Difficult, Middle, Easy, Very easy, Explanation, White space.

Our analysis is based on numerical data, including graphical representations, such as: *focus maps*, *scan paths*, *AOI charts*, *key performance indicators*.

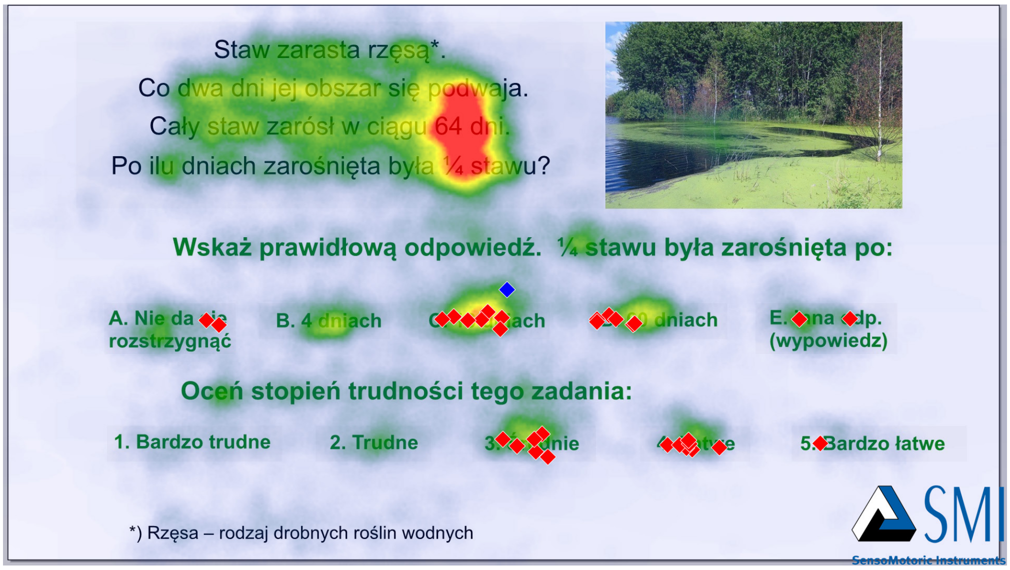


Figure 2: Heat map for all participants

RESEARCH RESULTS AND DISCUSSION

Overview of the general results

Only 5 students correctly solved the whole problem contained on the three slides. The correct answer, D, was provided only by 17 out of 52 students. The incorrect answer, C, "after 16 days," was selected by over half of the participants, i.e. 27 students. Despite the unsatisfactory general results, many participants rated the difficulty of the problem as "middle" (27) and "easy" (19).

The heat map for all participants is shown in Figure 2. Depending on the length of fixation time, the screen shows different colours – from blue (lack of fixations) through green, yellow, orange to red – representing the longest time of fixation.

The highest visual attention while reading the text of the problem was devoted to the most important phrases: "is doubled" („podwaja" at the end of the sentence); "64 days" and "¼ of the pond". The selected options are also visible as red symbols: ♦.

Two groups for comparison: „High Five" and „Low Six"

Only 5 students answered all of the questions from the three slides correctly. We call this group the "High Five". All of them were finalists of regional science competitions and they were recognized as gifted and interested in mathematics.

The second group in this comparative study was made by choosing a group of five the lowest performing participants from the ranking. This group consisted of 6 students and is called the „Low Six" group, as two of the students achieved the same mathematics score at the final external exam after finishing junior high school. The students from this group were the only respondents who did not pass the exam, achieving a result of below 30% of the available points.

A1. Total time of the analysis of the problem

The average total time for solving the problem by all participants of the study was 72 480 ms, the maximum time was 106 084 ms, and the minimum time was 32 890 ms.

The average total time for the "High Five" group was 72 150 ms, whereas the maximum time was 105 307 ms, and the minimum time was only 32 959 ms. Relevant individual differences can be observed (see Figure 3).

For the "Low Six" group, the corresponding values are the following: 57 275 ms; 69 981 ms, and 40 918 ms. The duration of solving the problem by the "Low Six" participants was more homogeneous (see Figure 3) and shorter than the average time of all participants.

A2. Time of the analysis of the problem's wording

In the two compared groups, we observe a crucial difference in the strategy of the analysis of the problem. The proportions between the visual attention devoted to analyzing the wording of the problem and the remaining text on the slide are significantly different.

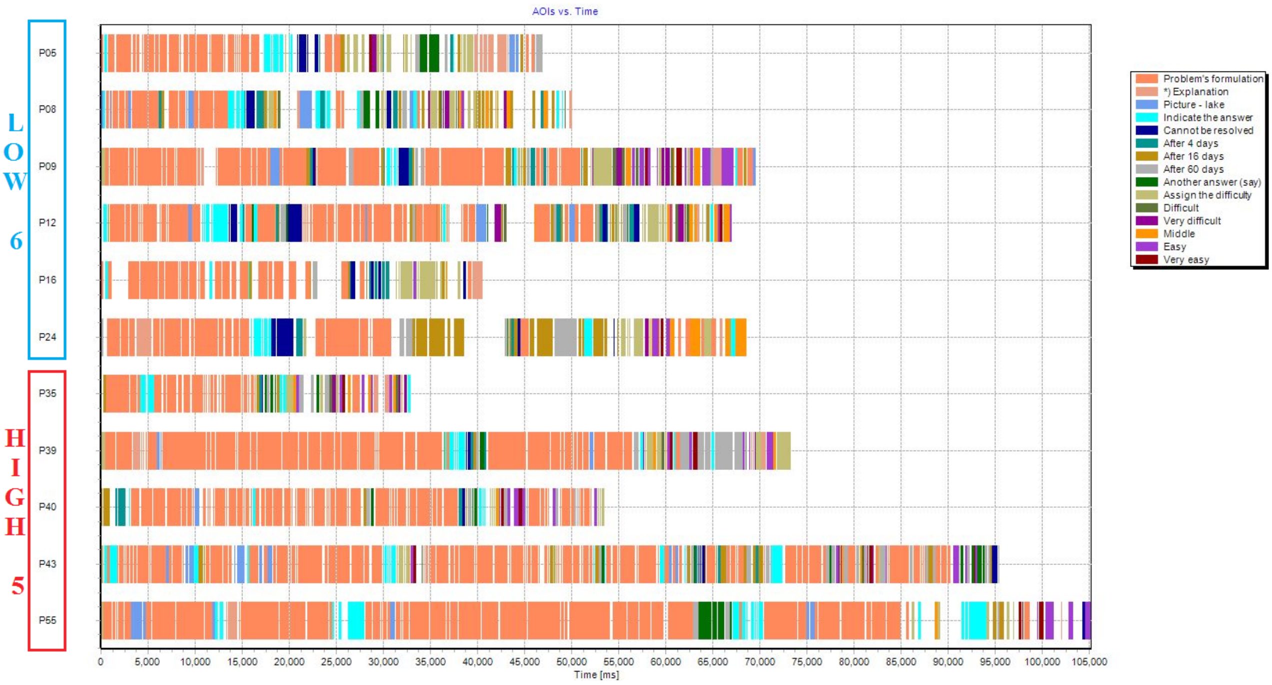


Figure 3: Sequence chart for "High Five" and "Low Six" groups

The average time percentage of the analysis of the wording of the problem in the "High Five" group is 65.9%. On the contrary, for the "Low Six" group it is only 36.5%. The proportions between visual attention devoted to the text of the problem and to the other parts of the screen are reversed for the two groups.

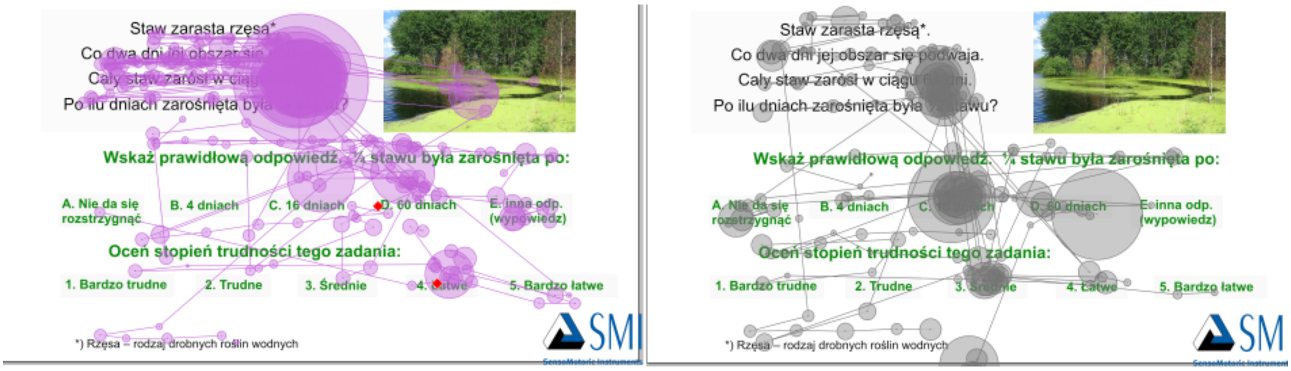
Figure 3 shows time (in milliseconds) spent by the participants' eyes at the defined AOIs respectively in the "High Five" and "Low Six" groups. The colors of the chart segments correspond to the sixteen respective AOIs described above. For example, the text of the problem is visible on the chart in orange. The sequence charts for both groups show an important difference in the way of looking at the screen.

A3. Number and placement of fixations while solving the whole problem

The respondents' visual attention is significantly different for the two compared groups. The heat maps (see Figure 4) show that students from the "High Five" group were concentrated on the wording of the problem and they achieved the maximum number of fixations on the area containing crucial information: "64 days", which had to be the starting point of discovering the correct answer.

On the other hand, the attention of the students from the "Low Six" group was more dispersed. They looked at the middle part of the screen as well – C and 3 an-

Figure 4: Heat maps for "High Five" and "Low Six" groups



P55 from HIGH FIVE group **P24 from LOW SIX group**

Figure 5: Scan paths of the representative participants from the "High Five" and "Low Six" groups

swers. This is a typical subconscious and intuitive way of looking.

The effect of dispersion can be observed individually, analyzing the students' looking paths, called *scan paths*. BeGaze™ software presents a clear graphic interpretation of data, showing the successive *fixations* (using circles) and *saccades,* i.e. paths of displacement between two consecutive *fixations* (using segments). In Figure 5, we present the *scan paths* of chosen two representative members from both groups.

A4. Number of fixations while analyzing the wording of the problem

The number of fixations while analyzing the text of the problem is the following for the "Low Six" and "High Five" groups respectively: Average number of fixations: 75,17 and 152,2; Maximum number of fixations: 110 and 225; Minimum number of fixations: 42 and 50.

SUMMARY

The following conclusions on the basis of the results of our research can be posed:

A1.

a) The average total time of solving the problem by the gifted students was the same as the average total time of all participants in our study. This parameter did not turn out to be a differentiator between gifted students and non-gifted students in the context of our research.

b) The total time of solving a problem by gifted students was very diversified. In this group we observed fast solvers (32 959 ms), average solvers, as well as slow ones (105 307 ms). Speed was not a parameter of mathematically gifted students.

A2.

The time of the analysis of the problem's wording was a differentiator between the group of gifted students and underperforming students in our research. Gifted students dedicated on average 65.9% of the total time of solving the problem to the analysis of the wording of the problem while the underperforming students devoted only 36.5% of their time for this purpose.

A3.

Analyzing the respondents' visual attention by observing the numbers and placement of fixations we observed significant differences between the two groups. The gifted students were concentrated on the text of the problem and they achieved maximum number of fixation at the area of the crucial information, which had to be a starting point to discover the correct answer. They did know where to look.

On the contrary underperforming students looked at various places of the screen, in a seemingly chaotic way. They also looked for longer periods in the middle of the screen, which is a natural way of looking. Their fixations were more often observed occurring at the areas on the slide without the wording of the problem.

A4.

The number of fixations while analyzing the text of the problem was also a differentiator between the groups of gifted students and non-gifted students. Both the average and the maximum number of fixations of the gifted students double those of the underperforming students.

On the basis of our research results, fixations were the visual symptoms of mental effort and motivation to solve the problem. It can be argued that the underperforming students were not sufficiently motivated to solve the problem or to make mental efforts.

The eye-tracking method allowed us to distinguish important differences in the strategy of reading a mathematical problem between gifted and underperforming students. It is be worth examining this topic further if the conclusions seem too broad, verifying them using different problems and a wider sample of students.

REFERENCES

Andrà, Ch. Arzarello, F., Ferrara, F., Holmqvist, K., Lindström, P., Robutti, O., & Sabena, C. (2009). How students read mathematical representations: an eye tracking study, In M. Tzekaki, M. Kaldrimidou, & C. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education, Vol. 2* (pp. 49-56). Thessaloniki, Greece: PME.

Ball, L. J., Lucas, E. J., Miles, J. N. V., & Gale, A. G. (2003). Inspection times and the selection task: What do eye-movements reveal about relevance effects? *Quarterly Journal of Experimental Psychology*, *56A*, 1053–1077.

Chesney, D. L., McNeil, N. M., Brockmole, J. R., & Kelley, K. (2013). An eye for relations: eye-tracking indicates long-term negative effects of operational thinking on understanding of math equivalence. *Memory & Cognition*, *41*, 1079–1095.

Epelboim, J., & Suppes, P. (2001). A model of eye movements and visual working memory during problem solving in geometry. *Vision Research*, *41*, 1561–1574.

Just, M. A., & Carpenter, P. A. (1976). Eye fixations and cognitive processes. *Cognitive Psychology*, *8*, 441–480.

Kahneman, D. (2011). *Thinking, fast and slow* (1st ed.). New York, NY: Farrar, Straus and Giroux.

Lai, M. L., Tsai, M. J., Yang, F. Y., Hsu, C. Y., Liu, T. C., Lee, S. W. Y., et al. (2013). A review of using eye-tracking technology in exploring learning from 2000 to 2012. *Educational Research Review*, *10*, 90–115.

Merkley, R., & Ansari, D. (2010). Using eye tracking to study numerical cognition: The case of the ratio effect. *Experimental Brain Research*, *206*, 455–460.

Schneider, E., Maruyama, M., Dehaene, S., & Sigman, M. (2012). Eye gaze reveals a fast, parallel extraction of the syntax of arithmetic formulas. *Cognition*, *125*, 475–490.

Stanovich, K. E., & West, R. F. (2000). Individual difference in reasoning: implications for the rationality debate? *Behavioural and Brain Sciences, 23*, 645–726.

Susac, A., Bubic, A., Kaponja, J., Planinic, M., & Palmovic, M. (2014). Eye movements reveal students' strategies in simple equation solving. *International Journal of Science and Mathematics Education*, *12*, 555–577.

Yoon, D., & Narayanan, N. H. (2004). Mental imagery in problem solving: An eye tracking study. In *Proceedings of the Eye Tracking Research and Applications Symposium 2004* (pp. 77–83). New York, NY: ACM Press.

Zelinsky, G., & Sheinberg, D. (1995). Why some search tasks take longer than others: Using eye movements to redefine reaction times. In J. M. Findlay, R. Walker, & R. W. Kentridge (Eds.), *Eye movement research: Mechanisms, processes and applications* (pp. 325–336). Amsterdam, The Netherlands: Elsevier.